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Convergence numerical results related to the vibration of the hydro-elastic system consisting of an anisotropic plate, compressible viscous fluid and rigid wall

Ilgar Sh. Jabbarov · Tarana V. Huseynova

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Abstract. *The forced vibration problems of the hydro-elastic systems consisting of the plate, compressible viscous fluid and rigid wall are solved analytical-numerical methods, according to which the final results are obtained by employing PC programs with the use of a certain algorithm for calculation. Namely, under this calculation procedure, it is necessary to establish convergence of the used calculation algorithm and PC programs. In this sense in the present paper it is made the attempt to examine the convergence of the mentioned calculation algorithm used in the case wherein the foregoing hydro-elastic system the plate material is anisotropic (orthotropic) one. Under this examination the motion of the plate is described with the exact equations of elastodynamics for anisotropic bodies, however, the flow of the fluid is described with the linearized Navier-Stokes equations. For the solution to the problem the Fourier integral transformation with respect to the coordinate which is on the coordinate axis which is directed along the plate length, is employed. Numerical results on the convergence of the calculation of the inverse Fourier transform the expressions of which are obtained in the solution procedure of the corresponding boundary value problems, are presented and discussed.*

Keywords. Convergence numerical results · hydro-elastic system · anisotropic material · compressible viscous fluid · Fourier transform.

Mathematics Subject Classification (2010): 74H55

1 Introduction.

The theoretical investigations of the problems related to the dynamics of hydro-elastic systems involves solutions to various type system of partial differential equations of continuum mechanics related to the motion of the deformable bodies which are in contact with various type fluids (i.e. viscous, inviscid, compressible incompressible and etc.) the flow of which is also described with the system of Navier-Stokes partial differential equations. Note that under solution procedure to these equations for obtaining final numerical results, according to which it is made the corresponding engineering recommendation, is based on the numerical calculation algorithm and PC programs. Consequently, the authenticity of the algorithm used under obtaining these result requires to establish of the convergence domain this algorithm. In connection with this, in the present paper it is made the attempt to

Ilgar Sh. Jabbarov, Tarana V. Huseynova
Ganja State University, Heyder Eliyev Ave., 425, Genje, Azerbaijan
E-mail: terane.huseynova.77@mail.ru

consider and analyze the convergence of the algorithm used under obtaining numerical results related to the forced vibration of the hydro-elastic system consisting of the anisotropic (orthotropic) plate, compressible viscous fluid and rigid wall. Note that the similar consideration and analyses for the case where the material of the mentioned hydro-elastic system is an isotropic one, was made in the works by Akbarov and Ismailov [1 - 5] and others the review of which is made in the paper by Akbarov [1] and in the monograph by Akbarov [6].

2 Mathematical formulation of the problem

Assume that the hydro-elastic system consists of the orthotropic plate, compressible viscous fluid and rigid wall as shown in Fig. 2 and this hydro-elastic system occupies the region $\{-\infty < x_1 < +\infty;\ -h - h_d < x_2 < 0;\ -\infty < x_3 < +\infty\}$ in the Cartesian coordinate system $Ox_1x_2x_3$ which is associated with the upper face plane of the plate. The part $\{-\infty < x_1 < +\infty;\ -h < x_2 < 0;\ -\infty < x_3 < +\infty\}$of this region is occupied by the plate and the remain part $\{-\infty < x_1 < +\infty;\ -h - h_d < x_2 < -h;\ -\infty < x_3 < +\infty\}$of that is filled with the fluid and the plane $x_2 = -h - h_d$is taken as the rigid wall.

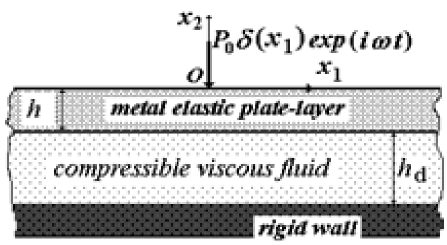

Fig. 1. The sketch of the hydro-elastic system consisting of orthotropic elastic plate, compressible viscous fluid and rigid wall

It is assumed that the coordinate axis Ox_3 is perpendicular to the plane of the Fig. 1 and therefore this axis is not shown in this figure. At the same time, we suppose that along to this line, i.e. on the line $-\infty < x_3 < +\infty$, $x_1 = 0$ and $x_2 = 0$ the uniformly distributed time-harmonic forces with intensity P_0 act and consequently we can reduce this three-dimensional problem to the corresponding two dimensional one, according to which, all the sought values will depend on the space coordinates x_1 and x_2 but not on the coordinate x_3. Finally, we assume that the material of the plate is the orthotropic one the elastic symmetry axes of which coincide with the coordinate axes Ox_1, Ox_2 and Ox_3, and this assumption is the main one, according to which, the investigations on the convergens of the numerical calculation algorithm differs from corresponding ones carried out in the papers by Akbarov and Ismailov [2, 3, 4, 5].

Thus, within the framework of the foregoing assumptions, we write the governing field equations and relations.

The equations of motion for the plate:

$$\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} = \rho\frac{\partial^2 u_1}{\partial t^2}, \frac{\partial\sigma_{12}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} = \rho\frac{\partial^2 u_2}{\partial t^2}. \tag{2.1}$$

The elasticity relation

$$\sigma_{11} = a_{11}\varepsilon_{11} + a_{12}\varepsilon_{22}, \sigma_{22} = a_{12}\varepsilon_{11} + a_{22}\varepsilon_{22}, \sigma_{12} = 2G_{12}\varepsilon_{12}, \tag{2.2}$$

where

$$a_{11}=\frac{A_{22}}{A_{11}A_{22}-A_{12}^2},a_{12}=\frac{A_{12}}{A_{11}A_{22}-A_{12}^2},a_{22}=\frac{A_{11}}{A_{11}A_{22}-A_{12}^2},$$

$$A_{11}=\frac{1-\nu_{13}\nu_{31}}{E_1},A_{12}=-\frac{\nu_{12}+\nu_{13}\nu_{32}}{E_1},A_{22}=\frac{1-\nu_{23}\nu_{32}}{E_1},$$

$$\nu_{13}E_1=\nu_{31}E_3,\nu_{21}E_2=\nu_{12}E_1,\nu_{32}E_3=\nu_{23}E_2. \tag{2.3}$$

In (2.3) the following notation is used: E_1, E_2 and E_3 are the modulus of elasticity of the plate material in the directions of the Ox_1, Ox_2 and Ox_3 axes, respectively, G_{12} is the shear modulus in the Ox_1x_2 plane, ν_{ij} $(i;j=1,2,3)$ is the Poisson's coefficient characterizing the shorting (the lengthening) of the material fibers in the Ox_i axis direction under stretching (under compressing) in the Ox_j axis direction; σ_{ij} and ε_{ij} $(ij=11;22;12)$ are the components of the stress and strain tensor, respectively; u_1 and u_2 are the components of the displacement vector in the Ox_1 and Ox_2 axes directions, respectively .

The strain-displacement relations:

$$\varepsilon_{11}=\frac{\partial u_1}{\partial x_1},\varepsilon_{22}=\frac{\partial u_2}{\partial x_2},\varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right). \tag{2.4}$$

The equations and relations which given in (2.1) – (2.4) are the closed system of the field equations related to the motion of the orthotropic plate.

Also, consider the mathematical modelling of the fluid flow which, according to the monograph by Guz [9], can be described with the linearized Navier-Stokes equations given below.

$$\rho_0^{(1)}\frac{\partial v_i}{\partial t}-\mu^{(1)}\frac{\partial v_i}{\partial x_j\partial x_j}+\frac{\partial p^{(1)}}{\partial x_i}-(\lambda^{(1)}+\mu^{(1)})\frac{\partial^2 v_j}{\partial x_j\partial x_i}=0,\frac{\partial\rho^{(1)}}{\partial t}+\rho_0^{(1)}\frac{\partial v_j}{\partial x_j}=0,$$

$$T_{ij}=\left(-p^{(1)}+\lambda^{(1)}\theta\right)\delta_{ij}+2\mu^{(1)}e_{ij},\theta=\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2},$$

$$e_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right).a_0^2=\frac{\partial p^{(1)}}{\partial\rho^{(1)}},i:j=1,2. \tag{2.5}$$

where $\rho_0^{(2.1)}$is the fluid density before perturbation, $\rho^{(2.1)}$ is the perturbation of the fluid density, $p^{(2.1)}$ is the perturbation of the hydrostatic pressure, v_1 and v_2 are the components of the fluid flow velocity vector in the directions of the Ox_1 and Ox_2axes, respectively, T_{ij} and e_{ij} are the components of the stress and strain velocity tensors in the fluid, a_0 is the sound velocity in the fluid, $\lambda^{(2.1)}$ and $\mu^{(2.1)}$ are the coefficients of the fluid viscosity. In (2.5) it is made summation with respect to the by repeating indices.

In the monograph the monograph by [9] it was established that the solution to the equations in (2.5) can be represented as follows

$$v_1=\frac{\partial\varphi}{\partial x_1}+\frac{\partial\psi}{\partial x_2},v_2=\frac{\partial\varphi}{\partial x_2}-\frac{\partial\psi}{\partial x_1},p^{(1)}=\rho_0^{(1)}\left(\frac{\lambda^{(1)}+2\mu^{(1)}}{\rho_0^{(1)}}\varDelta-\frac{\partial}{\partial t}\right)\varphi, \tag{2.6}$$

where the potentials φ and ψ satisfy the following equations.

$$\left[\left(1+\frac{\lambda^{(1)}+2\mu^{(1)}}{a_0^2\rho_0^{(1)}}\frac{\partial}{\partial t}\right)\varDelta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t}\right]\varphi=0,$$

$$\left(\nu^{(1)}\varDelta-\frac{\partial}{\partial t}\right)\psi=0,\varDelta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}. \tag{2.7}$$

Here $\nu^{2.1}$is the kinematic viscosity, i.e. $\nu^{2.1} = \mu^{2.1}\mu^{2.1}\rho_0^{2.1}\rho_0^{2.1}$ and $\lambda^{2.1} = -\frac{2}{3}\mu^{2.1}$ which follows from the assumption $p^{2.1} = -(T_{11} + T_{22} + T_{33})/3$.
We write also the following boundary, compatibility and impermeability conditions.

The boundary conditions on the upper face plane of the plate:

$$\sigma_{21}|_{x_2=0} = 0,\ \sigma_{22}|_{x_2=0} = -P_0 e^{i\omega t}. \tag{2.8}$$

The compatibility conditions on the interface plane between the fluid and plate:

$$\left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h} = v_1|_{x_2=-h},\ \left.\frac{\partial u_2}{\partial t}\right|_{x_2=-h} = v_2|_{x_2=-h},$$

$$\sigma_{21}|_{x_2=-h} = T_{21}|_{x_2=-h},\ \sigma_{22}|_{x_2=-h} = T_{22}|_{x_2=-h}. \tag{2.9}$$

The impermeability conditions on the rigid wall:

$$v_1|_{x_2=-h-h_{\mathrm{d}}} = 0,\ v_2|_{x_2=-h-h_{\mathrm{d}}} = 0. \tag{2.10}$$

This completes the mathematic formulation of the problem.

3 Employing the Fourier transform to solution of the field equations and determination of the expressions of these transforms

Taking the boundary conditions in (2.8) into consideration the sought values are presented as $g(x_1, x_2, t) = \bar{g}(x_1, x_2)e^{i\omega t}$, according to which, the derivatives $\partial(\cdot)/\partial t$ and $\partial^2(\cdot)/\partial t^2$ in the foregoing equations are replaced with the with $i\omega(\bar{\cdot})$ and $-\omega^2(\bar{\cdot})$, respectively. After this replacing, we obtain corresponding equations and relations for the amplitudes of the sought values for determination of which the Fourier transform $f_F(s, x_2) = \int\limits_{-\infty}^{+\infty} f(x_1, x_2)e^{-isx_1}dx_1$ is employed with respect to the coordinate x_1 (see, for instance, the monograph by Sneddon [10]) . Thus, taking the problem symmetry with respect to the $x_1 = 0$ plane the sought quantities can be presented as follows.

$$u_1 = \frac{1}{\pi}\int_0^\infty u_{1F}(s, x_2)\sin(sx_1)ds,\ u_2 = \frac{1}{\pi}\int_0^\infty u_{2F}(s, x_2)\cos(sx_1)ds,$$

$$\sigma_{11} = \frac{1}{\pi}\int_0^\infty \sigma_{11F}(s, y_2)\cos(sy_1)ds,$$

$$\sigma_{22} = \frac{1}{\pi}\int_0^\infty \sigma_{22F}(s, x_2)\cos(sx_1)ds, \sigma_{12} = \frac{1}{\pi}\int_0^\infty \sigma_{12F}(s, x_2)\sin(sx_1)ds,$$

$$\varphi = \frac{1}{\pi}\int_0^\infty \varphi_F(s, x_2)\cos(sx_1)ds, \psi = \frac{1}{\pi}\int_0^\infty \psi_F(s, x_2)\sin(sx_1)ds,$$

$$v_1 = \frac{1}{\pi}\int_0^\infty v_{1F}(s, x_2)\sin(sx_1)ds, v_2 = \frac{1}{\pi}\int_0^\infty v_{2F}(s, x_2)\cos(sx_1)ds,$$

$$T_{11} = \frac{1}{\pi}\int_0^\infty T_{11F}(s, x_2)\cos(sx_1)ds,$$

$$T_{22} = \frac{1}{\pi}\int_0^\infty T_{22F}(s, x_2)\cos(sx_1)ds, T_{12} = \frac{1}{\pi}\int_0^\infty T_{12F}(s, x_2)\sin(sx_1)ds. \tag{3.1}$$

Substituting the expressions in (3.1) into the field equations detailed above we obtain the equations and relations satisfied by the Fourier transforms of the unknown functions. Doing some mathematical manipulations the following equations are obtained for the Fourier transforms related to the plate, i.e. with respect to the u_{1F} and u_{2F}.

$$Au_{1F} - B\frac{du_{2F}}{dx_2} + \frac{d^2u_{1F}}{dx_2^2} = 0, Du_{2F} + B\frac{du_{1F}}{dx_2} + G\frac{d^2u_{2F}}{dx_2^2} = 0, \tag{3.2}$$

where

$$A = X^2 - s^2a_{11}/G_{12}, B = sa_{12}/G_{12} + s, D = X^2 - s^2, G = a_{22}/G_{12},$$
$$X^2 = \omega^2h^2/c_2^2, c_2 = \sqrt{G_{12}/\rho}. \tag{3.3}$$

As in the paper by Akbarov and Ismailov [3], introducing the notation

$$A_0 = \frac{AG + B^2 + D}{G}, B_0 = \frac{BD}{G}, k_1 = \sqrt{-\frac{A_0}{2} + \sqrt{\frac{A_0^2}{4} - B_0}},$$
$$k_2 = \sqrt{-\frac{A_0}{2} - \sqrt{\frac{A_0^2}{4} - B_0}}, \tag{3.4}$$

we can write the solution of the equation (3.2) as follows:

$$u_{2F} = Z_1e^{k_1x_2} + Z_2e^{-k_1x_2} + Z_3e^{k_2x_2} + Z_4e^{-k_2x_2},$$
$$u_{1F} = Z_1a_1e^{k_1x_2} + Z_2a_2e^{-k_1x_2} + Z_3a_3e^{k_2x_2} + Z_4a_4e^{-k_2x_2}, \tag{3.5}$$

where

$$a_1 = \frac{-D - Gk_1^2}{Bk_1^2}, a_2 = -a_1, a_3 = \frac{-D - Gk_2^2}{Bk_2^2}, a_4 = -a_3. \tag{3.6}$$

According to equations (3.5) and (2.2) – (2.4), we obtain also the following expressions for the Fourier transforms of the quantities related to the stresses related to the plate.

$$\frac{\sigma_{21F}}{G_{12}} = Z_1\left(k_1a_1 - s\right)e^{k_1x_2} + Z_2\left(-k_1a_2 - s\right)e^{-k_1x_2} +$$
$$+Z_3\left(k_2a_3 - s\right)e^{k_2x_2} + Z_4\left(-k_2a_3 - s\right)e^{-k_2x_2},$$
$$\frac{\sigma_{22F}}{G_{12}} = Z_1\left(s\frac{a_{12}}{G_{12}}a_1 + k_1\frac{a_{22}}{G_{12}}\right)e^{k_1x_2} + Z_2\left(s\frac{a_{12}}{G_{12}}a_2 - k_1\frac{a_{22}}{G_{12}}\right)e^{-k_1x_2} +$$
$$Z_3\left(s\frac{a_{12}}{G_{12}}a_3 + k_2\frac{a_{22}}{G_{12}}\right)e^{k_2x_2} + Z_2\left(s\frac{a_{12}}{G_{12}}a_4 - k_2\frac{a_{22}}{G_{12}}\right)e^{-k_2x_2}. \tag{3.7}$$

Using the approach developed in the papers by Akbarov and Ismailov [3], Akbarov and Huseynova [7] and others listed therein, for determination of the expressions of the Fourier transforms for quantities related to the fluid motion, we use the representations $\varphi_F = \omega h^2\tilde{\varphi}_F$ and $\psi_F = \omega h^2\tilde{\psi}_F$, after which it is obtained the following equations for the functions $\tilde{\varphi}_F$and $\tilde{\psi}_F$.

$$\frac{d^2\tilde{\varphi}_F}{dx_2^2} + \left(\frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)} - s^2\right)\tilde{\varphi}_F = 0, \frac{d^2\tilde{\psi}_F}{dx_2^2} - \left(s^2 + iN_w^2\right)\tilde{\psi}_F = 0, \tag{3.8}$$

where $\Omega_{\text{-}1} = \omega h/a_0$, $N_w^2 = \omega\, h^2/\nu^{(2.1)}$.

Thus, the solution to the equations in (3.8) we determine as follows.

$$\tilde{\varphi}_F = Z_5 e^{\delta_1 x_2} + Z_7 e^{-\delta_1 x_2}, \tilde{\psi}_F = Z_6 e^{\gamma_1 x_2} + Z_8 e^{-\gamma_1 x_2}, \tag{3.9}$$

where $\delta_1 = \sqrt{s^2 - \Omega_1^2/(1 + i4\Omega_1^2/(3N_w^2))}$ and $\gamma_1 = \sqrt{s^2 + iN_w^2}$.
Using the expressions (3.9), (2.6) and (2.5) we obtain:

$$v_{1F} = \omega \_\_h \left[-Z_5 s e^{\delta_1 x_2} - Z_7 s e^{-\delta_1 x_2} + Z_6 e^{\gamma_1 x_2} + Z_8 e^{-\gamma_1 x_2} \right],$$

$$v_{2F} = \omega \_\_h \left[Z_5 \delta_1 e^{\delta_1 x_2} - Z_7 \delta_1 e^{-\delta_1 x_2} - Z_6 s e^{\gamma_1 x_2} - Z_8 s e^{-\gamma_1 x_2} \right],$$

$$T_{22F} = \mu^{(1)} \omega \left[Z_5 \left(\frac{4}{3}\delta_1^2 + \frac{2}{3}s^2 - R_0 \right) e^{\delta_1 x_2} + Z_7 \left(\frac{4}{3}\delta_1^2 + \frac{2}{3}s^2 - R_0 \right) e^{-\delta_1 x_2} + \right.$$

$$\left. Z_6 \left(-s\gamma_1 - \frac{2}{3}s\gamma_1 \right) e^{\gamma_1 x_2} + Z_8 \left(s\gamma_1 + \frac{2}{3}s\gamma_1 \right) e^{-\gamma_1 x_2} \right],$$

$$T_{21F} = -\mu^{(1)} \omega \left[2s\delta_1 Z_5 e^{\delta_1 x_2} - 2s\delta_1 Z_7 e^{-\delta_1 x_2} + (s^2 + \gamma_1^2) Z_6 e^{\gamma_1 x_2} + (s^2 + \gamma_1^2) Z_8 e^{-\gamma_1 x_2} \right],$$

$$p_F^{(1)} = \mu^{(1)} \omega R_0 \left(Z_5 e^{\delta_1 x_2} + Z_7 e^{-\delta_1 x_2} \right), \tag{3.10}$$

where $R_0 = -4\Omega_1^2/(3(1 + i4\Omega_1^2/(3N_w^2))) - iN_w^2$.

Substituting the expressions (3.5), (3.7) and (3.10) into the conditions (2.8) – (2.10) we obtain the following system of algebraic equations for the unknown constants Z_1, Z_2,…,Z_8 which enter the expressions of the Fourier transforms of the sought values.

$$(\sigma_{21F}/G_{12})|_{x_2=0} = Z_1\alpha_{11} + Z_2\alpha_{12} + Z_3\alpha_{13} + Z_4\alpha_{14} = 0,$$

$$(\sigma_{22F}/G_{12})|_{x_2=0} = Z_1\alpha_{21} + Z_2\alpha_{22} + Z_3\alpha_{23} + Z_4\alpha_{24} = -P_0/G_{12},$$

$$\left.\frac{\partial u_{1F}}{\partial t}\right|_{x_2=-h} - v_{1F}|_{x_2=-h} = i\omega(Z_1\alpha_{31} + Z_2\alpha_{32} + Z_3\alpha_{33} + Z_4\alpha_{34}) -$$

$$\omega h(Z_5\alpha_{35} + Z_6\alpha_{36} + Z_7\alpha_{37} + Z_8\alpha_{38}) = 0,$$

$$\left.\frac{\partial u_{2F}}{\partial t}\right|_{x_2=-h} - v_{2F}|_{x_2=-h} = i\omega(Z_1\alpha_{41} + Z_2\alpha_{42} + Z_3\alpha_{43} + Z_4\alpha_{44}) -$$

$$\omega h(Z_5\alpha_{45} + Z_6\alpha_{46} + Z_7\alpha_{47} + Z_8\alpha_{48}) = 0,$$

$$(\sigma_{21}/G_{12})|_{x_2=-h} - (T_{21}/G_{12})|_{x_2=-h} = Z_1\alpha_{51} + Z_2\alpha_{52} + Z_3\alpha_{53} + Z_4\alpha_{54} -$$

$$M(Z_5\alpha_{55} + Z_6\alpha_{56} + Z_7\alpha_{57} + Z_8\alpha_{58}) = 0,$$

$$(\sigma_{22}/G_{12})|_{x_2=-h} - (T_{22}/G_{12})|_{x_2=-h} = Z_1\alpha_{61} + Z_2\alpha_{62} + Z_3\alpha_{63} + Z_4\alpha_{64} -$$

$$M(Z_5\alpha_{65} + Z_6\alpha_{66} + Z_7\alpha_{67} + Z_8\alpha_{68}) = 0,$$

$$v_{1F}|_{x_2=-h-h_{\mathrm{d}}} = \omega h(Z_5\alpha_{75} + Z_6\alpha_{76} + Z_7\alpha_{77} + Z_8\alpha_{78}) = 0,$$

$$v_{2F}|_{x_2=-h-h_{\mathrm{d}}} = \omega h(Z_5\alpha_{85} + Z_6\alpha_{86} + Z_7\alpha_{87} + Z_8\alpha_{88}) = 0, \tag{3.11}$$

where $M = \mu^1\omega/G_{12}$.
The explicite expressions of the coefficients $\alpha_{nm}(n; m = 1, 2, ..., 8)$ in the equations in (3.11) can be easily determined from the Eqs. (3.5), (3.7) and (3.10), and the unknowns $Z_1, Z_2, ..., Z_8$ in can be determined via the formula $Z_k = \det \left\| \beta_{nm}^k \right\| / \det \left\| \alpha_{nm} \right\|$, where the matrix (β_{nm}^k) is obtained from the matrix (α_{nm}) by replacing the $k - th$ column of the

latter with the column $(0, -P_0/G_{12}, 0, 0, 0, 0, 0, 0)^T$. Consequently, according to this statement, integrated expressions in (3.1) may have singular points with respect to the Fourier transform parameter s, according to which, $\det \|\alpha_{nm}\| = 0$, $n; m = 1, 2, ..., 8$. Note that the equation $\det \|\alpha_{nm}\| = 0$ coincides with the dispersion equation of the waves with the velocity ω/s propagated in the hydro-elastic system under consideration in the Ox_1 axis direction. As a result of the viscosity of the fluid this dispersion equation has not real roots and therefore the mentioned singular points do not appear in the integrated function in the integrals in (3.1). Thus, using the representation $g(x_1, x_2, t) = \bar{g}(x_1, x_2)e^{i\omega t}$, the sought values are determined through the following two types of relations:

$$\{\sigma_{22}, \sigma_{11}, u_2, T_{22}, T_{11}, v_2\} =$$

$$= \frac{1}{\pi} Re \left\{ e^{i\omega t} \int_0^\infty [\sigma_{22F}, \sigma_{11F}, u_{2F}, T_{22F}, T_{11F}, v_{2F}] \cos(sx_1) ds \right\}$$

$$\{\sigma_{21}, \sigma_{12}, u_1, T_{21}, v_1\} =$$

$$= \frac{1}{\pi} Re \left\{ e^{i\omega t} \int_0^\infty [\sigma_{21}, \sigma_{12F}, u_{1F}, T_{21F}, v_{1F}] \sin(sx_1) ds \right\}. \quad (3.12)$$

Note that under calculation procedures, the improper integrals $\int\limits_0^\infty f(s)\cos(sx_1)ds$ and $\int\limits_0^\infty f(s)\sin((s)x_1)ds_1$ in (3.12) are replaced by the corresponding definite integrals $\int\limits_0^{S_1^*} f(s)\cos(sx_1)ds$ and $\int\limits_0^{S_1^*} f(s)\sin((s)x_1)ds$, respectively. The values of S_1^* are determined from the convergence requirement of the numerical results.

In this way, we determine completely the Fourier transforms of the sought values, after which it is require to calculate the integrals $\int\limits_0^{S_1^*} f(s)\cos(sx_1)ds$ and $\int\limits_0^{S_1^*} f(s)\sin((s)x_1)ds$ for which it is required to develop the converging algorithm and PC programs for the problem under consideration. Namely, this development is considered in the following section.

4 The algorithm for calculation of the integrals in (1) and its convergence

In the calculation procedure the integration interval $[0, S_1^*]$ is divided into N number subintervals $[0, S_{1,1}^*]$, $[S_{1,1}^*, S_{1,2}^*]$, ..., $[S_{1,N-1}^*, S_{1,N}^*]$, where $\bigcup\limits_{k=1}^{N} [S_{1,k-1}^*, S_{1,k}^*] = [0, S_1^*]$ and $S_{1,0}^* = 0$, $S_{1,N}^* = S_1^*$. After this dividing in each integration interval $[S_{1,k-1}^*, S_{1,k}^*]$ the integrals $\int\limits_{S_{1,k-1}^*}^{S_{1,k}^*} f(s)\cos(sx_1)ds$ and $\int\limits_{S_{1,k-1}^*}^{S_{1,k}^*} f(s)\sin(sx_1)ds$, using the transform $s = \tau_k(S_{1,k}^* - S_{1,k-1}^*)/2 + (S_{1,k}^* + S_{1,k-1}^*)/2$, are reduced to the integrals $\int\limits_{-1}^{+1} f(s(\tau_k))\cos(s(\tau_k)x_1)d\tau_k$ and $\int\limits_{-1}^{+1} f(s(\tau_k))\sin(s(\tau_k)x_1)d\tau_k$. For calculation of the latter ones it is used the Gauss integration technique with ten sample points and for calculation the values of the integrated function $f(s(\tau_k))$, which represent one of the functions σ_{22F}, σ_{11F}, u_{2F}, T_{22F}, T_{11F}, v_{2F}, σ_{21}, σ_{12F}, u_{1F},T_{21F} and v_{1F}, requires to know the values of the unkown constants $Z_1, Z_2, ..., Z_8$ at these sample points. The values of these constants are calculated from the equations (3.11) and this calculation procedure is made authomatically through the PC program in MATLAB the user manual of which is detailed in many references (see, for instance, the book by Palm [11]). In all these calculation procedures the number N and the value of S_1^* are selected in advance and final final values of those (denote they by N_* and S_{1*}^*) are determined from the convergence requirement of the numerical resluts. The aim of the present numerical investigation is to determine how the anisotropy properties of the plate material acts on the values of the N_* and S_{1*}^*. For this determination we consider the calculation the values of stress-pressure T_{22} at point $x_1/h = 0$ acting on the interface plane between the plate and fluid.

For obtaining concrete numerical results we assume that the material of the fluid is Glycerin with viscosity coefficient $\mu^{2.1} = 1,393kg/(m \cdot s)$, density $\rho_0^{2.1} = 1260kg/m^3$ and sound speed $a_0 = 1927m/s$ (Guz (2009)) and introduce the notation

$$\rho/\rho_0^{(1)} = k_1, c_2/a_0 = k_2, G_{12} = (c_2)^2\rho, \tag{4.1}$$

through which we determine the density and shear modulus of elasticity in the Ox_1x_2 plane of the plate material. Consequently, if we know the density of the fluid, then giving the values for the k_1 we determine the density of the plate material, as well as if we know the sound speed in the fluid, then giving the values for the k_2 we determine the values for the shear modulus G_{12}.

In other words selecting the values for the constants k_1 and $k_2$we determine the density and shear modulus of the plate material through the density and sound speed of the fluid material, and an increase in the values of the k_1(of the k_2) means an increase in the values of the density (of the shear modulus) of the plate material and under fixed value of the fluid density (under fixed sound speed in the fluid).

Moreover, we introduce the following ratios which characterize the anisotropy of the plate material.

$$E_1/G_{12}, E_1/E_2, E_2/E_3, E_1/E_3, \tag{4.2}$$

$E_3 = E_3$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$, and the values of the ν_{21}, ν_{31} and ν_{32} we determine through the relations $\nu_{21} = \nu_{12}E_1/E_2$, $\nu_{31} = \nu_{13}E_1/E_3$, $\nu_{32} = \nu_{23}E_2/E_3$.

In this way, it remains two ratios E_1/G_{12} and E_1/E_2 through which we characterize the influence of the anisotropy of the plate material on the convergence of the numerical results, i.e. on the values of the N_* and S_{1*}^*.

In the present investigations we will consider the numerical results illustrating the influence of the k_1, k_2, and E_1/G_{12} on the frequency response of the interface dimensionless stress $T_{22}h/P_0$. As the influence of the ratio E_1/E_2on the convergence of the numerical results is insignificant, therefore this influence is not considered here.

First, we assume that $S_1^* = 5.0$, $\omega\tau = 0$, $E_1/E_2 = 1.5$, $h = 0.001\,m$ and $h_d/h = 2$, and consider the convergence of the numerical results with respect to the number N and for this purpose consider the graphs of the frequency response of the $T_{22}h/P_0$ calculated in the cases where $E_1/G_{12} = 3$, 10 and 50 under $k_1 = k_2 = 1$and are illustrated in Fig. 2a, 2b and 2c, respectively.

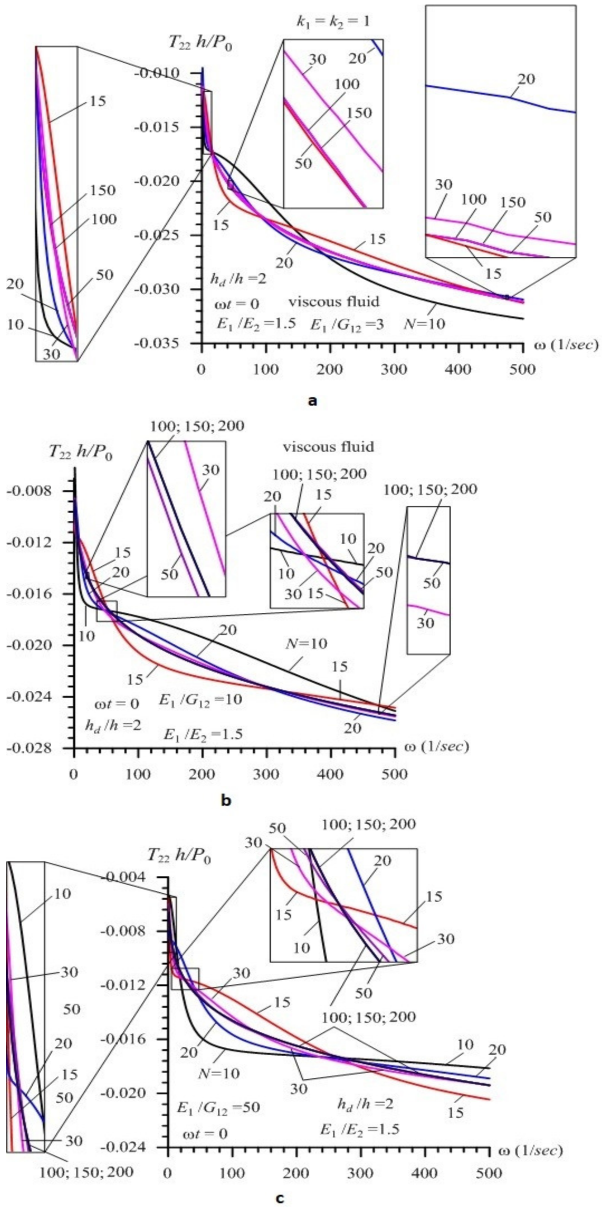

Fig. 2. Convergence of the numerical results with respect to the number N in the cases where $E_1/G_{12} = 3$, 10 and 50

It follows from these results that the convergence of the numerical results with respect to the number Nimproves with decreasing of the ratio E_1/G_{12} and in all the considered cases under $N \geq 100$ (i.e. $N_* = 100$) numerical results obtained for each N coincide with each other with accuracy 10^{-6}.

Consider also the influence of the coefficients k_1 and k_2 the convergence of the numerical result with respect to the number N. Assume that $E_1/G_{12} = 50$ and analyze the graphs given in Fig. 3 which are constructed for the pairs $\{k_1 = 1; k_2 = 2\}$ (Fig.3a) $\{k_1 = 2; k_2 = 2\}$(Fig. 3b), $\{k_1 = 7; k_2 = 2\}$(Fig. 3c) and $\{k_1 = 1; k_2 = 0.1\}$.

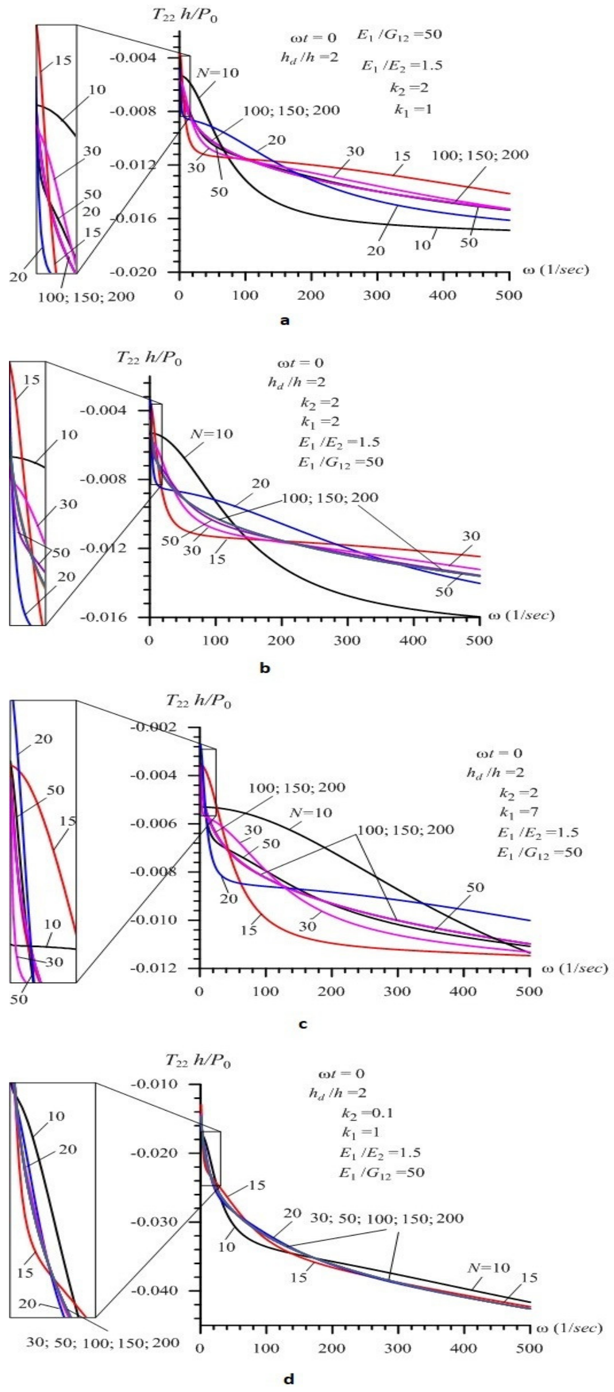

Fig. 3. Convergence of the numerical results with respect to the number N in the cases where $\{k_1 = 1; k_2 = 2\}$ (Fig.3a) $\{k_1 = 2; k_2 = 2\}$(Fig. 3b), $\{k_1 = 7; k_2 = 2\}$(Fig. 3c) and $\{k_1 = 1; k_2 = 0.1\}$ (Fig. 3d)

Thus, it follows from Fig. 3 that an increase in the values of the coefficient k_1 causes to increase the difference between the results obtained in the cases where $N = 50$ and $N = 100$. Nevertheless, all the numerical results which are illustrated in Fig. 3 and are obtained in the cases where $N \geq 100$ coincide with each other. In other words, it is also established that $N_* = 100$for the cases considered in Fig. 3.

Now we consider numerical results shown in Fig. 4 which illustrate the convergence of these results with respect to the S_1^*in the case where $k_1 = 7$, $k_2 = 2$, $E_1/G_{12} = 50$and $N = 200$. Thus, it follows from Fig. 4 that for the case under consideration all the numerical results obtained in the cases where $S_1^* \geq 1.0$ coincide with each other with accuracy 10^{-6}, i.e. i.e. $S_{1*}^* = 1.0$.

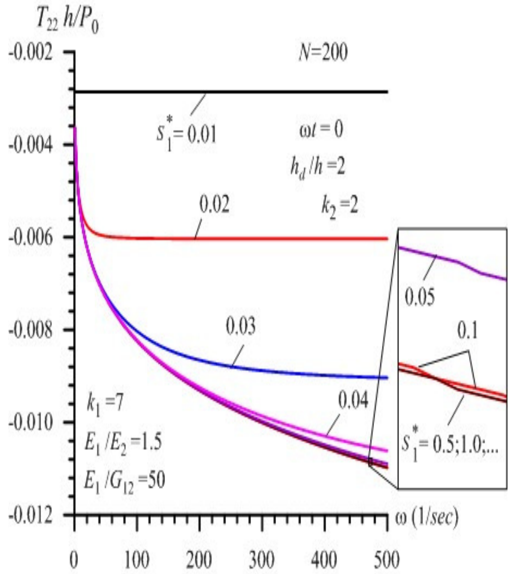

Fig. 4. The influence of the S_1^* on the convergence of the numerical results in the case where $k_1 = 7$, $k_2 = 2$, $E_1/G_{12} = 50$ and $N = 200$

Finally, note that the foregoing convergence requirements are taken into consideration under obtaining numerical results obtained in the papers by Akbarov and Huseynova [7] and Huseynova [8].

5 Conclusions

Thus, in the present paper it is examined the convergence of the algorithm and PC programs employed for obtaining numerical results related to the forced vibration of the hydro-elastic system consisting of the orthotropic plate, compressible viscous fluid and rigid wall. Under this examination the main attention is focused on the influence of the anisotropy properties of the plate material on the number of sub-intervals which are introduced under calculation of the inverse Fourier transforms, according to which, the frequency response of the system under consideration, is studied. Numerical results illustrated the mentioned convergence are presented and discussed. As a result of this discussions it is established the validity and effectiveness of the algorithm and PC programs used in the related investigations. It is also established the approach for controlling the convergence of the numerical results under employing the proposed investigation method.

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