APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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NON-STATIONARY FILTRATION IN NON-HOMOGENEOUS-POROUS MEDIUM WITH DISCONTINUOUS JUMP OF PENETRABILITY

Abstract

In the paper we study the influence of discontinuous jump of penetrability of porous medium on its filtration ability at one-dimensional non-stationary filtration subject to the finite length of bed. The influence of discontinuous jump of penetrability of porous medium on consumption of liquid is considered at changing the direction of filtration flow.

For description of fluid filtration in non-homogeneous stratum we accept it as piecewise-homogeneous one with its mean characteristics [8].

Such a model allows practically to describe any form of change of characteristics of porous medium. But in this case the exact solution of received non-linear filtration equations is not a simple mathematical problem.

Their solution by approximate methods can be accompanied by the qualitative losses. Therefore the finding of exact analytical solution of nonstationary filtrationar in non-homogeneous porous medium is of both practical and scientific interest.

In the paper on the base of theoretical investigations we study the influence of discontinuous jump of penetrability of porous medium on its filtration capability at one-dimensional non-stationary filtration subject to finite length. The influence of discontinuous jump of penetrability of porous medium on flow is considered at change of direction of filtration flow.

The papers of many authors [1-8] are devoted to the investigation of this problem.

But in the above-mentioned papers by solving such problems the stratums are ssumed as semi-infinite domain, or they are solved by approximate or numerical methods. In the given paper we attempt to consider this question analytically subject to finite length to stratum at one-dimensional non-stationary filtration.

Then the filtering equation [2, 3]:

$$\left.\begin{aligned} \frac{\partial^2 p_1}{\partial X^2} &= \frac{1}{\chi_1}\frac{\partial P_1}{\partial t}, \\ \frac{\partial P_2}{\partial x^2} &= \frac{1}{\chi_2}\frac{\partial P_2}{\partial t}, \end{aligned}\right\}, \tag{1}$$

where $\chi_1 = \dfrac{K_1}{(\mu\beta_1^*)}$, $\chi_2 = \dfrac{K_2}{(\mu\beta_2^*)}$; $\beta_1^* = m_1\beta_f + \beta_c$; $\beta_2^* = m_2\beta_f + \beta_c$.

(β_1^*, β_2^* are elasticity coefficients; K_1 and K_2 are permeability indices , m_1 and m_2 are stratum porosity, μ is a dynamic fluid viscosity; χ_1, χ_2 are piezoconductivity coefficients; P_1, P_2 are corresponding pressures).

The initial and boundary conditions (fig. 1)

$$P_1 = P_{01}, \quad P_2 = P_{01} \quad \text{at} \quad t = 0, \tag{2}$$

$$\begin{cases} P_1 = P_2 \quad \text{at} \quad x = l_1 \\ K_1 \dfrac{\partial P_1}{\partial x} = K_2 \dfrac{\partial P_2}{\partial x} \quad \text{at} \quad x = l_1 \\ P_1|_{x=0} = P_l, \quad P_2|_{x=l} = P_0 \end{cases} \tag{3}$$

From (1) subject to (3) applying Laplace transformation we'll obtain [8]

$$\bar{P}_1(x,s) = -\frac{e^{-\sqrt{\frac{s}{\chi_1}}l_1}(P_{01}-P_l)\sqrt{S}\left(K_1\sqrt{\chi_2}+K_2\sqrt{\chi_1}\right)}{S\sqrt{\chi_1}\left(K_1\sqrt{\chi_2}-K_2\sqrt{S}l\right)} - \frac{2e^{-\sqrt{\frac{s}{\chi_2}}l_1}K_2\sqrt{S}}{\left(K_1\sqrt{\chi_2}-K_2\sqrt{S}l_1\right)}\times$$

$$\times\left[\frac{\left(K_1\sqrt{\chi_2}+K_2\sqrt{\chi_1}\right)(P_{01}-P_l)\left(e^{-2\sqrt{\frac{s}{\chi_1}}l_1}-1\right)}{2S\left[e^{-\sqrt{\frac{s}{\chi_1}}l_1}\left(K_1\sqrt{\chi_2}+K_2\sqrt{S}l_1\right)-e^{-\sqrt{\frac{s}{\chi_2}}(2l-l_1)}\left(K_1\sqrt{\chi_2}+K_2\sqrt{S}l_1\right)\right]}-\right.$$

$$\left.-\frac{\left[e^{-\sqrt{\frac{s}{\chi_1}}l_1}(P_{01}-P_l)-e^{\sqrt{\frac{s}{\chi_2}}(l_1-l)}(P_0+P_{01})\right]\left(K_1\sqrt{\chi_2}-K_2\sqrt{S}l_1\right)}{S\left[e^{-\sqrt{\frac{s}{\chi_2}}l_1}\left(K_1\sqrt{\chi_2}+K_2\sqrt{S}l_1\right)-e^{-\sqrt{\frac{s}{\chi_2}}(2l-l_1)}\left(K_1\sqrt{\chi_2}+K_2\sqrt{S}l_1\right)\right]}\right]+$$

$$+\frac{\sqrt{S}}{S\sqrt{\chi_1}}(P_{01}-P_l). \tag{4}$$

The flow rate is determined by the following equality:

$$Q = \frac{K_1}{\mu}\cdot f\left.\frac{\partial\bar{P}_1}{\partial x}\right|_{x=0}. \tag{5}$$

Passing from the image to pre-image and allowing for equality (5) and formula (4) we'll obtain [8]:

$$\frac{\partial\bar{P}_1}{\partial x} = -\left[(P_{01}-P_l)\frac{K_1\sqrt{\chi_1}\left(K_1\chi_2+K_2\sqrt{\chi_1\chi_2}\right)}{\chi_1K_2^2l_1^2}\left(-\frac{e^{at}}{\sqrt{a}}\operatorname{erf}\sqrt{at}\right)+\right.$$

$$+(P_{01}-P_l)\frac{K_1\left(K_1\chi_2+K_2\sqrt{\chi_1\chi_2}\right)}{\chi_1K_2^2l_1}e^{at}-(P_{01}-P_l)\frac{K_1\sqrt{x_2}+K_2\sqrt{\chi_1}}{\chi_2K_2}\times$$

$$\left.\times\left(-\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{\tau^2}{4t}}ch\,a_1\tau d\tau\right)+(P_{01}-P_l)\frac{\sqrt{\chi_1}\left(K_1\sqrt{\chi_2}+K_2\sqrt{\chi_1}\right)}{\chi_1K_2l_1}\left(-e^{at}\right)\right]-$$

$$-\left[\frac{-3\chi_2K_1at^2}{2l_1^3K_2}+\frac{5\chi_2^2K_1at^3}{2l_1^5K_2}+\frac{K_1at}{l_1K_2}-\frac{15\chi_2^3K_1at^4}{4l_1^7K_2}\right]\times$$

$$\times\left[\frac{(P_{01}-P_l)\sqrt{\chi_2}\left(K_1\sqrt{\chi_2}+K_2\sqrt{\chi_1}\right)}{2\sqrt{\chi_1}lK_2}\left(\frac{1}{\sqrt{\pi t}}-a_2e^{a_2^2t}+\frac{2a_2^2\sqrt{t}}{\sqrt{\pi}}+\right.\right.$$

$$\left.+\frac{4a_2^4}{3}t^{\frac{3}{2}}\frac{e^{a_2^2t}}{\sqrt{\pi}}-\frac{4a_2^6}{5}t^{\frac{5}{2}}\frac{e^{a_2^2t}}{\sqrt{\pi}}+\frac{2a_2^8}{7}t^{\frac{7}{2}}\frac{e^{a_2^2t}}{\sqrt{\pi}}-\frac{2a_2^{10}}{27}t^{\frac{9}{2}}\frac{e^{a_2^2t}}{\sqrt{\pi}}\right)-$$

$$-\left(\left[\frac{K_1^4\chi_2^2}{K_2^4l_1^4}te^{at}-\frac{4K_1^3\chi_2\sqrt{\chi_2}}{K_2^3l_1^3}\left(\frac{2\sqrt{t}}{\sqrt{\pi}}+\frac{46a^2}{15}t^{\frac{5}{2}}\frac{e^{at}}{\sqrt{\pi}}-\frac{34a^3}{105}t^{\frac{7}{2}}\frac{e^{at}}{\sqrt{\pi}}+\right.\right.\right.$$

$$\left.+\frac{26a^4}{189}t^{\frac{9}{2}}\frac{e^{at}}{\sqrt{\pi}}+\frac{4a}{3}t^{\frac{3}{2}}\frac{e^{at}}{\sqrt{\pi}}+\frac{4a}{3\sqrt{\pi}}t^{\frac{3}{2}}-\frac{16a^5}{297}t^{\frac{11}{2}}\frac{e^{at}}{\sqrt{\pi}}\right)+$$

$$\left.+\frac{4K_1^2\chi_2}{K_2^2l_1^2}\left(e^{at}-tae^{at}\right)+e^{at}\frac{K_1^2\chi_2}{K_2^2l_1^2}\left(2+\frac{K_1^2\chi_2}{K_2^2l_1^2}t\right)\right]\times$$

$$\times\left[\frac{e^{-\frac{4(l-l_1)^2}{\chi_2t}}\sqrt{\chi_2t}}{2(l-l_1)\sqrt{\pi}}\left(\frac{64(l-l_1)^4-8(l-l_1)^2\chi_2t+3\chi_2^2t^2}{64(l-l_1)^4}\right)\right]\times$$

$$\times\frac{2(P_{01}-P_l)}{l_1\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)\sqrt{\pi}}\left(t^{\frac{3}{2}}-\frac{4}{5}\frac{t^{\frac{3}{2}}}{l_1^2\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)^2}-\frac{l_1^2\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)^2t^{\frac{1}{2}}}{2}\right)-$$

$$-\frac{4(P_{01}-P_l)}{l_1\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)\pi}\left(\frac{3\pi t}{4}\frac{K_2l_1}{K_1\sqrt{\chi_2}}-\frac{3\pi t^2}{4}\frac{K_2}{K_1\sqrt{\chi_2}l_1\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)^2}-\right.$$

$$\left.-\frac{\pi^2}{4}\frac{K_2l_1^3}{K_1\sqrt{\chi_2}}\left(\frac{1}{\sqrt{\chi_1}}-\frac{1}{\sqrt{\chi_2}}\right)^2\right)+\frac{4(P_0+P_{01})}{\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)\sqrt{\pi}}\left(\frac{3\pi t}{4}\frac{K_2l_1}{K_1\sqrt{\chi_2}}-\right.$$

$$\left.-\frac{3}{4}\frac{K_2l_1}{K_1\sqrt{\chi_2}}\frac{\pi t^2}{\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)^2}-\frac{\pi}{4}\frac{K_2l_1}{K_1\sqrt{\chi_2}}\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)^2\right)-$$

$$-\frac{2(P_0+P_{01})}{\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)\sqrt{\pi}}\left(t^{\frac{3}{2}}-\frac{4}{5}\frac{t^{\frac{5}{2}}}{\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)^2}-\frac{\left(-\frac{2l_1}{\sqrt{\chi_2}}+\frac{l}{\sqrt{\chi_2}}\right)^2}{2}t^{\frac{1}{2}}\right)\Bigg)\Bigg)\Bigg]+$$

$$+\frac{P_{01}-P_1}{\sqrt{\chi_1}\sqrt{\pi t}}, \tag{6}$$

where $a=\frac{K_1^2\chi_2}{K_2^2l_1^2}$; $\left(a_1^2=a\right)$; $a_2=\frac{(-K_2l_1+K_1l_1-K_1l)\sqrt{\chi_2}}{K_2l_1l}$.

From (5) subject to (6) we'll obtain the analytical expression for determination amount of consumption in porous, non-homogeneous medium with discontinuous jump of penetrability of finite length.

The numerical computation of the value of consumption in direct and backward directions is carried out by formula (5) at the following values of parameters:

$$l_1=l_2=1\text{ m};\ K_1=0,1\times10^{12}\text{ m}^2,\quad K_2=0,005\times10^{-12}\text{m}^2;\quad \mu=1\text{cП}.$$

The results of computation are represented in fig. 2.

As we see from figure at filtration from smaller penetrability to the big one the consumption becomes bigger than at inverse filtering which has an important practical value at exploitation and in well injection.

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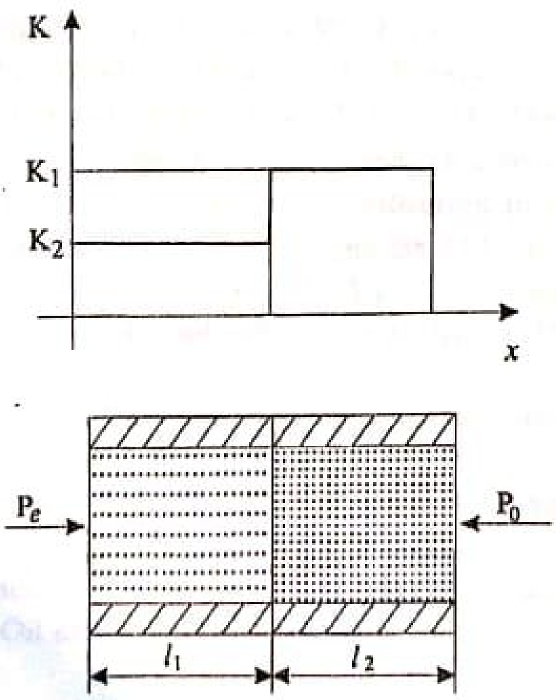

Fig. 1. Scheme of design of non-homogeneous porous medium

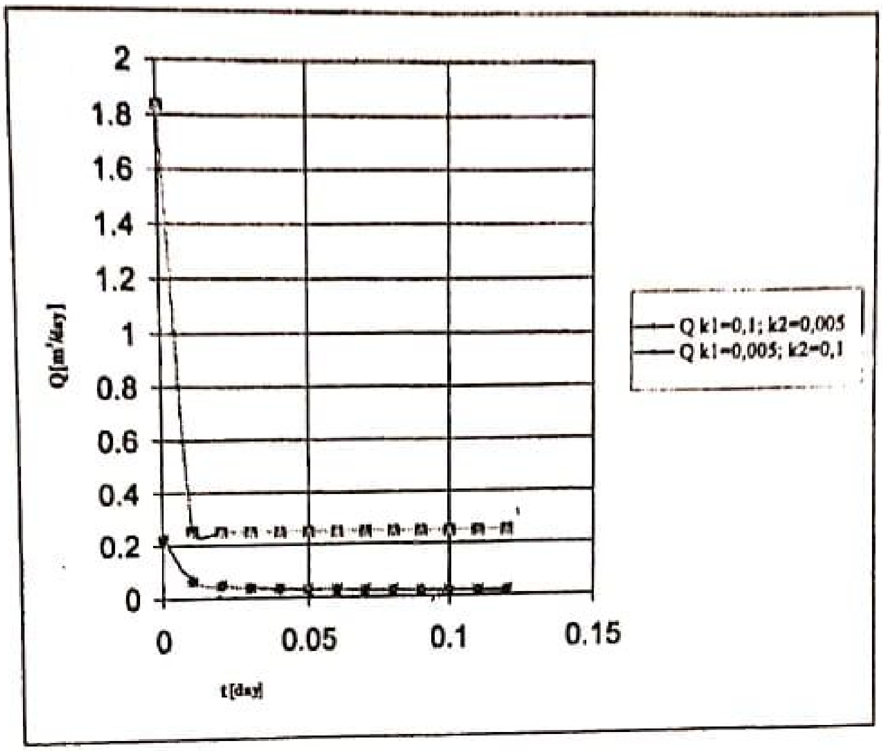

Fig. 2. Dependence of fluid consumption Q on the time t.

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