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Structural synthesis of the robot manipulators with general constraint one using the transformation of the higher kinematic pare

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Abstract. *Many robot manipulators with less than six degrees of freedom are extended for industrial welding processing, painting, industrial machines, etc. New types of the overconstrained robot manipulators need to be synthesized to reduce existing production errors. In this research, structural synthesis of the robot manipulators with general constraint one $d = 1, \lambda = 5$ is investigated. Synthesized new types of the robot manipulators considered are constructed from the transformation of the higher kinematic pare "plane - plane inside of the sphere" with five degrees of freedom. The space mobility of the gripper for the designed robot manipulators realizes its movement with the same degrees of freedom (DOF) of the diad.*

Keywords. higher kinematic pair · subspace $\{\lambda_i\}_1^5$ · mobility (λ) · constraint (d) · "plane - plane inside of the sphere" (FF).

Mathematics Subject Classification (2010): 70B15, 70Q05

1 Introduction

In the last decade, robot manipulators which the mobility of the executive body is in subspace $\{\lambda_i\}_1^5$, have become the main objective of investigation for researchers. Robot manipulators with less than 6 DOF are considered more suitable for many cases. Such overconstrained robot manipulators have advantages in terms of straightness, accuracy, load-weight ratio, dynamic performance, as well as fewer errors. The structural synthesis allows analytical research to find new ways and formulas when designing new types of robot manipulators. Research works related to the structural synthesis of robot manipulators are considered as follows.

In 1853 Sarrus [10] formulas for turning the linear movement to arc and vice versa in the mechanism with an angular constraint and in 1905 Bennet [3] formulas for a new type of spatial mechanism with linear and angular constraints were discovered. The valuable method concerns general kinematic chains and mechanisms (degree of freedom independent of metric restrictions), in which some chains have plane motions and others spatial motions was investigated by Boden [5]. In the article [9] three-dimensional mechanism with providing the requirement for application in industry have designed by Harrisberger

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and Soni. Correspondingly, Waldron in the article [8] hybrid over constrained linkages, Wohlhart in [7] and Bagci in [1] over constrained mechanisms and their degrees of freedom (DOF) were investigated. In 1975 Freudenstein, Alizade [2] published an article: ”On the degree of freedom of mechanisms with variable general constraint”. In this investigation a precise definition of the term ”general constraint” and the general DOF formula for the mechanism was given.

The theory of bonds II: Closed 6R linkages with maximal genus was investigated by Hegedüsa, Lib, Schichoc and Schörckerd [4] in 2015. In this research closed linkages with six rotational joints (6R) that allow a one-dimensional set of motions. In additional, they have demonstrated that the class of the configuration curve of such a linkage is at most five, and give a complete classification of the linkages with a configuration curve of genus four or five. In the year 1966, Bałchanowski [6] have presented a method of the structural synthesis of spatial or planar parallel mechanisms. Thus, the successive steps in the procedure for generating a structural form of a closed branch, the opening of the branch, the way of constructing a parallel mechanism from branches with negative and zero DOF, and the connection of the drives separated from the branches were described. In 2019 [11] Structural Synthesis of Robot Manipulators by Using Screw with Variable Pitch was investigated by Alizade. In this approach, new structural formulas, schemes of kinematic pairs are described for parallel Euclidian platform robot manipulators with fixed and variable general constraints. In addition, 6 DOF Euclidean special docking manipulators of the spacecraft with the same general constraints of each legs $\lambda = 3$ were given. In 2021, [12] Alizade, Soltanov, and Hamidov are described structural synthesis of lower-class robot manipulators with general constraint one. They presented new methodology and 6 axioms which is helpful in desgn of robot manipulators with general constraint one. Furthermore, snthesized robot manipulators with general constraint one are illustrated in the tables.

In this paper, the DOF of the FF dyad combination constructed by a higher kinematic pair with $\lambda = 5$ is found using the analytical method. In addition, the structural classification for the expression of robot manipulators with general constraint one, structural groups, and their kinematic pairs are described in the tables. The use of such robot manipulators with general constraint one is considered to be more useful in reducing production errors and ensuring reliability. Excessive freedom can lead to excessive movements, which makes it difficult to evaluate and control the movement of robot manipulators in certain directions. During operation, there is a need for overconstrained robot manipulators that moving in the subspaces, and for these cases presented robot manipulators with general constrained one can be used.

2 Structural synthesis of robot manipulators with general constraint one

The mobility number of robot manipulators should be provided during structural synthesis. It is known that the free solid object in the space has 6 DOF, and the mobility number of the space is $\lambda = 6$. In three-dimensional space, this complex motion corresponds to the combination of rotation and translation movements. The mobility number of the robot manipulator’s executive body in subspace can be $\{\lambda_i\}_1^5$. The geometry of higher kinematic pairs with the maximum DOF $f = 5$ is analytically analyzed for the moving of the executive body of robot manipulators with mobility number in subspace $\lambda = 5$. The “plane - plane inside of the sphere” FF kinematic pair are studied as the object of the research. Mobility equations (1) of platform robot manipulators are described as follows:

$$M = \sum_{i=1}^{j} f_i - \lambda L \tag{2.1}$$

where f_i – DOF of kinematic pairs;

λ – closed loop motion parameter that describes the positions and orientations of the couple in the loop;

L – the number of independent loops.

The number of revolute joints placed on each leg and general constraint for the motion of a rigid body in space are determined by using the following formulas (2):

$$j_i = C_i^{-1} \sum_{i=1}^{j} \lambda(C - B),\ d = 6 - \lambda \tag{2.2}$$

3 Determination of the space mobility number and general constraint for FF dyad

During the study, the "plane - plane inside of the sphere" FF kinematic pair (Fig. 1) is examined. This kinematic pair is created by touching a plane with another plane inside of the sphere to each other. It is known that the plane and the plane (obtained by cutting from the inside of the sphere) touch each other only at one point. In this case, investigated FF kinematic pair has a constraint. As a result, the spatial mobility of the dyad combination corresponds to the number $\lambda = 5$. Two different approaches are used in the analytical calculations used to find the DOF for the FF higher kinematic pair during the study. The analytical solutions of this number are found as follows.

The first case: Let's write an equation denoting the plane π_1 (Fig.1).

$$\pi_1 : A_0x + B_0y + C_0z + D_0 = 1$$

For the given plane equation the quantities A_0, B_0, C_0, D_0 are the known parameters. The sphere with center O has a common point $M_0(x_0, y_0, z_0)$ with the plane . Furthermore, plane π_2 intersects the sphere and passes through the point $M_1(x_1, y_1, z_1)$.

$$\pi_2 : A_1x + B_1y + C_1z + D_1 = 0$$

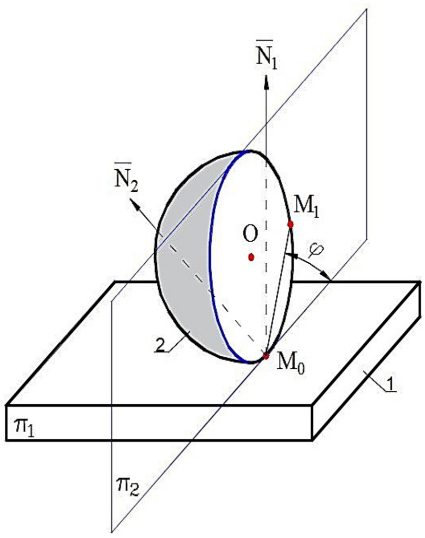


Fig. 1. Plane-plane inside of the sphere (FF). 1 – plane, 2 – sphere.

Let's show the angle between the planes by φ. Here, $\varphi = (\pi_1 \wedge \pi_2)$. If we choose degree for the given angle $\varphi = \pi/2$ that is, π_2 passed through the center of the sphere, then the sphere has a common point π_1 and π_2. Then it is known that the planes π_1 and π_2 intersect in a straight line. Let‘s assume that $\varphi \neq \pi/2$. Later, we can write the expressions for the FF diad combination in mathematical form like, $M_0(x_0, y_0, z_0) \in \pi_2$ and $M_1(x_1, y_1, z_1) \in \pi_2$. Based on the above considerations, we can write the following expressions:

$$\overline{M_0M_1} = \{x_1 - x_0, y_1 - y_0, z_1 - z_0\} \subset \pi_2, \quad \overline{N_2} = \{A_1, B_1, C_1\} \perp \pi_2 \Longrightarrow \overline{N_2} \perp \overline{M_0M_1}$$

Then we can write for the angle between the vector $\overline{M_0M_1}$ and the vector $\overline{N_2}$: $\left(\overline{M_0M_1} \wedge \overline{N_2}\right) = \frac{\pi}{2} - \varphi$.

Hence

$$\cos\left(\frac{\pi}{2} - \varphi\right) = \sin\varphi = A_1(x_1 - x_0) + B_1(y_1 - y_0) + C_1(z_1 - z_0)$$

Thus, the number of independent parameters for the FF kinematic pair is found 6. $A_1, B_1, C_1, x_1, y_1, z_1$ and there is one equation that connects the unknown parameters:

$$A_1(x_1 - x_0) + B_1(y_1 - y_0) + C_1(z_1 - z_0) = \sin\varphi \tag{3.1}$$

Thus, we find the mobility number of the investigated system by subtracting the number of balanced equation (2) from the number of unknown $(A_1, B_1, C_1, x_1, y_1, z_1)$ parameters written plane - plane inside of the sphere higher kinematic pair. As a result, the space mobility number of the studied system is found $\lambda = 8 - 3 = 5$ and the general constraint $d = 6 - \lambda = 1$.

The second case: Now, let’s look at the two intersecting planes and separately. Suppose that the planes that do not pass through the origin 0 are given.

$$\pi_1 : A_1x_1 + B_1y + C_1z + D_1 = 0$$

$$\pi_2 : A_2x_2 + B_2y + C_2z + D_2 = 0$$

Since $D_1 \neq 0$ and $D_2 \neq 0$ satisfy the condition, we can write these equations as follows:

$$\pi_1 : A_1^*x_1 + B_1^*y + C_1^*z + 1 = 0 \tag{3.2}$$

$$\pi_2 : A_2^*x_2 + B_2^*y + C_2^*z + 1 = 0 \tag{3.3}$$

It can be seen from Equations (3) and (4) that, DOF of each of these planes is 3: $A_1^*, B_1^*, C_1^*$for the first plane π_1, A_2^*, B_2^*, C_2^* for the second plane π_2. The degree of freedom of the two planes together becomes 6. Assume that these planes intersect at a given angle φ . Obviously, this linear angle will be the remaining angle between the normal vectors $\overline{N_1} = \{A_1^*, B_1^*, C_1^*\}$ and $\overline{N_2} = \{A_2^*, B_2^*, C_2^*\}$ of the planes under consideration.

$$\cos\varphi = \frac{(N_1, N_2)}{|N_1||N_2|} = \frac{A_1^*, A_2^* + B_1^*B_2^* + C_1^*C_2^*}{\sqrt{(A_1^*)^2 + (B_1^*)^2 + (C_1^*)^2} + \sqrt{(A_2^*)^2 + (B_2^*)^2 + (C_2^*)^2}} \tag{3.4}$$

Therefore, equation (5) is an additional condition that creates a relationship between the 6 parameters that determine the DOF of the two planes. Thus, we find the mobility number of the investigated system by subtracting the number of balanced equation (5) from the number of unknown ($A_1^*, B_1^*, C_1^*, A_2^*, B_2^*, C_2^*$) parameters written plane - plane inside of the sphere higher kinematic pair. As a result, the space mobility number of the studied system is found $\lambda = 6 - 1 = 5$ and the general constraint $d = 6 - \lambda = 1$.

In the next step, a structural synthesis of the robot manipulators with general constraint one using the transformation of a higher kinematic pair FF is investigated.

At this point we can give the following definitions for structural classification:

$\overline{S}$ – vector, $(\overline{SS})$ and $(\overline{SSS})$ – planar expression with the two and three intersecting noncoplanar vectors,

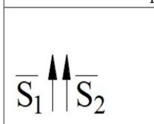
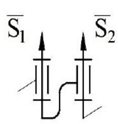
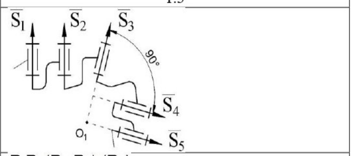
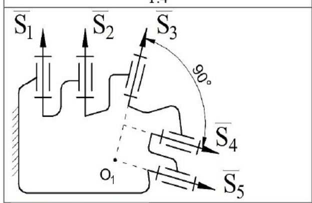
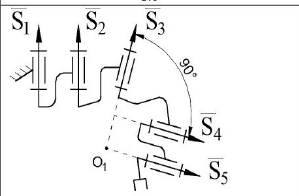
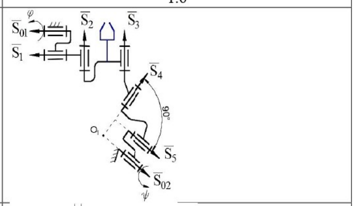
R – revolute kinematic pair, $(\underline{\overline{RR}})$, $(R\perp R)_1$ and $(R\cap R)_2-$ the writing form of the parallel, perpendicular and intersecting planes with the revolute kinematic pairs.

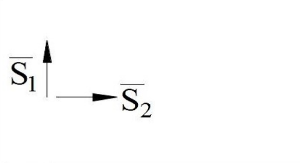
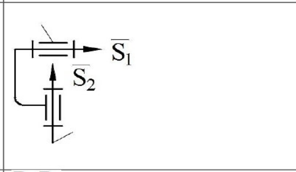
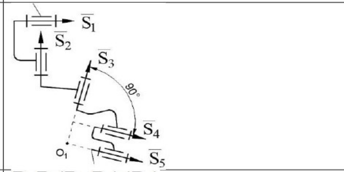
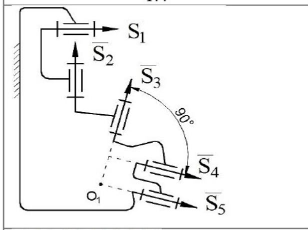
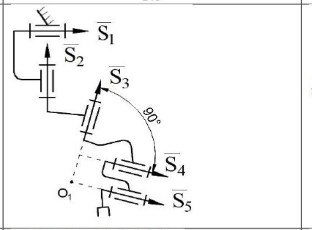
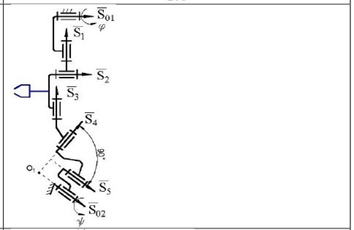
In Table 1, the structural synthesis for FF robot manipulators with general constraint one is performed in the following sequence.

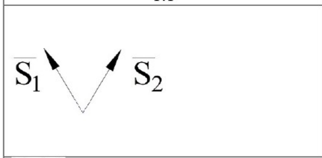
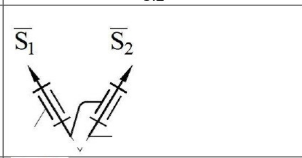
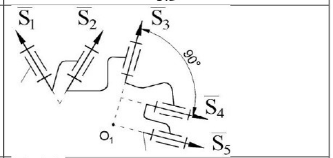
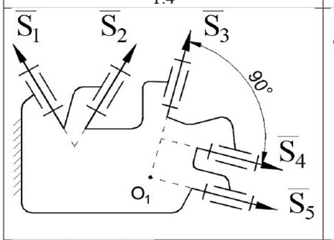
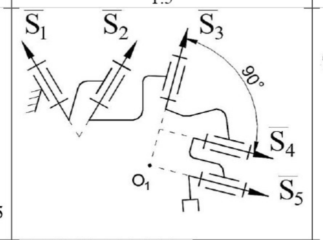
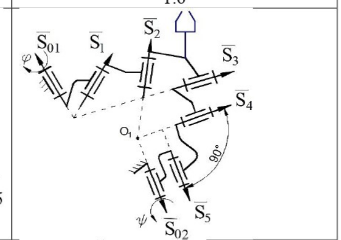
Initially, a description of the plane with parallel vectors is shown, and the revolute pairs of the plane are obtained by adding kinematic pairs to the parallel vectors (Table 1, 1.1 and 1.2). The FF kinematic chain is designed by connecting the kinematic pairs of revolution of the two planes (Table 1, 1.3). Then, a second-class structural group is synthesized from the projected kinematic chain $M = 0, \lambda = 5, d = 1$ (Table 1, 1.4). In the next stage, the structural synthesis of the serial type robot manipulator with general constraint one is synthesized $M = 5, \lambda = 5, d = 1$ (Table 1, 1.5). At the end the structural schemes of synthesis of closed loop robot manipulator $M = 1, \lambda = 5, d = 1$ is shown (Table 1, 1.6).

In the second stage, the plane is continuously represented by perpendicular vectors (Table 1, 2.1). In the third stage, a description of the plane with intersecting vectors is shown (Table 1, 3.1). The following processes are analogous with the above design for the FF type robot manipulator. Here, S_{01} and S_{02} show motors for each leg of the synthesized robot manipulators. The functional dependence for a closed-loop type robot manipulator is shown as $\psi = f(\overline{C}, \varphi)$. Here, $\overline{C}-$ is the constant parameter for the synthesized robot manipulators.

Table 1. Structural synthesis of robot manipulators by transformations kinematic pair plane - plane inside of the sphere (FF)

№	Vectors of the plane	Revolute pairs of the plane	Kinematic chain of the pairs plane and plane inside of the sphere
1	1.1	1.2	1.3
			
	$\overline{S}_1\overline{S}_2$	R_1R_2	$R_1R_2(R_3 \perp R_5)_1(R_4)$
	Structural group	Structure of serial manipulator	Closed loop robot manipulator
	M=0, λ=5, d=1	M=5, λ=5, d=1	M=2, λ=5, d=1
	1.4	1.5	1.6
			
	$R_1R_2(R_3 \perp R_5)_1(R_4)$	$R_1R_2(R_3 \perp R_5)_1(R_4)$	$R_{01}R_1 \perp R_2\, R_3(R_4 \perp R_{02})_1(R_5)$

№	Vectors of the plane	Revolute pairs of the plane	Kinematic chain of the pairs plane and plane inside of the sphere
2	1.1	1.2	1.3
			
	$S_1 \perp S_2$	$R_1 \perp R_2$	$R_1 \perp R_2(R_3 \perp R_5)_1(R_4)$
	Structural group	Structure of serial manipulator	Closed loop robot manipulator
	M=0, λ=5, d=1	M=5, λ=5, d=1	M=2, λ=5, d=1
	1.4	1.5	1.6
			
	$R_1 \perp R_2(R_3 \perp R_5)_1(R_4)$	$R_1 \perp R_2(R_3 \perp R_5)_1(R_4)$	$R_{01} \perp R_1 \perp R_2 \perp R_3(R_4 \perp R_{02})_1(R_5)$

№	Vectors of the plane	Revolute pairs of the plane	Kinematic chain of the pairs plane and plane inside of the sphere
3	1.1	1.2	1.3
			
	$S_1 \cap S_2$	$R_1 \cap R_2$	$S_1 \cap S_2$
	Structural group	Structure of serial manipulator	Closed loop robot manipulator
	M=0, λ=5, d=1	M=5, λ=5, d=1	M=2, λ=5, d=1
	1.4	1.5	1.6
			
	$R_1 \cap R_2(R_3 \perp R_5)_1(R_4)$	$R_1 \cap R_2(R_3 \perp R_5)_1(R_4)$	$R_{01} \cap R_1(R_2)_1 \cap R_3(R_4 \perp R_{02})_1(R_5)$

4 Conclusions

As a result of the research process, the mobility number in subspace $\lambda = 5$ of higher kinematic pair which have 5 DOF, for structural synthesis of robot manipulators with general constraint one is determined by analytical approach. The structural synthesis of robot manipulators which are constructed from geometrical transformations of the higher dyads (FF) are described. At this point, the geometrical and mutual relationships of kinematic pair of robot manipulators with general constraint one are shown. During the research, new types of 3 structural groups, and 6 second class robot manipulators with general constraint one are structurally synthesized. Designed robot manipulators with general constraint one are suitable for use in industry, medicine, the military, aerospace and other places.

References

1. C. Bagci: Degrees of freedom of motion in mechanisms, *ASME J. Eng.Ind.* 93B, (1971) 140-148.
2. F. Freudenstein, R. Alizade: On the degree of freedom of mechanisms with variable general constraint, *in IV World IFToMM Congress, England,* (1975) 51–56.
3. G.T. Bennett: The parallel motion of Sarrus and some allied mechanisms, *Philosophical Magazine, 6th Series 9*, (1905) 803–810.
4. Gábor Hegedüsa, Zijia Lib, Josef Schichoc, Hans-Peter Schröckerd, The theory of bonds II: Closed 6R linkages with maximal genus, Elsevier, *Journal of Symbolic Computation* 68 (2015) 167–180.
5. H. Boden: Zum zwanglauf genuscht raümlichebener getriebe, Maschinenbautechnic (11) (1962) 612-615.
6. J. Bałchanowski: General method of structural synthesis of parallel mechanisms, *Elsevier, Archives of civil and mechanical engineering* 16 (2016) 256-268.
7. K. Wohlhart: A new 6R space mechanism, in: *Proceedings of the Seventh World Congress on the Theory of Machines and Mechanisms, Sevilla, Spain,* vol. 1, (1987) 193–198.
8. K.J. Waldron, Hybrid over constrained linkages, *Journal of Mechanisms 3*, (1968) 73–78.
9. L. Harrisberger, A.H. Soni: A survey of three-dimensional mechanisms with general constraint one, ASME, (1966), No. 66-Mech-44.
10. P.F. Sarrus: On the transformation of alternative movements rectilinear, circular motions, and conversely, *Comptes Rendus Weekly meetings of the Academy of Sciences,* (1853) 1036–1038.
11. Rasim Alizade: Structural Synthesis of Robot Manipulators by Using Screw with Variable Pitch, *Universal Journal of Mechanical Engineering* **7**(2), USA, (2019) 50-63.
12. Rasim Alizade, Suleyman Soltanov, Abusalat Hamidov: Structural synthesis of lower-class robot manipulators with general constraint one, *MDPI journal Robotics* **10**(1),2021, 14.