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# Hydrodynamic simulation of fluid flow in the "deformable reservoir-pipeline" system

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**Abstract.** *A model of the process of unsteady motion of fluid in the coupled "deformable reservoir-pipeline" system is structured and connected equations are solved. An analytic expression admitting to determine the influence of pressure change law on the bottomhole, harnesses to it and deformation of the reservoir on the dynamics of pressure at the outlet of the main pipeline, was obtained. Numerical calculations were carried out for various values of the system's parameters.*

**Keywords.** Laplace transform · deformation · fluid motion · differential equation · pressure · continuity equation · Volterra type integral equation.

**Mathematics Subject Classification (2010):** 76S05

## 1 Introduction

Oil production process consists of three interconnected motion of fluid: in a stratum, in lifting pipes and in the main pipeline. Any change that happens in one of them, is reflected in other flows. This, in the initial period leads the violation of the steady state of wells. Further, after some time, the wells switch to a different steady state, but with a different return.

Determination of the influence of this transition process on the existing mode of wells is of important applied and scientific value. Therefore, when modelling the oil production process, it is necessary to consider the "reservoir-well" and main line as a single system. Furthermore, in addition other factors, deformation of the formation matrix also may have essential influence on the fluid flow hydrodynamics. In spite of significant number of hydrodynamic studies of oil production process [1-9], the issue of hydrodynamics of flow in the coupled system of "reservoir-pipeline" was not given due attention.

Therefore, simulation and study of hydrodynamics process in a "reservoir-pipeline" system allowing for deformation of the formation matrix is of scientific and applicational significance.

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## 2 Statement and solution of the problem

Let us consider the process of homogeneous fluid filtration in a uniform annular deformable reservoir (Fig. 1). In the first approximation we accept that permeability of reservoir due to its deformation depending on the pressure change linearly [5].

$$k(P) = k_0 - \frac{k_0 - k_c}{P_k - P_c(0)}(P_k - P), \tag{2.1}$$

where $k_0$ and $k_c$ are initial permeabilities on the contour and pore channel wall.

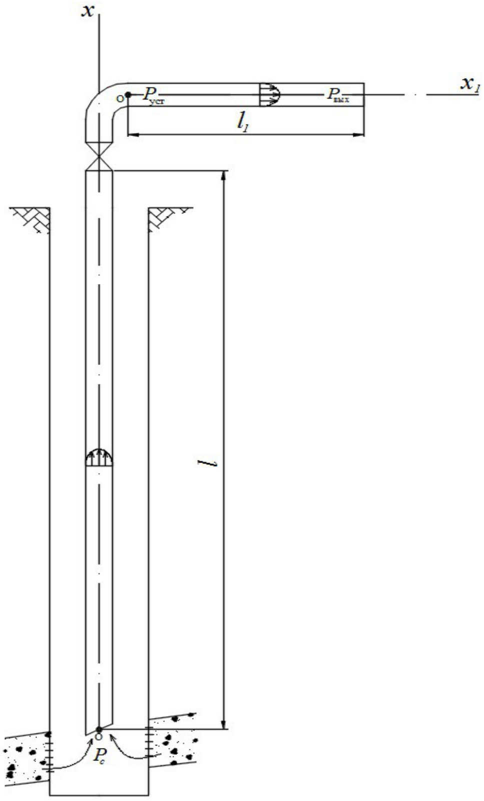


**Fig. 1.** Calculation scheme

Within the accepted assumptions, the differential equation of flat-radial filtration of fluid will have the form [3]

$$\frac{\partial \Delta P}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial\, r}\,\left[\chi(P) r\,\frac{\partial \Delta P}{\partial\, r}\right], \tag{2.2}$$

where

$$\Delta P = P - P_k\,, \quad \chi(P) = \frac{k(P)}{\mu\beta^*}. \tag{2.3}$$

The initial and boundary conditions

$$\Delta P\mid_{\,t=0} = \frac{P_k - P_c(0)}{\ln\left(\frac{R_k}{r_c}\right)}\,\ln\left(\frac{R_k}{r}\right), r_c \leq r \leq R_k\,, \tag{2.4}$$

$$\Delta P\ \mid_{r=R_k}\ = 0\,,\ \ t\ >\ 0\,, \tag{2.5}$$

$$\Delta \mathrm{P}|_{\mathrm{r}=r_c} = P_k - \mathrm{P_c}(t) \quad , \qquad t > 0. \tag{2.6}$$

We solve the equation (2.2) approximately. For that accepting that pressure change per unit time weakly depends on the coordinate, we average the right-hand side of the equation (2.2) with respect to $r$ [10]

$$\varphi\left(t\right) = \frac{2}{R_k^2 - r_c^2} \int_{r_c}^{R_r} \frac{\partial \Delta P}{\partial t} r dr \tag{2.7}$$

Substituting expressions (2.1) and (2.7) in equation (2.2), allowing for boundary conditions (2.5) and (2.6) we get

$$a_1 \Delta P + b_1 P \Delta P = \frac{r^2}{4} \varphi\left(t\right) + c_1 \ln(r) + c_2 \tag{2.8}$$

where $c_1$ and $c_2$ are integration constants, $a_1 = \frac{1}{\mu\beta^*}\left(k_0 - \frac{k_0 - k_c}{P_k - P_c(0)} P_k\right)$,
$b_1 = \frac{1}{\mu\beta^*}\left(\frac{k_0 - k_c}{P_k - P_c(0)}\right)$.

Following Leibenzon, in the first approximation we accept [8]

$$P^2 = \frac{P}{2}(P_k + P_c(0)) \tag{2.9}$$

Then from the expression (2.8) allowing for (2.3) and (2.9) we get

$$P = \frac{1}{A_1}\left[\frac{r^2 - R_k^2}{4}\varphi(t) + \ln\left(\frac{r}{R_k}\right)\left[A_2(P_c(t) - P_k) - A_3\varphi(t)\right] + A_1 P_k\right] \tag{2.10}$$

where

$$A_1 = a_1 + 0.5 b_1 P_c(0) - 0.5 b_1 P_k,$$
$$A_2 = \frac{A_1}{\ln\left(\frac{R_k}{r_c}\right)}, A_3 = \frac{R_k^2 - r_c^2}{4\ln\left(\frac{R_k}{r_c}\right)} \tag{2.11}$$

Substituting expression (2.10) in formula (2.7) and integrating, we get

$$\frac{2}{R_k^2 - r_c^2} \dot{\varphi}(t)\left[\frac{A_4}{4A_1} - \frac{A_3 A_5}{A_1}\right] - \varphi(t) + \frac{2}{R_k^2 - r_c^2} \frac{A_2 A_5}{A_1} \dot{P}_c(t) = 0 \tag{2.12}$$

where

$$A_4 = \frac{R_k^4 - r_c^4}{4} - \frac{R_k^2(R_k^2 - r_c^2)}{2},$$
$$A_5 = \frac{r_c^2}{2} \ln\left(\frac{R_k}{r_c}\right) - \frac{(R_k^2 - r_c^2)}{4} \tag{2.13}$$

Applying the Laplace transform, convolution and inversion theorems, from equation (2.12) we get:

$$\varphi(t) = \varphi_0 \exp\left(\frac{1}{A_6} t\right) + \frac{A_7}{A_6} P_c(0) \exp\left(\frac{1}{A_6} t\right) -$$
$$-\frac{A_7}{A_6} P_c(t) - \frac{A_7}{A_6^2} \int_0^t P_c(\tau) \exp\left(\frac{t - \tau}{A_6}\right) d\tau \tag{2.14}$$

where

$$A_6 = \frac{2}{R_k^2 - r_c^2}\left(\frac{A_4}{4A_1} - \frac{A_3 A_5}{A_1}\right)$$

$$A_7 = \frac{2}{R_k^2 - r_c^2}\frac{A_2 A_5}{A_1} \tag{2.15}$$

Substituting expression (2.14) in formula (2.10), we get

$$P = \left(\varphi_0 \exp\left(\frac{1}{A_6}t\right) + \frac{A_7}{A_6}P_c(0)\exp\left(\frac{1}{A_6}t\right) - \right.$$
$$\left. -\frac{A_7}{A_6}P_c(t) - \frac{A_7}{A_6^2}\int_0^t P_c(\tau)\exp\left(\frac{t-\tau}{A_6}\right)d\tau\right) \times$$
$$\times\left[\frac{1}{4A_1}(r^2 - R_k^2) + \frac{A_3}{A_1}\ln\left(\frac{R_k}{r}\right)\right] - \left[\frac{A_2}{A_1}(P_c(t) - P_k)\left(\ln\left(\frac{R_k}{r}\right)\right)\right] \tag{2.16}$$

$\varphi_0$ is determined from the expression (2.16) allowing for the initial condition (2.4)

$$\varphi_0 = \frac{1}{\frac{1}{4A_1}(r_c^2 - R_k^2) + \frac{A_3}{A_1}\ln\left(\frac{R_k}{r_c}\right)} \times$$
$$\times\left[2P_k + P_c(0) + \frac{A_2}{A_1}(P_c(0) - P_k)\ln\left(\frac{R_k}{r_c}\right)\right]. \tag{2.17}$$

The flow rate at the moment $t$ through the lateral surface of the well of radius $r_c$ is determined by the formula

$$Q|_{r=r_c} = -2\pi r_c h\frac{k}{\mu}\left.\frac{\partial \Delta P}{\partial r}\right|_{r=r_c}. \tag{2.18}$$

Having substituted expression (2.16) in formula (2.18), we get

$$Q|_{r=r_c} = -2\pi r_c h\frac{k(P)}{\mu}\left\{A_8\left(\varphi_0\exp\left(\frac{1}{A_6}t\right) + \frac{A_7}{A_6}P_c(0)\exp\left(\frac{1}{A_6}t\right) - \right.\right.$$
$$\left.\left. -\frac{A_7}{A_6}\,P_c(t) - \frac{A_7}{A_6^2}\int_0^t P_c(\tau)\exp\left(\frac{t-\tau}{A_6}\right)d\tau\right) + A_9\left(P_c\left(t\right) - P_k\right)\right\} \tag{2.19}$$

where

$$A_8 = -\frac{r_c}{2A_1} - \frac{A_3}{A_1 r_c}, A_9 = \frac{A_2}{A_1 r_c} \tag{2.20}$$

$$k(P)|_{r=r_c} = a_2 + b_2 P_c(t) \tag{2.21}$$

$$\begin{array}{l} a_2 = k_0 - \frac{k_0 - k_c}{P_k - P_c(0)}P_k \\ b_2 = \frac{k_0 - k_c}{P_k - P_c(0)} \end{array} \tag{2.22}$$

Now in the first approximation we accept that the bottomhole pressure in the course of time drops linearly

$$P_c(t) = P_c(0) - \frac{P_c(0) - P_{cT}}{T}t \tag{2.23}$$

where $P_c(0)$ and $P_{cT}$ are bottomhole pressures at the beginning and at the end of well operation.
Then, substituting expression (2.17) and (2.23) in formula (2.19), we get

$$Q|_{r=r_c} = -2\pi r_c h \frac{(a_2 + b_2(P_c(0) - A_{10}t))}{\mu} \left( A_{11} \exp\left(\frac{1}{A_6}t\right) - A_{12}t - A_{13} \right) \quad (2.24)$$

where,

$$A_{10} = \frac{P_c(0) - P_{cT}}{T}$$

$$A_{11} = A_7 A_8 A_{10} - \frac{A_7 A_8}{A_6} P_c(0) + A_8 \varphi_0$$

$$A_{12} = A_9 A_{10}$$

$$A_{13} = A_7 A_8 A_{10} - A_9 P_c(0) \quad (2.25)$$

## 3 Fluid flow in the tubing

Now we consider fluid flow in a tubing. Taking the fluid as dropping, compressible, homogeneous, for the equation of its flow in the pipe and continuity equation we have [9-12]

$$-\frac{\partial P}{\partial x} = \frac{\partial Q_1}{\partial t} + 2aQ_1,$$

$$-\frac{1}{c^2}\frac{\partial P}{\partial t} = \frac{\partial Q_1}{\partial x}, \quad (3.1)$$

where $c^2 = \frac{\partial P}{\partial \rho}$; $c$ is the sound speed in fluid, $Q_1 = \rho u$ is the mass flow rate in the unit area of flow section of the pipe $\rho$ is fluid density, $u$ is fluid flow speed averaged along cross section of the pipe, $a$ is a resistance factor.

Differentiating both hand sides of the first equation with respect to $x$, the second equation with respect to $t$ of the expression (3.1) and subtracting them term by term, we get:

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} - 2a\frac{\partial P}{\partial t}. \quad (3.2)$$

The initial and boundary conditions

$$P(x,0)|_{t=0} = P_c(0) - 2aQ_{10}x, 0 \le x \le l, \quad (3.3)$$

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = 0, 0 \le x \le l, \quad (3.4)$$

$$P|_{x=l} = P_y(t), t > 0, \quad (3.5)$$

$$P|_{x=0} = P_c(t), t > 0. \quad (3.6)$$

We shall look for the solution of the equation (3.2) allowing for conditions (3.5) and (3.6) in the form:

$$P = P_c(t) - \frac{P_c(t) - P_y(t)}{l}x + \sum_{i=1}^{n} \varphi_i(t) \sin\left(\frac{i\pi x}{l}\right), \quad (3.7)$$

where $\varphi_i\,(t)$ is an unknown function dependent on time $t$;$l$ is the pipe length. Substituting expression (3.7) in the equation (3.2), multiplying the both hand sides of the obtained expression by $\sin\left(\frac{i\pi x}{l}\right)$ and integrating it from 0 to $l$, we get the equation:

$$\frac{l}{2}\ddot{\varphi}_i\,(t) + al\dot{\varphi}_i\,(t) + \frac{c^2\pi^2 i^2}{2l}\varphi_i\,(t) + \frac{l}{\pi i}\ddot{P}_c\,(t) - (-1)^i\frac{l}{\pi i}\ddot{P}_y\,(t) + \\ + \frac{2al}{\pi i}\dot{P}_c\,(t) - (-1)^i\frac{2al}{\pi i}\dot{P}_y\,(t) = 0. \tag{3.8}$$

Applying the Laplace transform, convolution and inversion theorems, allowing for initial conditions (3.3) and (3.4) from the equation (3.8) we get:

$$\varphi_i = \frac{\varphi 0}{\xi_1 - \xi_2}(\exp(\xi_1 t)(2a+\xi_1) - \exp(\xi_2 t)(2a+\xi_2)) + \dot{\varphi}_0\frac{\exp(\xi_1 t) - \exp(\xi_2 t)}{\xi_1 - \xi_2} - \\ - \frac{2}{\pi i}\left(P_c(t) + \frac{\xi_1(2a+\xi_1)\int_0^t P_c(\tau)\exp(\xi_1(t-\tau))d\tau}{\xi_1-\xi_2} - \right. \\ \left. - \frac{\xi_2(2a+\xi_2)\int_0^t P_c(\tau)\exp(\xi_2(t-\tau))d\tau}{\xi_1-\xi_2}\right) - \\ - \frac{2}{\pi i}\left(P_y(t) + \frac{\xi_1(2a+\xi_1)\int_0^t P_y(\tau)\exp(\xi_1(t-\tau))d\tau}{\xi_1-\xi_2} - \right. \\ \left. - \frac{\xi_2(2a+\xi_2)\int_0^t P_y(\tau)\exp(\xi_2(t-\tau))d\tau}{\xi_1-\xi_2}\right) + \\ + \frac{2(P_c(0)+P_y(0))}{\pi i(\xi_1-\xi_2)}(\exp(\xi_1 t)(2a+\xi_1) - \exp(\xi_2 t)(2a+\xi_2)) + \\ + \frac{2(\dot{P}_c(0)+\dot{P}_y(0))}{\pi i(\xi_1-\xi_2)}(\exp(\xi_1 t) - \exp(\xi_2 t)), \tag{3.9}$$

where $\xi_1$ and $\xi_2$ are the roots of the equation

$$s^2 + 2as + \frac{c^2\pi^2 i^2}{l^2} = 0. \tag{3.10}$$

Substituting expression (3.10) in formula (3.7) allowing for initial conditions (3.3) and (3.4) we have $\varphi_0 = 0, \quad \dot{\varphi}_0 = 0$.

Differentiating formula (3.7) with respect to $x$ and then substituting to the first equation in (3.1), we get

$$\frac{P_c(t)}{l} - \frac{P_y(t)}{l} + \sum_{i=1}^{n}\varphi_i\,(t)\,\frac{\pi i}{l}\cos\left(\frac{i\pi x}{l}\right) = -\frac{\partial Q_1}{\partial t} - 2aQ_1. \tag{3.11}$$

Applying the Laplace transform, and then convolution and inversion theorems, from equation (3.11) we get

$$\bar{Q}_1 = \frac{Q_1(0)}{s+2a} + \frac{\bar{P}_c}{l(s+2a)} - \frac{\bar{P}_y}{l(s+2a)} - \sum_{i=1}^{n}\bar{\varphi}_i\frac{\pi i}{l(s+2a)}\cos\left(\frac{i\pi x}{l}\right),$$

$$Q_1 = Q_1(0)\exp(-2at) - \frac{1}{l}\int_0^t P_{\text{y}}(\tau)\exp\left[-2a(t-\tau)\right]d\tau +$$
$$+\frac{1}{l}\int_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau -$$
$$-\sum_{i=1}^{n}\left(\frac{i\pi}{l}\cos\frac{i\pi x}{l}\right)\left(\int_0^t \varphi_i(\tau)\exp\left[-2a(t-\tau)\right]d\tau\right). \tag{3.12}$$

The continuity condition

$$Q|_{r=r_c} = \frac{f}{\rho}Q_1\bigg|_{x=0}. \tag{3.13}$$

Substituting expressions (2.19) and (3.12) in formula (3.13) allowing only for one term of the series, in the first approximation we get the

$$-2\pi r_c h\frac{k(P)}{\mu}\left\{A_8\left(\varphi_0\exp\left(\frac{1}{A_6}t\right) + \frac{A_7}{A_6}P_c(0)\exp\left(\frac{1}{A_6}t\right) -\right.\right.$$
$$\left.-\frac{A_7}{A_6}P_c(t) - \frac{A_7}{A_6^2}\int_0^t P_c(\tau)\exp\left(\frac{t-\tau}{A_6}\right)d\tau\right) + A_9\left(P_c(t) - P_k\right))\} =$$
$$= \frac{f}{\rho}\left(Q_1(0)\exp(-2at) - \frac{1}{l}\int_0^t P_{\text{y}}(\tau)\exp\left[-2a(t-\tau)\right]d\tau +\right.$$
$$+\frac{1}{l}\int_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau -$$
$$\left.-\sum_{i=1}^{n}\left(\frac{i\pi}{l}\right)\left(\int_0^t \varphi_i(\tau)\exp\left[-2a(t-\tau)\right]d\tau\right)\right). \tag{3.14}$$

Applying the Laplace transform, from equation (3.14) we get

$$\overline{P}_y = \frac{\rho l(s+2a)(s-\xi_1)(s-\xi_2)}{f(s-\eta_1)(s-\eta_2)}\left[\frac{A_{14}}{s^3} + \frac{A_{15}}{\left(s-\frac{1}{A_6}\right)^2} + \frac{A_{16}}{s^2} -\right.$$
$$\left.-\frac{\frac{A_{17}}{A_6}}{s\left(s-\frac{1}{A_6}\right)} - \frac{f\overline{P}_c}{\rho l(s+2a)} + \frac{f\pi}{\rho l(s+2a)}\overline{\Phi}\right] \tag{3.15}$$

where

$$\bar{\Phi} = \frac{\varphi_0(s+2a)}{(s-\xi_1)(s-\xi_2)} + \frac{\dot{\varphi}_0}{(s-\xi_1)(s-\xi_2)} -$$
$$-\frac{2s\bar{P}_c(s+2a)}{\pi(s-\xi_1)(s-\xi_2)} + \frac{2P_c(0)(s+2a)}{\pi(s-\xi_1)(s-\xi_2)} +$$
$$+\frac{2\dot{P}_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \frac{2P_y(0)(s+2a)}{\pi(s-\xi_1)(s-\xi_2)} + \frac{2\dot{P}_y(0)}{\pi(s-\xi_1)(s-\xi_2)},$$
$$A_{14} = -\frac{4\pi r_c h b_2 A_{10}A_{12}}{\mu}, A_{15} = \frac{2\pi r_c h b_2 A_{10}A_{11}}{\mu},$$
$$A_{16} = \frac{2\pi r_c h a_2 A_{12}}{\mu} + \frac{2\pi r_c h b_2 P_c(0) A_{12}}{\mu} - \frac{2\pi r_c h b_2 A_{10}A_{13}}{\mu},$$

$$A_{17} = \frac{2\pi r_c h a_2 A_{13}}{\mu} + \frac{2\pi r_c h b_2 P_c(0) A_{13}}{\mu} \tag{3.16}$$

$\eta_1$ and $\eta_2$ are the roots of the equation

$$s^2 + s(4a + \xi_1 + \xi_2) - \xi_1\xi_2 = 0 \tag{3.17}$$

## 4 Fluid flow in the main pipeline

We consider the fluid flow in the main pipeline. We locate the origin of the coordinate axis $x_1$ at the inlet of the pipeline and direct it in the direction of fluid flow. Assume that at some time moment oil line with flow rate $G$ is connected to the main pipeline at the distance $l_2$ from the origin of the coordinate axis $x$.

Then fluid flow in the main pipeline will be of the form

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} - 2a_3 \frac{\partial P}{\partial t} - \frac{2a_3 c^2 G}{f_1}\delta(x_1 - l_2) \tag{4.1}$$

Initial and boundary conditions:

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = -c^2 \frac{G}{f_1}\delta\left(x_1 - l_2\right) \tag{4.2}$$

$$P(x,0)|_{t=0} = P_{yc}(0) - 2a_3 Q_{20} x_1 \tag{4.3}$$

$$P|_{x_1=0} = P_y(t) \tag{4.4}$$

$$P|_{x_1=l_1} = P_{outlet}(t) \tag{4.5}$$

where $l_2$ is the distance from the wellhead to the point of connection to the main pipe.

We will look for the solution of equation (4.1) allowing for boundary conditions (4.4) and (4.5) in the form:

$$P = P_y(t) - \frac{P_y(t) - P_{2KE}(t)}{l_1} x + \sum_{i=1}^{n} \psi_i\left(t\right)\left(\sin\frac{i\pi x_1}{l_1}\right) \tag{4.6}$$

where $\psi_i\left(t\right)$ is an unknown function dependent on time $t$, $l_1$ is the pipeline length. Having substitused expression (4.6) in formula (4.1) allowing for initial conditions similar to the solution of equations (3.2)-(3.12), (4.2),(4.3) and applying the Laplace transform, we get

$$\bar{Q}_2 = \frac{f_1 Q_2(0)}{\rho(s + 2a_3)} + \frac{f_1 \bar{P}_y}{\rho l_1 (s + 2a_3)} - \frac{f_1 \bar{P}_{outlet}}{\rho l_1 (s + 2a_3)} -$$
$$- \sum_{i=1}^{n} \frac{f_1 \pi i}{\rho l_1 (s + 2a_3)} \cos\left(\frac{i\pi x_1}{l_1}\right)\left(\bar{\Psi} - \frac{2s\bar{P}_y(s + 2a_3)}{\pi i (s - \xi_3)(s - \xi_4)}\right) \tag{4.7}$$

where

$$\bar{\Psi} = \frac{\psi_0 (s + 2a_3)}{(s - \xi_3)(s - \xi_4)} + \frac{\dot{\psi}_0}{(s - \xi_3)(s - \xi_4)} +$$
$$+ \frac{2P_y(0)(s + 2a_3)}{\pi(s - \xi_3)(s - \xi_4)} + \frac{2\dot{P}_y(0)}{\pi(s - \xi_3)(s - \xi_4)} - \frac{4a_3 c^2 G}{l_1 f_1 s (s - \xi_3)(s - \xi_4)} \sin\left(\frac{\pi l_2}{l_1}\right) \tag{4.8}$$

$\xi_3$ and $\xi_4$ are the roots of the equation $s^2 + 2a_3 s + \frac{c^2\pi^2 i^2}{l_1^2} = 0$ while $\psi_0, \ \dot{\psi}_0$ are found from the initial conditions (4.2) and (4.3) and equal $\psi_0 = 0, \quad \dot{\psi}_0 = 0$.

From the continuity condition on the well, allowing for expressions (3.12) and (4.7), we get

$$\left. \left( \frac{Q_1(0)}{s+2a} + \frac{\bar{P}_c}{l(s+2a)} - \frac{\bar{P}_y}{l(s+2a)} - \sum_{i=1}^{n} \bar{\varphi}_i \frac{\pi i}{l(s+2a)} \cos\left(\frac{i\pi x}{l}\right) \right) \right|_{x=l_1} =$$

$$\left( \frac{f_1 Q_2(0)}{\rho(s+2a_3)} + \frac{f_1 \bar{P}_y}{\rho l_1(s+2a_3)} - \frac{f_1 \bar{P}_{outlet}}{\rho l_1(s+2a_3)} - \right.$$

$$\left. \left. - \sum_{i=1}^{n} \frac{f_1 \pi i}{\rho l_1(s+2a_3)} \cos\left(\frac{i\pi x_1}{l_1}\right) \left( \bar{\Psi} - \frac{2s\bar{P}_y(s+2a_3)}{\pi i(s-\xi_3)(s-\xi_4)} \right) \right) \right|_{x_1=0} \tag{4.9}$$

$\bar{P}_{outlet}(t)$ is determined from formula (4.9).

$$\bar{P}_{outlet} = \bar{P}_y \left( \frac{3(fl_1 + f_1 l)(s-j_1)(s-j_2)s-j_3)(s-j_4)s-j_5)}{f_1 l(s+2a)(s-\xi_1)(s-\xi_2)(s-\xi_3)(s-\xi_4)} \right) -$$

$$- \frac{l_1 f \bar{P}_c(s+2a_3)}{l f_1(s+2a)} - \frac{f l_1 \pi (s+2a_3)}{l f_1 (s+2a)} \bar{\Phi} - \pi \bar{\Psi} \tag{4.10}$$

Taking into attention convolution and inversion theorems, from the expression (4.10) allowing for formulas (2.23) and (3.15) for the following numerical values of the parameters of the system

$c = c_1 = 1000 m \cdot c^{-1};\ \mu = 10^{-3} Pa \cdot c;\ h = 5m;\ k = 10^{-13} m^2;\ \rho = 860 kg \cdot m^{-3}$
$l = 2000m,\ l_1 = 20000m;\ P_c(0) = 24 \cdot 10^6 Pa;\ P_0 = 10^6 Pa;\ P_{wellhead}(0) = 3 \cdot 10^6 Pa;$
$P_k(0) = 27 \cdot 10^6 Pa; P_{atm} = 10^5 Pa;\ R_k = 100m;\ \pi = 3,14;\ a = 10^{-3} c^{-1},\ a_3 = 10^{-3} c^{-1};$
$m = 0.2;\ T = 90 days;\ P_{cT} = 120 Pa;\ d = 6 \cdot 10^{-2} m; d_1 = 20 \cdot 10^{-2} m;$
$r_c = 7.5 \cdot 10^{-2} m,$

we get

$$P_{outlet} = 2.4223 \cdot 10^7 - 28.324\ t + 9.3361 \cdot 10^{-8} t^2 - 1977.7852 \exp(-0.001\ t) \sin(0.156987\, t) -$$
$$-91929.9404 \exp(-0.001\, t) \cos(0.156987\, t) +$$
$$+91968.52273 \exp(-0.001\, t) \cos(1.81288\, t) +$$
$$+204.9836 \exp(-0.001\, t) \sin(1.81288\, t) -$$
$$-9.33599 \cdot 10^7 \exp(-0.001\, t) \cos(3.1399\, t) -$$
$$-2.97325 \cdot 10^5 \exp(-0.001\ t) \sin(3.1399\, t) \tag{4.11}$$

The results of numerical calculations are in Fig. 2 and 3. As can be seen from Fig. 2, connection of a new line to the main pipeline leads to strong pulse pressure at the outlet of the main line at the initial stage, then it damps and after certain time stabilizes and approaches to the initial value.

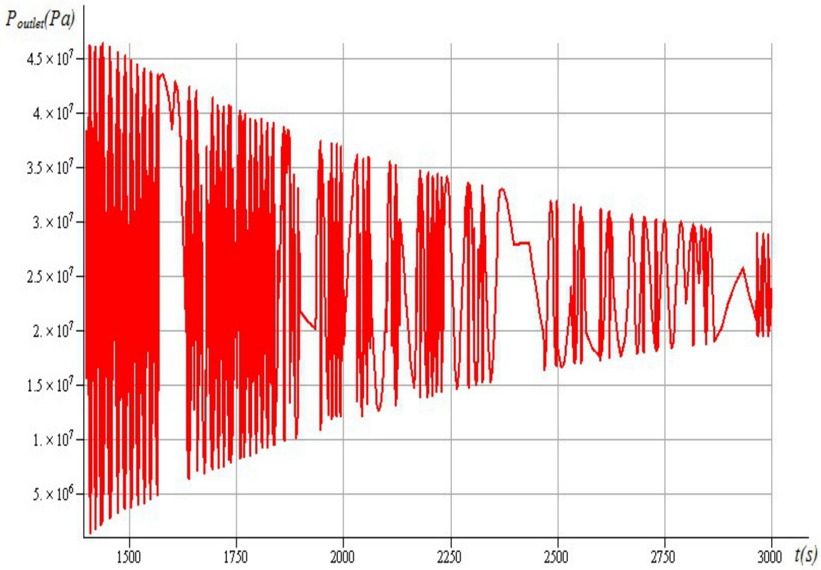


**Fig. 2.** Dynamics of pressure change at the outlet of the pipe, for $t$=3000 s.

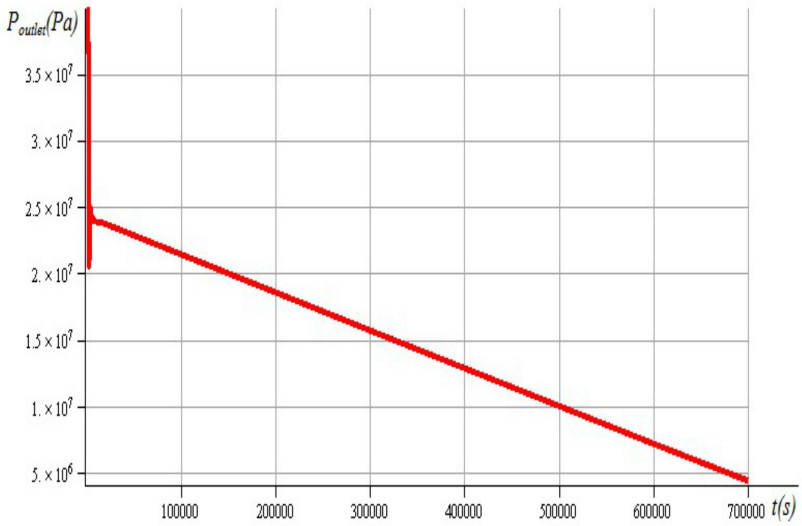


**Fig. 3.** Dynamics, of pressure change at the outlet of the pipe, for $t$=700000 s.

## 5 Conclusion

We constructed a model of unsteady flow of fluid in the coupled system of a “reservoir-pipeline” allowing for deformation of formation and connections to it. Analytic expression allowing to determine dynamics of pressure at the outlet of the main pipeline for the given law of pressure change at the bottomhole and deformation of formation skeleton that is of important applicational significance.

## 6 Denotation

$P$ is pressure at any point of the stratum, $MPa$;
$P_k$ is pressure on the stratum contour $MPa$;
$P_c$ is pressure at the well wall, $MPa$;
$k$ is efficient permeability, $m^2$;
$k_0$ is initial permeability on the contour, $m^2$;
$k_c$ is pore channel wall permeability $m^2$;
$\rho_j$ fluid density, $kg/m^3$;
$P_0$ is initial pressure $MPa$;

$R_c$ is radius of the stratum contour, $m$; $r$ is a coordinate, $m$; $r_w$ well's radius, $m$;

$T$ period of fluctuations in the stratum, $c$; $\tau, t$ is time, $A$; $\beta^*$ is a compressibility factor,$1/Pa$; $\chi$ is a piezoconductivity factor, $m^2/c$;

$\mu$ fluid's dynamic viscosity factor, $MPa \cdot c$; $h$ is stratum's power, $m$; $a$,$a_3$ is a resistance factor,$c^{-1}$; $\varphi_i$, $\varphi_{1i}$ is a time -dependent unknown function;

$f$ is the flow section of the piping string, $m^2$;

$f_1$ is the area of flow section of a transport pipeline , $m^2$; $a_1, a_2, b_1, b_2, A_1...A_{17}$ are denotations, $n = \nu = 1, 2, 3, ....$ are natural numbers. The indices: $*$ is an upper index,

0 is a lower index; $k$ is a contour; $c$ is a well.

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