Tran. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.
Mechanics, **40** (7), 37–41 (2020).

# On a method solving many-dimensional problems of mathematical physics

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Received: 27.11.2019 / Revised: 22.05.2020 / Accepted: 28.07.2020

**Abstract.** *A method for solving partial differential equations in the integral form with a kernel satisfying a wave equation, remaining unchangeable ( to a complex variable and multiplier) after Laplace- Fourier transformations, possessing interchangeable derivatives in all variables and by the same token allowing to reduce the number of variables in auxiliary equation, in suggested.*

**Keywords.** many dimensionality · partial equations · kernel · wave equation · Laplace-Fourier thatsform · analytic function.

**Mathematics Subject Classification (2010):** 35A25

## 1 Introduction.

One of the method for solving partial differential equations is an operator method based on transition from the sought-for function to its image [2]. This admits to decrease the number of independent variables in the auxiliary equation. But application of this method is connected with some difficulties consisting of finding of inverse transformations. Recently there has been found a class of functions remaining unchangeable ( to a complex variable and multiplier) after Laplace-Fourier transforms. Therewith they acquire some properties typical for transform of functions. The use of such functions allows to decrease the number of independent variables in the auxiliary equation to two and thereby to construct the solutions of an input equation with arbitrary number of unknowns. Consider the equality

$$\int_0^{k(t-y)} J_0\left(\sqrt{k^2t^2-(\tau+yk)^2}\right) f(\tau)\, d\tau$$
$$=\int_0^{k(t-y)} J_0\left(a\sqrt{(k-\tau b)^2-\tau^2c^2}\right) f(\tau)\, d\tau$$

where $J_0$ is the Bessel function, $k$ is the Fourier transform parameter with respect to $x$,

$$a=\sqrt{t^2-y^2};\ b=\frac{y}{t^2-y^2};\ c=\frac{t}{t^2-y^2};$$

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Application of the Laplace transform with respect to $t$ to the left, the inverse Fourier transform with respect to $x$ to the right side, reduce them to the Laplace integral [1]

$$\frac{e^{-y\sqrt{s^2+k^2}}}{\sqrt{s^2+k^2}}\int_0^\infty e^{-\tau\sqrt{\frac{s^2}{k^2}+1}}f(\tau)d\tau$$

$$\div\frac{1}{\sqrt{t^2-y^2-x^2}}\int_0^\infty e^{-\tau\frac{t\sqrt{t^2-y^2-x^2}+xyi}{t^2-y^2}}f\left(\tau\right)d\tau$$

Here $s$ is Laplace's transformation parameter, $t^2-y^2>x^2$. This relation is equivalent to the expression

$$\int_0^\infty\int_{-\infty}^\infty e^{-st+ikx}\frac{F_0\left(\theta\right)}{\sqrt{t^2-x^2-y^2}}dtdx=\frac{e^{-y\sqrt{s^2+k^2}}}{\sqrt{s^2+k^2}}F_0\left(\bar{\theta}\right) \tag{1.1}$$

$$\theta=\frac{t\sqrt{t^2-x^2-y^2}+xyi}{t^2-y^2} \tag{1.2}$$

$$\bar{\theta}=\sqrt{\frac{s^2}{k^2}+1} \tag{1.3}$$

$$F_0(\theta)=\int_0^\infty e^{-\tau\theta}f(\tau)d\tau \tag{1.4}$$

In order to study the behavior of the integrand function in expression (1.1) under differentiation (or integration), at first from relation (1.3) we define

$$ik=\frac{s}{\sqrt{1-\bar{\theta}^2}} \tag{1.5}$$

$$\sqrt{s^2+k^2}=\frac{i\bar{\theta}s}{\sqrt{1-\overline{\theta}^2}}. \tag{1.6}$$

Since the differentiation of the integrand function in (1.1) with respect to $x$ and $y$ reduces, to multiplication of the riht side by (1.5) and (1.6), respectively, then using (1.2) we determine expressions for $(1-\theta^2)^{-\frac{1}{2}}$ and $\theta(1-\theta^2)^{-\frac{1}{2}}$

$$\frac{1}{\sqrt{1-\theta^2}}=\frac{1}{x^2+y^2}(tx+iy\sqrt{t^2-x^2-y^2}) \tag{1.7}$$

$$\frac{\theta}{\sqrt{1-\theta^2}}=\frac{1}{x^2+y^2}(x\sqrt{t^2-x^2-y^2}+iyt). \tag{1.8}$$

Having substituted for simplicity in expressions (1.5) and (1.6) $\overline{\theta}=th\overline{p}$ and in expressions (1.7) and (1.8) $\theta=thp$, taking into account zero initial conditions and expression (1.1), we find formulas for determining the derivatives with respect to $x$ and $y$

$$\begin{cases}\frac{\partial^n\psi}{\partial x^n}=\frac{\partial^n}{\partial t^n}((-chp)^n\psi)\\ \frac{\partial^n\psi}{\partial y^n}=\frac{\partial^n}{\partial t^n}((ishp)^n\psi)\end{cases} \tag{1.9}$$

where

$$\psi=\frac{F_0(\theta)}{\sqrt{t^2-x^2-y^2}}. \tag{1.10}$$

It is easy to prove that formulas (1.9) are valid for $F_0(\theta) = \int_0^\infty e^{-\tau\theta}\delta(\tau)d\tau = 1$, where $\delta(\tau)$ is Dirac's function. Having solved equation (1.2), with respect to $x$, we get

$$x - iy\theta = t\sqrt{1-\theta^2}$$

Substitution of $\theta = thp$ gives

$$xchp - iyshp = t \tag{1.11}$$

Having accepted $x = r\cos\varphi$ and $y = r\sin\varphi$ we get

$$rch(p - i\varphi) = t.$$

Hence we find

$$p = \ln(\frac{t}{r} + \sqrt{\frac{t^2}{r^2} - 1}) + i\varphi = \ln\frac{z}{R^-} \tag{1.12}$$

where

$$R^{\pm} = t \pm \sqrt{t^2 - |z|^2}$$

$$z = x + iy, |z| = (x^2+y^2)^{\frac{1}{2}}, \varphi = \arg z = arctg\frac{y}{x} < 2\pi$$

Allowing for expression (1.12), the formulas for $chp$ and $shp$ in (1.9) take the form

$$\begin{cases} chp = \frac{1}{2}(\frac{z}{R^-} + \frac{R^-}{z}) \\ shp = \frac{1}{2}(\frac{z}{R^-} - \frac{R^-}{z}) \end{cases}$$

Differentiating expression (1.11) with respect to $t$, we find

$$\frac{\partial p}{\partial t} = \frac{1}{xshp - iychp} = \frac{1}{rsh(p - i\varphi)} = \frac{1}{\sqrt{t^2 - x^2 - y^2}}. \tag{1.13}$$

Allowing for expression (1.13) we can represent the integrand function in (1.1) in the form of a derivative of an analytic function with respect to parameter

$$\psi = \frac{F_0(\theta)}{\sqrt{t^2 - x^2 - y^2}} = \frac{\partial F(p)}{\partial t}.$$

For $t \leq (x^2+y^2)^{\frac{1}{2}}$ in formulas (1.9) hyperbolic formulas are substituted by appropriate trigonometric ones.

## 2 Main result

Analyzing (1.9) it is easy see that inter changeablity of derivaties may be used to decrease the number of independent variables. To this end we construct a function in the form

$$U(x_1, x_2, x_3) = \int_0^\infty U_0(x_3, \tau)\psi(p(x_1, x_2, \tau))d\tau \tag{2.1}$$

where $U_0(x_3, \tau)$ is an unknown function to be determined, $\psi(p(x_1, x_2, \tau))$ is a function determined from (1.10) by substitution of $x_1 = x$, $x_2 = y$, $\tau = t$. Besides, $\psi$ is a rather finite function

$$\left.\frac{\partial^n \psi}{\partial \tau^n}\right|_{\tau=0}^{\tau=\infty} = 0 \tag{2.2}$$

Having substituted (2.1) in the Laplace equation

$$\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_2^2} + \frac{\partial^2 U}{\partial x_3^2} = 0.$$

Allowing for expression (1.9) we get

$$\int \left( U_0 \frac{\partial^2 \psi}{\partial \tau^2} + \frac{\partial^2 U_0}{\partial x_3^2} \psi \right) d\tau = 0$$

Integrating by parts, allowing for expression (2.2) we get

$$\int (\frac{\partial^2 U_0}{\partial \tau^2} + \frac{\partial^2 U_0}{\partial x_3^2})\psi d\tau = 0$$

Having substituted (2.1) into the biharmonic equation

$$\Delta\Delta U = 0$$

and repeating the similar procedure, we get

$$\int \Bigg( U_0 \frac{\partial^4}{\partial \tau^4} \left(\psi(ch^4 p - 2ch^2 p \cdot sh^2 p + sh^4 p)\right)$$

$$+2\frac{\partial^2 U_0}{\partial x_3^2}\frac{\partial^2}{\partial \tau^2}\left(\psi\left(ch^2 p - sh^2 p\right)\right) + \frac{\partial^4 U_0}{\partial x_3^4}\psi \Bigg) d\tau$$

$$= \int \left( U_0 \frac{\partial^4 \psi}{\partial \tau^4} + 2\frac{\partial^2 U_0}{\partial x_3^2}\frac{\partial^2 \psi}{\partial \tau^2} + \frac{\partial^4 U_0}{\partial x_3^4}\psi \right) d\tau$$

$$= \int \left( \frac{\partial^4 U_0}{\partial \tau^4} + 2\frac{\partial^4 U_0}{\partial \tau^2 \partial x_3^2} + \frac{\partial^4 U_0}{\partial x_3^4} \right) \psi d\tau = 0$$

Consider now the function

$$U(x_1, x_2, x_3, x_4) = \int_0^\infty \int_0^\infty U_0(x_4, \tau_2) \cdot \psi_2(p_2(x_3, \tau_1, \tau_2))$$
$$\times \psi_1(p_1(x_1, x_2, \tau_1)) d\tau_1 d\tau_2, \tag{2.3}$$

where $U_0(x_4, \tau_2)$ is an unknown function to be determined, $\psi_1$ and $\psi_2$ are arbitrary functions determined also from expressions (1.4), (1.10), (1.11) by appropriate change of variables. Substitute (2.3) into the wave equation

$$L_c U = \Delta U - \frac{1}{c^2}\frac{\partial^2 U}{\partial x_4^2} = 0$$

and get

$$\int\int\left(U_0\psi_2\frac{\partial^2\psi_1}{\partial\tau_1^2} + U_0\frac{\partial^2}{\partial\tau_2^2}(\psi_2(ch^2 p_2))\psi_1 - \frac{1}{c^2}\frac{\partial^2 U_0}{\partial x_4^2}\psi_2\psi_1\right)d\tau_2 d\tau_1$$

$$= \int\int(U_0\frac{\partial_1^2}{\partial\tau_2^2}\left(\psi_2\left(-sh^2 p_2\right)\right)\psi_1$$

$$+U_0\frac{\partial^2}{\partial\tau_2^2}\left(\psi_2\left(ch^2 p_2\right)\right)\psi_1 - \frac{1}{c^2}\frac{\partial^2 U_0}{\partial x_4^2}\psi_2\psi_1\Bigg)d\tau_2 d\tau_1$$

$$= \int\int\left(U_0\frac{\partial^2\psi_2}{\partial\tau_2^2}\psi_1 - \frac{1}{c^2}\frac{\partial^2 U_0}{\partial x_4^2}\psi_2\psi_1\right)d\tau_2 d\tau_1$$

$$= \int\int\left(\frac{\partial^2 U_0}{\partial\tau_2^2} - \frac{1}{c^2}\frac{\partial^2 U_0}{\partial x_4^2}\right)\psi_2\psi_1 d\tau_2 d\tau_1 = 0$$

Having substituted expression (2.3) in the equation

$$L_{c_1}L_{c_2}U = 0$$

we get

$$\int\int\left(\frac{\partial^4 U_0}{\partial\tau_2^4} - \left(\frac{1}{c_1^2} + \frac{1}{c_2^2}\right)\frac{\partial^4 U_0}{\partial\tau_2^2\partial x_4^2} + \frac{1}{c_1^2}\frac{1}{c_2^2}\frac{\partial^4 U_0}{\partial x_4^4}\right)\psi_2\psi_1 d\tau_2 d\tau_1 = 0.$$

As seen from the solutions in all the cases many-dimensional problems of mathematical physics are reduced to one -dimensional problem

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