

New wave function and its new differential equation in quantum physics. The fuzzy logic and fuzzy sets theory of Lutfi Zadeh

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Abstract. *It is known the quantum mechanics is different from the classical mechanics. The reason is the principle of Heisenberg's Uncertainty in quantum mechanics. The uncertainty as principle of mechanics result in wave function in it, square of which is probability. Therefore principal stochasticity of microworld which noted Bohr in famous dispute with Einstein about a God playing dice results in the quantum logic. In frame this logic the Schrodinger cat is alive and dead the same time. The probability theory with its elementary events in its topological probability space cannot extend its definitions. Fuzzy logic and fuzzy sets of Zadeh with focal elements in topological possibility space can makes it.*

Keywords. wave function · Schrodinger wave equation · potential wall · simplex .

Mathematics Subject Classification (2010): 03B52

1 Introduction

Why are there two different mechanics, classical and quantum? May be the reason is hid in human perception itself. Wigner noted [8] that separation of our perception from the laws of a nature is no more than simplification and although we are convinced that it has a harmless character, to nevertheless forget about it does not follow. May be the time has come to see on problem of perception of man directly not afraiding be original. As mathematicians we perceive occur events in their number. Often seeing that the same event repeat many time we say it is ought in next time. But Hume's law or Hume's guillotine tell us from "is" does not follow "ought be" David Hume raised the is-ought problem in his "Treatise of Human Nature". We can present is-ought as probability vector P in complex space where length of P is number of all events ("is") and on the real axis there are occurred events ("ought"). All events, which were, are and will be on this real "ought" axis, are

known to only hypothetic Laplace's demon because everything that happens in the world has a cause and effect. Laplace's Determinizm has led to absolutization of classical mechanics. Laplace's demon could repeat words famous russian matematicians Ostrogradski that in the nature there are no uncertainties, they are for human due to his unknowledge all things in world. Einstein according with him say about probabilistic determinizm and local realizm in physics [2]. He doesn't accept Bohr's quantum theory and Heisenberg's Uncertainties in it saying, that a God doesn't play dice. He even gives a paradox in his hypothetic EPR experiment in result of it the entanglement [1]. But Bohr insists on principal stochasticity of microworld that is microworld is probabilistic not from our unknowledge bur it is principally [5]. The Aspect's experiment [5] for testing Bell's inequalities showed Bohr was right in dispute with Einstein. About probability we will going to say in mechanics of microworld.

2 Probability in known wave function and Zadeh's Possibility in new wave function

Frequently probability can be presented as the vector P (fig.1), where on axis x and y it is noted "No" and "Yes" events for $N = (a + b)$ objects.

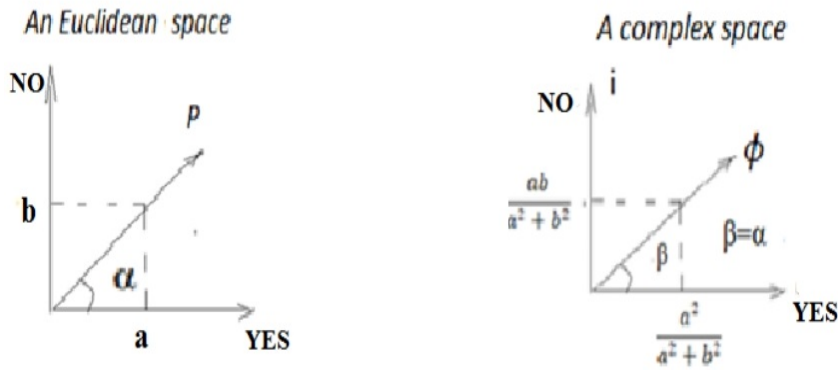


Fig. 1. Probability representation

$$P_+ = \frac{a}{a+b} \text{ (yes) , } P_- = \frac{b}{a+b} \text{ (not) and } P_+ + P_- = 1.$$

Thus

$$P_+ = \frac{1}{1 + tg\alpha} \text{ and } P_- = \frac{1}{1 + ctg\alpha}$$

But there is a new measure

$$M_+ = \cos^2\alpha = \frac{a^2}{a^2 + b^2} = \frac{a}{a - ib} \times \frac{a}{a + ib} = \phi \times \phi^* \text{ is called by us a new probability.}$$

The new wave function will be presented

$$\phi = \frac{a}{a - ib} = \frac{a^2 + iab}{a^2 + b^2} = A + iB \text{ as complex number,}$$

where $A = \frac{a^2}{a^2 + b^2}$ and $B = \frac{ab}{a^2 + b^2}$.

Here $M_- = \frac{b^2}{a^2 + b^2} = \sin^2\alpha$ and $M_+ + M_- = 1$ too as P .

In the quantum physics the wave function is complex number

$$\psi = Ce^{i\gamma}$$

where γ is the phase of wave and it is disappeared at multiplication ψ on conjugate

$$\psi^* = Ce^{-i\gamma}.$$

The mean of ψ is that it is the probability

$$P = \psi\psi^* = C^2 = \text{const.}$$

So, our new wave function

$$\varphi = A + iB = \cos\alpha (\cos\alpha + i\sin\alpha) = \cos\alpha e^{i\alpha}$$

contains itself the variable

$$C = \cos\alpha,$$

where phase α depends on “where” and “when”, that is

$$\alpha = \alpha(x, t).$$

We can find α , knowing that on axis x ($\alpha = 0$ and $P = 1$) all objects, figuratively speaking, in the right place at the right time, like a frozen world with its hard cause and effect relationships that is $x \equiv t$. In it there is a essence of Laplace’s determinism. For equality $x \equiv t$ let’s put coefficients p (impulse) and E (energy) in front of x and t that is $px = Et$. In the complex space the real axis x reflects the events of determine world with its classical physics. But there is unknowledge of human about all influences on given object and man sees in it randomness and use theory of probabilities. It means the Einstein’s local realism which isn’t suitable for Bohr’s principal stochasticity of microworld. Einstein’s local realizm is on real axis of complex space. Always $px = Et$ but due to human physical nonability ”seeing” all cause and effect relationships, due to we don’t know them all we use probability for prognosing future. But for Bohr it is principally that in the invisible by us microworld there is no Laplace’s determinism and therefore $px \neq Et$ and there is an angle or phase

$$a = \frac{px - Et}{h} \neq 0$$

where h is Planck’s constant. Let’s note from new probability

$$M_+ = \cos^2\alpha$$

appears the L.Zadeh’s measure of possibility $\Pi_+ = \cos\alpha$ [9]. Because of

$$\cos\alpha > \cos^2\alpha, \left(0 = \alpha = \frac{\pi}{2}\right),$$

$$\Pi_+ + \Pi_- > 1$$

where $\Pi_- = \sin\alpha$ is the possibility of opposite event.

Such situation takes place with the Schrodinger’s cat, which alive and dead in same time. Our new probability is

$$M = \varphi \times \varphi^* = \Pi e^{i\alpha} \times \Pi^{-i\alpha} = \Pi^2 = \cos^2\alpha.$$

Thus we have the new wave function

$$\varphi = \Pi e^{i\alpha} = \cos\alpha e^{i\alpha} = \frac{1}{2} + \frac{1}{2}e^{2i\alpha} = \frac{1}{2}\cos 2\alpha + \frac{1}{2}i \sin 2\alpha + \frac{1}{2}$$

for which there is a non homogenous differential equation second order

$$\frac{\partial^2}{\partial \alpha^2} \varphi + 4\varphi = 2.$$

We know $\alpha = \alpha(x, t) = \frac{px-Et}{h}$ and $\Delta = \frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial}{\partial x} = \left(\frac{\partial}{\partial \alpha} \frac{\partial \alpha}{\partial x} \right)^2 = \left(\frac{p}{h} \frac{\partial}{\partial \alpha} \right)^2 = \frac{p^2}{h^2} \frac{\partial^2}{\partial \alpha^2}$, but $\frac{p^2}{h^2} = \frac{m^2 v^2}{h^2} = \frac{2mE}{h}$. Therefore $\frac{\partial^2}{\partial \alpha^2} = \frac{h^2}{2mE} \Delta$. So we get the new non homogeneous differential equation

$$\frac{h^2}{2m} \Delta \pi + 4E\phi = 2E,$$

with decision

$$\phi = \cos \frac{px-Et}{h} e^{i \frac{px-Et}{h}} = \frac{1}{2} + \frac{1}{2} e^{2i \frac{px-Et}{h}}.$$

It looks like on Schrodinger equation $\frac{h^2}{2m} \Delta \psi - E\psi = 0$ for free particle with decision $\psi = A e^{i \frac{px-Et}{h}}$, where $A = \text{const.}$ If $A = \frac{1}{2}$ and phase of wave will be increased 2 time then $\phi = \psi + \frac{1}{2}$, that is Zadeh's possibility $\Pi^{i\alpha} = \phi$ more than wave function ψ in the quantum physics.

3 Superposition in quantum mechanics

It is known that by using his imaginal experiment "shrodinger cat" [1] Shrodinder proved that in the box the superposition of two microstate (atom decayed and atom didn't decay) transform in the superposition of two macrostate (cat is alive and cat is dead). In open system, observer outside, this paradox is solved by opening the box by him, it is called the "decoherence" [5]. But in the close system no one open the box and see the state of cat. Therefore there are many theories. One of such theory is many-world interpretation of quantum mechanics of Evverett-Willer [3]. Menski [6] says the objective real world existing in the form of the parallel worlds can be presented as volumetric figure. The classical world is one of parallel world and therefore it is illusion depending on consciousness of the observer. From the beginning of existence of the quantum mechanics the famous scientists Pauli, Wigner, Shredinger [1] said about the necessity of inclusion of the observer's consciousness in the quantum theory of measurements. Wigner papers [8] contains more stronger statement: the consciousness not only should be included in the theory of measurements but the consciousness may influence on the reality.

In this paper Menski's volumetric figure can be by a simplex from functional analyses [4].

4 Simplex

Simplex is very interesting figure in the functional analyse [4]. To create n -dimensional simplex it is necessary that $x_1, x_2, \dots, x_n$ points belong $(n-1)$ -dimensional space. For example, the three-dimensional simplex – the tetrahedron formed from x_1, x_2, x_3, x_4 - has 4 two-dimensional faces (triangles), 6 one-dimensional faces (pieces) and 4 zero-dimensional faces (points). In total the sum of the faces equals 14. They are simplex too.

Table 1.

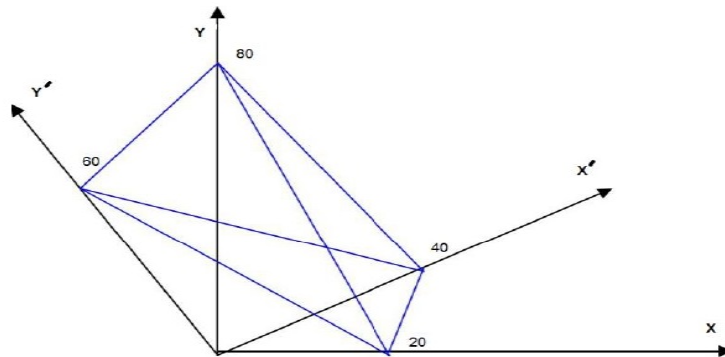
THE SIMPLEX					
0-dimensional	1-dimensional	2-dimensional	3-dimensional	4-dimensional	etc.
Point	Segment	Triangle	Tetrahedron	It isn't possible	
THE NUMBER OF FACES OF SIMPLEX					
1	2	6	14	30	

Let's note that a cube are not the simplex because for creation of the simplex from the 8 points it is necessary all these points are in the 6-dimensional space. Let's consider the 4-dimensional simplex. Here the number of the points are 5. All of them should not be located in the three-dimensional space. It is impossible to imagine such figure. This 4-dimensional simplex has 30 faces: 5 three- dimensional faces (tetrahedron), 10 two-dimensional faces (triangles), 10 one- dimensional faces (pieces) and 5 zero-dimensional faces (points). Thus the formed from more than four points simplex cannot be presented in our three-dimensional space. It is the complex volumetric figure (Table 1). The simplex in n -dimensional space is the minimal convex set, i.e. all points of a kind $\sum a_n x_n$, where $\sum a_n = 1$ belong to this simplex. It is very suitable figure for our arm.

From the theory of probabilities it is known that the probability of event is closely connected to random and average value .The belonging to a simplex points are the set of all average value if we count that the tops of the simplex $\{x_n\}$ are the random value and a_n are probabilities of x_n . In this point of view the consideration of a simplex is expedient. Also from the point of view of A.Einstein who wrote [2]: "We should refuse from the description of the nuclear phenomena as phenomena in the space and the time . We should go away from old mechanical review. The quantum physics formulates the laws which manager by set of particles but not single particle. It is described the probabilities and not the properties".

5 Construction of the simplex

So the above mentioned complex volumetric figure is the simplex.

**Fig. 2.** Simplex - Tetrahedron

Let's imagine that on two-coordinate plane xoy , on an axis $o\tilde{o}$ the number of dead "Schrodinger cats" and on an axis y the number of alive cats are marked. Let's suppose in experiment with 100 "Schrodinger cats" 80 cats are alive and 20 cats are dead. It is may be different numbers alive and dead cats from the total number all cats. But we consider numbers 100, 80, 20. In this case the probabilities are approximately equal 0,8 (alive cat) and 0,2 (dead cat). The points 20 and 80 are two tops of the simplex. In the other case or at other moment of time with 100 "Schrodinger cats" there are 60 alive cats and 40 dead cats. These points we put in other system of coordinates xoy in 3-dimensional space. If we connect given 4 points we receive three-dimensional simplex - tetrahedron. (fig.1). If we would consider a lot of points we shall receive the complex volumetric figure – n-dimensional simplex. Tetrahedral is the final simplex that we can represent in our three-dimensional space. The simplex of higher order have faces taken from these tetrahedron. The ribs of the tetrahedral indicate to various probabilities. For example, the rib linking the points 80 alive cats and 40 dead cats point out at $80/120 = 2/3$ of probability of case in that cat is alive. In the case 60 alive and 20 dead cats the rib of the simplex shows the probability that is equal $60/80 = 3/4$ and etc. The rib linking the points 20 dead and 40 dead cat and the rib linking the points 80 alive and 60 alive cats point out to probability of 1. All these ribs are the above vectors of state P from fig. 1. Let's consider the faces of the simplex. In the case of alive cat on one of them the probability changes from $2/3$ to 0,8 ; on another face – from $3/4$ to 0,6; on third - from $2/3$ up to 0,6; on fourth – from $3/4$ to 0,8 etc. So, these faces relate to events from fuzzy sets L. Zadeh.

6 The directing cosines and the probabilities

From the quantum mechanics [1] it is known the following. Let's assume that e is the energy eigenvalue of energy operator $\hat{E}$.

$$e = \sum C_k e_k$$

where e_k are eigenvectors (the basis),

C_k are coefficients and $\sum C_k e_k = 1$.

$$(e_k, e) = |e_k| * |e| * \cos\alpha,$$

where $\cos^2\alpha$ is the corresponding to e_k part of e .

From the physical point of view this means that if we measure the energy of the identical systems that are in the e state then the part of the number of E_k energy measurements equals $|\cos\alpha|$. Therefore $\cos\alpha$ is called directing cosine and it is the probability that the system have E_k energy in the state e .

It is known that for many operators $\cos\alpha$ is the complex number $\cos\alpha = x + iy = Ae^{i\beta}$ where $A = \text{const}$ is the length of the vector $\cos\alpha$. Then $\cos^2\alpha = x^2 + y^2$. Using the mentioned about new wave function, Zadeh's possibility and simplex, the above can be imagined and presented as the following. Let's assume that e_1 and e_2 are eigenvectors. E_1 and E_2 are the eigenvalue energy E (E_1 is energy of decayed atom, E_2 is energy of normal atom).

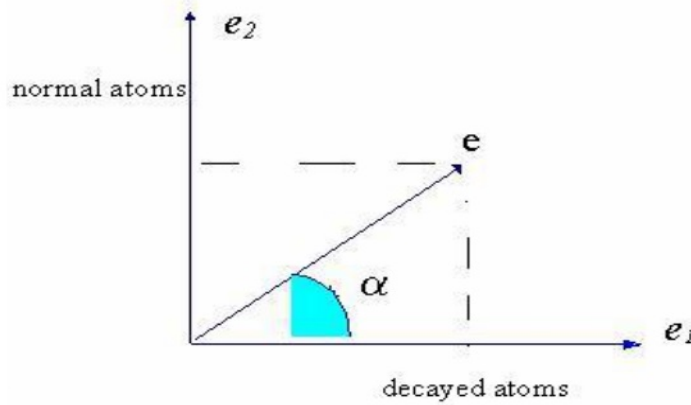


Fig.3. Directing cosine

The information about an amount of atoms with energy E_1 and E_2 are given us on the axis e_1 , e_2 , respectively. The exact information about the amounts of normal or decayed atoms are presented on the e_1 and e_2 axes. The statement of physical system and uncertainty of information are presented by vector e and angle α that is formed by rotation of e on the plane e_1oe_2 , respectively (fig.2). It is clear the statements of physical system can be pictured by unit rays. The rotation of unit vector was considered by Orlov [8] and called by him “the intention” of the quantum system. For Zadeh it would be “the possibility $\cos\alpha$ ” of the quantum world which in the classical world turn into probability $\cos^2\alpha$ with known $\cos\alpha > \cos^2\alpha$ that is the possibility is more than probability.

Thus angle α is the uncertainty measure. The vector e is non-eigenvector and one may be written in the form superposition of e_1 and e_2 .

$$e = C_1e_1 + C_2e_2.$$

The projections of e on e_1 and e_2 axes are the number of decayed and normal atoms, respectively and $\cos\alpha$ is the part of decayed atoms and $\sin\alpha$ ($\sin\alpha = \cos(90-\alpha)$) is part of normal of total number of all atoms. Let's assume that e_1 is a real axis and e_2 is a imaginary one of a complex number.

Let's consider the case which we used at constricting the simplex: from 100 “schrodinger cats” ($c = a + b = 100$) 80 cats are alive ($a = 80$) and 20 cats are dead ($b = 20$) (fig.4).

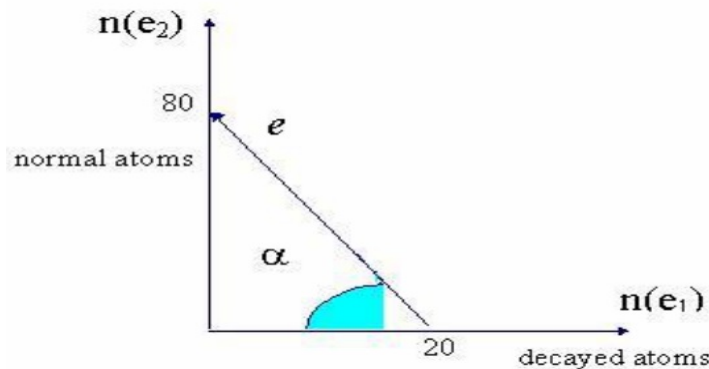


Fig.4. Rib of simplex as a state

So in the quantum mechanics the directing cosines can be presented not as $\cos\alpha = \alpha + ib$, but as $\cos\alpha = \frac{a}{a+ib} = \frac{a^2}{a^2+b^2} - i\frac{ab}{a^2+b^2}$. In this case the directing cosine is in fact the part of

the total amount. The ribs of our simplex as the directing cosines show us different states and probabilities.

7 The new wave function in a potential well

For the particle in a potential well

$$U(0) = U(L) = 0 \text{ and } \phi(0) = \phi(L) = 0$$

our new equation is

$$\ddot{\phi} + 8k^2 = 4k^2, \text{ where } k^2 = \frac{2m(E - U)}{h^2}.$$

The decision is

$$\begin{aligned} \phi(x) &= Ae^{2\sqrt{2} kxi} + Be^{-2\sqrt{2} kxi} + \frac{1}{2} = \\ &= (A + B) \cos 2\sqrt{2} kx + i(A - B) \sin 2\sqrt{2} kx + \frac{1}{2}. \end{aligned} \quad (7.1)$$

From the initial and boundary conditions

$$\phi(0) = (A + B) + \frac{1}{2} = 0$$

we find

$$(A + B) = -\frac{1}{2}.$$

Let's find $C' = (A - B)$ from

$$\begin{aligned} \phi(x) &= -\frac{1}{2} \cos 2\sqrt{2} kx + iC' \sin 2\sqrt{2} kx + \frac{1}{2} = \sin^2 \sqrt{2} kx + iC' \sin 2\sqrt{2} kx = \\ &= \sin \sqrt{2} kx \left(\sin \sqrt{2} kx + iC \cos \sqrt{2} kx \right), \text{ where } C = 2C'. \end{aligned}$$

We said above

$$\phi(L) = \sin \sqrt{2} kL \left(\sin \sqrt{2} kL + iC \cos \sqrt{2} kL \right) = 0.$$

Therefore

$$\sin \sqrt{2} kL = 0 \text{ and } k = \frac{\pi n}{\sqrt{2}L}.$$

Thus

$$\phi_n(x) = \sin^2 \sqrt{2} \frac{\pi n}{\sqrt{2}L} x + iC \sin 2\sqrt{2} \frac{\pi n}{\sqrt{2}L} x = \sin^2 \frac{\pi n}{L} x + iC \sin \frac{2\pi n}{L} x.$$

The conjugate $\phi_n^*(x) = \sin^2 \frac{\pi n}{L} x - iC \sin \frac{2\pi n}{L} x$.

We know the normalization condition

$$\int_0^L \|\phi\|^2 dx = \int_0^L \phi_n(x) \phi_n^*(x) dx = \int_0^L \left(\sin^4 \frac{\pi n}{L} x + C^2 \sin^2 \frac{2\pi n}{L} x \right) dx = 1.$$

There are ready formulae

$$\begin{aligned} \int \sin^2 y dy &= \frac{y}{2} - \frac{1}{4} \sin 2y \text{ and } \int \sin^4 y dy = -\frac{1}{4} \sin^3 y \cos y + \frac{3}{4} \int \sin^2 y dy = \\ &= -\frac{1}{4} \sin^3 y \cos y + \frac{3}{4} \left(\frac{y}{2} - \frac{1}{4} \sin 2y \right). \end{aligned}$$

If $\frac{2\pi n}{L}x = y$ and $dx = \frac{L}{2\pi n}dy$, then

$$\begin{aligned} \int_0^L \|\phi\|^2 dx &= C^2 \left(\frac{2\pi n}{2L}L - \frac{1}{4} \sin \frac{4\pi n}{2L}L \right) \frac{L}{2\pi n} - \sin^3 \frac{\pi n}{2L} L \cos \frac{\pi n}{2L} L + \\ &+ \left(\frac{3}{4} \frac{\pi n}{L}L - \frac{3}{16} \sin \frac{4\pi n}{L}L \right) \frac{L}{2\pi n} = C^2 (\pi n - 0) \frac{L}{2\pi n} + \left(\frac{3}{4} \pi n - 0 \right) \frac{L}{2\pi n} = \\ &= \frac{C^2 L}{2} + \frac{3}{8} L = \left(\frac{C^2}{2} + \frac{3}{8} \right) L = 1. \end{aligned}$$

So, $C^2 = \frac{8-3L}{4L}$ and $C' = A - B = \frac{C}{2} = \sqrt{\frac{8-3L}{8L}}$.

Thus for equation above (1) we get

$$\phi = -\frac{1}{2} \cos 2\sqrt{2} kx + i \sqrt{\frac{8-3L}{8L}} \sin 2\sqrt{2} kx + \frac{1}{2}.$$

From above $k = \frac{\pi n}{\sqrt{2}L}$ let's find

$$E_n = \frac{p^2}{2m} = \frac{(hk)^2}{2m} = \frac{\pi^2 h^2}{4mL^2} n^2.$$

Our new E_n are the quantum of energy too as in quantum mechanics $E_n = \frac{\pi^2 h^2}{2mL^2} n^2$.

But unlike the wave function there $\psi_n = \sqrt{\frac{2}{L}} i \sin \frac{\pi n}{L} x$ our wave function is

$$\phi_n = \sin^2 \frac{\pi n}{L} x + \sqrt{\frac{8-3L}{8L}} i \sin \frac{2\pi n}{L} x,$$

which contains real part $\sin^2 \frac{\pi n}{L} x$ of complex number that means its join with reality.

Here we can say about dispute Albert Einstein with Nils Bohr. Bohr said that in microworld our measurement itself open result. Really the wave function for him $\psi_n = \sqrt{\frac{2}{L}} i \sin \frac{\pi n}{L} x$ is only virtual. But Einstein answered him reason uncertainty in microworld is our unknowledge. Really the wave function ϕ_n consists real part $\sin^2 \frac{\pi n}{L} x$.

In the quantum mechanics the probability is

$$P = \int_0^L \psi_n \psi_n^* = \frac{2}{L} \sin^2 \frac{\pi n}{L} x,$$

but our new probability is

$$\begin{aligned} M &= \int_0^L \phi_n \phi_n^* = \sin^4 \frac{\pi n}{L} x + \frac{8-3L}{8L} \sin^2 \frac{2\pi n}{L} x = \\ &= \left(\sin^4 \frac{\pi n}{L} x - \frac{3}{8} \sin^2 \frac{\pi n}{L} x \right) + \frac{1}{L} \sin^2 \frac{2\pi n}{L} x, \end{aligned}$$

in which new member $(\sin^4 \frac{\pi n}{L} x - \frac{3}{8} \sin^2 \frac{\pi n}{L} x)$ is added and if it is more than 0 then new probability become possibility and that less than 0 then that become necessary from Zadeh's fuzzy sets.

8 Simplex interpretation of quantum physics

So a point is zero-dimensional, a line is one-dimensional, a triangle is bi-dimensional, a tetrahedron a three-dimensional simplex. The three-dimensional simplex, a tetrahedron, has 4 bi-dimensional sides (triangles), 6 one-dimensional sides (lines) and 4 zero dimensional sides (points), giving a total of 14 sides. It is impossible to imagine a four-dimensional simplex in our three-dimensional space. The parallelepiped or cube is not a simplex because for this purpose it is necessary that all 8 points were in six measured space. Thus, formed from more than four points, is a complex volumetric figure. In our experiment with 100 Schrodinger cats, 80 cats are alive and 20 are dead. Points 20 and 80 are two ends of a simplex. At other moment of time or in another experiment let's assume from 100 cats that 60 are alive and 40 are dead. These two points are also ends of a simplex. We can continue our tests, but we shall stop with these two, and thus, we consider a three-dimensional simplex – a tetrahedron. The ribs of our tetrahedron indicate various probabilities and angles α . For example, the rib linking the points 80 live cats and 40 dead cats give $80/120=2/3$ of probability with angle α ($\text{tg } \alpha = 2$) of the case in which a cat is alive. In the case 60 live and 20 dead cats, the rib of the simplex shows that the probability is $60/80=3/4$ with angle α ($\text{tg } \alpha = 3$), etc. These ribs have determined angles and show on stochastic event. As to points of a tetrahedron they specify determinism of an event. For example, the point of 80 live cats specifies that in fact all 80 cats are alive. The rib linking the points 20 dead and 40 dead cats gives a probability of 0 and angle 0. The rib linking the points 80 live and 60 live cats gives a probability of 1 with angle $\alpha = 90$. These latest two rib show also on the determinism of events. Let's consider the faces of the simplex. In the case of a live cat on one of them the probability changes from $2/3$ to 0.8; on another face, from $3/4$ to 0.6; on third face, from $2/3$ to 0.6; on fourth, from $3/4$ to 0.8 etc. On all these faces angle α has not determined value but it is changed. One event belonging to the face can have many value of probability. It is already principal stochastic of event. Its principality not due to acknowledge but itself nature. Clear in this case there will be not probability, but Zadeh's possibility.

We could construct the simplex with various probabilistic ribs and sides because we are observers from outside. In this case we built a physical picture of the real world. Only in this world is the probabilistic interpretation of the quantum mechanics given by Bohr true. In a physical picture of the sentient world, we cannot construct a simplex. We can only perceptions as projections, i.e. sides of a simplex. After that, classical probability is applied, but it is applied, we shall repeat, not for a whole simplex, but only for one of its sides. This side, perceived by us as the sentient world, is an illusion because it not unique: there exists a set of worlds alternative to it. With a physical picture of the world, we can even count the number of parallel worlds. As our world is three-dimensional and our consciousness exists in it we can count only sides of a three-dimensional simplex — a tetrahedron, which, as shown above, has only 14 sides. Returning now to the dispute between Einstein and Bohr, in the real and sentient worlds, of course Einstein was right — really, God does not play dice. However, in the physical picture of the world, Bohr had the right to apply probability and statistics. Usually in a game of dice we mean only the act of throwing dice. However, dice consists of acts before (we build forecasts) and after (realization of one forecast from possible results). This situation can be likened to a court case; there is a hearing of a case, a verdict and a process after the verdict. In the physical picture of the real world, a game of dice by consciousness is a game up to the act of throwing the dice. Our consciousness can only imagine all sides of a three-dimensional simplex, i.e. all alternative results. But the choice of one of them depends on “active” consciousness. In our sentient world, in the act of throwing the dice, we shall see this choice. In the physical picture of the sentient world, a game of dice by consciousness is a game after the act of throwing the dice. Having these outcomes allow us to statistically forecast. Thus, uncertainty of the real world qualitatively differs from uncertainty of the sentient world. Thus, uncertainty of the sentient world is

not present and, as a matter of fact, the finding of the probability of some casual event has no connection with uncertainty because this probability exists beforehand, a priori, and by doing a series of tests we simply find it. It becomes clear then why quantum statistics essentially differs from the classical. This simplex, with various probabilistic ribs and sides, we could construct with the help of epistemological analysis.

Knowledge which was analyzed in this case is knowledge of active consciousness. In the case when the simplex from a volumetric figure is converted into one of its projections, we see only one of its sides (a point, a line, a triangle). Knowledge appropriate to this case is knowledge of passive consciousness. In a simplex the lines (80, 20) and (60, 40) where points 80, 60 are live, and 20, 40 are dead cats, correspond to *usual* knowledge. In this case we use classical statistics (after we have looked in the box, Schrodinger's cats became simple cats, and we already have data, for example, from 100 cats in one case 80 alive, and in the other case 6, etc.). With the help of this data we find an average and dispersion of a random variable. But when the ensemble consists not of simple cats, but Schrodinger cats we deal with a microcosm, with a world, the perception of which, is multiple-valued. In this case, for example, the point 80 is already fixed simultaneously and with the point 20, and with the point 40. Therefore the triangle (20, 80, 40) is examined. Similarly, the triangle (40, 60, 20) is also considered. These triangles correspond to *unusual* knowledge. In this case we cannot apply classical statistics. Therefore we use quantum statistics. There is a question: But what in a simplex will correspond to *transcendental* and *transient* knowledge? We can answer that transcendental knowledge is knowledge of active consciousness in the case of a macrocosm, and corresponds to the entire simplex. Transcendental knowledge can be acquired by us a priori (because we could construct the simplex), but for transient knowledge this is not possible. Knowledge of active consciousness appropriate to transition from a microcosm to macrocosm, i.e. to our world, will be transcendental, and from a microcosm to a microcosm it will be transient. There is no sharp border between macro-world and microcosms, but in fact there is a sharp border between knowledge about them.

9 Conclusion

It is difficult to answer on these old questions of quantum mechanics. But it is necessary to do because without good theory and interpretation we can't understand the Universe. It is necessary for mathematician too. For mathematics there was and is an old, understood by all, a classical mechanics. But new quantum mechanics bring us understanding that in mechanics itself there will be many problem and one of them is the phenomena of observer.

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