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The influence of the magnitude of initial inhomogeneous stresses in the hollow cylinder immersed in fluid on the ZGV modes

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Abstract. *The work deals with the investigation of the influence of the magnitude of the inhomogeneous initial stresses in the hollow cylinder immersed in the compressible, inviscid fluid on the frequency and the dimensionless wave number at which the ZGV points occur. This study is carried out for the first lowest mode in the case where the material of the cylinder is Lucite and the fluid is water.*

Keywords. Axisymmetric waves · ZGV modes · initial inhomogeneous stresses · inviscid fluid · immersed hollow cylinder · hydrostatic pressure · wave dispersion

Mathematics Subject Classification (2010): 74F10

1 Introduction

In the works [1, 2], it was found that in the case where the hollow cylinder has axisymmetric inhomogeneous initial stresses caused by the hydrostatic pressure on the outer surface of the cylinder, the axisymmetric waves may have the zero group velocity (ZGV) modes (or points). This means that there are such values of wave frequency and length at which the group velocity is zero under the non-zero wave number. The question of how the magnitude of the initial inhomogeneous stresses in the cylinder affects the frequency and velocity of the ZGV points was studied in [1] only for the case where the material of the cylinder is steel. In the present investigation, this study is developed for the case where the material of the cylinder is Lucite (Plexiglas) and the fluid is water.

2 Formulation of the problem and on the solution method

We study wave propagation in a hydro-elastic system consisting of a hollow cylinder immersed in a compressible, barotropic, inviscid fluid. We determine the position of the points of the cylinder and the fluid in the cylindrical coordinate system of $Or\theta z$ using the Lagrange

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and Euler coordinates in this system. We assume that this coordinate system is connected to the central axis of the cylinder, which coincides with the Oz axis, and that the length of the cylinder is infinite. We distinguish between two states of the hydro-elastic system, which we call the initial state and the perturbed state. Furthermore, we assume that in the initial state (before wave propagation) a fluid with density ρ_0 is at rest and the hydrostatic pressure p in this fluid acts on the outer surface of the cylinder and this pressure causes the inhomogeneous initial stresses in the cylinder. The expressions of the initial stresses, according to the monograph [3], are determined by the following expressions:

$$\sigma_{rr}^0 = \frac{(1+h/R)^2 p}{(1+h/R)^2 - 1}\left(\frac{R^2}{r^2} - 1\right),$$

$$\sigma_{\theta\theta}^0 = -\frac{(1+h/R)^2 p}{(1+h/R)^2 - 1}\left(\frac{R^2}{r^2} + 1\right), \sigma_{zz}^0 = \nu(\sigma_{rr}^0 + \sigma_{\theta\theta}^0), \tag{2.1}$$

where h is the cylinder's thickness, R is the inner radius of the cylinder's cross section.

We assume that the waves propagate in the system under consideration after appearing in that the aforementioned initial stresses. The problem is to determine how the mentioned inhomogeneous initial stresses influence the dispersion of these waves. To determine this influence, we use the exact linearized 3D equations and relations of the theory of elastic waves in bodies with initial stresses and the linearized equations of the flow of barotropic inviscid fluids.

These equations for the cylinder are:

$$\frac{\partial t_{rr}}{\partial r} + \frac{\partial t_{zr}}{\partial z} + \frac{1}{r}(t_{rr} - t_{\theta\theta}) = \rho\frac{\partial^2 u_r}{\partial t^2}, \frac{\partial t_{rz}}{\partial r} + \frac{1}{r}t_{rz} + \frac{\partial t_{zz}}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2}, \tag{2.2}$$

where

$$t_{rr} = \sigma_{rr} + \sigma_{rr}^0(r)\frac{\partial u_r}{\partial r}, t_{rz} = \sigma_{rz} + \sigma_{rr}^0(r)\frac{\partial u_z}{\partial r}, t_{\theta\theta} = \sigma_{\theta\theta} + \sigma_{\theta\theta}^0(r)\frac{u_r}{r},$$

$$t_{zr} = \sigma_{zr} + \sigma_{zz}^0(r)\frac{\partial u_r}{\partial z}, t_{zz} = \sigma_{zz} + \sigma_{zz}^0(r)\frac{\partial u_z}{\partial z}. \tag{2.3}$$

The elasticity relations:

$$\sigma_{rr} = \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{rr}, \sigma_{\theta\theta} = \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{\theta\theta},$$

$$\sigma_{zz} = \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{zz}, \sigma_{rz} = 2\mu\varepsilon_{rz}. \tag{2.4}$$

The strain-displacement relations:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \varepsilon_{\theta\theta} = \frac{u_r}{r}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right). \tag{2.5}$$

Thus, equations (2.2) – (2.5) compose the complete system of the linearized field equations of the motion of the cylinder. Note that in (2.2) and (2.3), the notation t_{rr}, t_{rz}, $t_{\theta\theta}$, t_{zr} and t_{zz} shows the components of the non-symmetric Kirchhoff stress tensor. The other notation used in (2.2) – (2.5) is conventional.

According to [4], we now write the linearized field equations for the barotropic compressible inviscid fluid flow.

The continuity equation:

$$\frac{\partial \rho'}{\partial t} + \rho_0\left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right) = 0. \tag{2.6}$$

Linearized Euler equations:

$$\frac{\partial V_r}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial r}, \frac{\partial V_z}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z}. \tag{2.7}$$

The linearized state equation:

$$p' = a_0^2 \rho', a_0^2 = \frac{\partial p'}{\partial \rho'}, \tag{2.8}$$

where a_0 is the sound speed in the fluid, ρ_0 is the density of the fluid in the initial state, p' and ρ' are perturbations of the pressure and density in the fluid, V_r and V_z are the perturbations of the velocity vector of the fluid.

Consider the formulation of the boundary conditions on the inner surface and compatibility conditions on the outer surface of the cylinder. For the case considered here, which can be formulated as follows:

$$t_{rr}|_{r=R} = 0, \, t_{rz}|_{r=R} = 0. \tag{2.9}$$

The compatibility conditions between the fluid and cylinder are:

$$t_{rr}|_{r=R+h} = -p', \, t_{rz}|_{r=R+h} = 0, \, \left.\frac{\partial u_r}{\partial t}\right|_{r=R+h} = V_r|_{r=R+h}\,. \tag{2.10}$$

The condition on boundedness of the quantities related to the fluid at the central axis of the cylinder is:

$$\left\{\left|p'\right|, \left|\rho'\right|, \left|V_r\right|, \left|V_z\right|\right\}\Big|_{r\to\infty} < \infty. \tag{2.11}$$

This completes the mathematical formulation of the problem under consideration.

For the solution of the problem, all the sought values are presented as $g(r,z,t) = \bar{g}(r)e^{i(kz-\omega t)}$. If we substitute this representation into the above field equations, we obtain the complete system of ordinary differential equations for these amplitudes. If we solve the equations with respect to the cylinder, we obtain the system of ordinary differential equations with variable coefficients. To solve this system we use the discrete analytical method. This method for the problem under consideration was developed in the papers [2, 3] and other papers cited therein.

The essence of this method is based on the following idea. The region $[R, R+h]$ occupied by cylinder is divided into a number of N sub-regions (or sublayers). The thickness of these sublayers is h/N and in the $n-th$ sublayer the relation $(R+(n-1)h/N) \leq r \leq (R+nh/R)$ holds, where $1 \leq n \leq N$. Within the framework of this notation, the inhomogeneous initial stresses determined by the expressions in (2.1) in each of the aforementioned sublayers are assumed to be constants defined by the following relations:

$$\sigma_{rr}^0(r) \approx \sigma_{rr}^0(r_n), \sigma_{\theta\theta}^0(r) \approx \sigma_{\theta\theta}^0(r_n), \sigma_{zz}^0(r) \approx \sigma_{zz}^0(r_n),$$
$$r_n = R + (n-1)h/N + h/(2N). \tag{2.12}$$

After the above preparations, we solve the equations resulting from equations (2.2) - (2.5) within the framework of the above assumptions. We take into account that during the mathematical transformations, the equations in (2.4) and (2.5) remain valid and unchanged in each sublayer. Thus, taking into account the above statements and the relations in (2.12), we obtain from equations (2.2) and (2.3) the following equations of motion, which are satisfied separately in each sublayer:

$$\frac{\partial \sigma_{rr}^n}{\partial r} + \sigma_{rr}^0(r_n)\frac{\partial^2 u_r^n}{\partial r^2} + \frac{\partial \sigma_{zr}^n}{\partial z} + \sigma_{zz}^0(r_n)\frac{\partial^2 u_r^n}{\partial z^2} + \frac{1}{r}(\sigma_{rr}^n - \sigma_{\theta\theta}^n) +$$

$$+\sigma_{rr}^0(r_n)\frac{1}{r}\frac{\partial u_r^n}{\partial r}-\sigma_{\theta\theta}^0(r_n)\frac{u_r^n}{r^2}=\rho\frac{\partial^2 u_r^n}{\partial t^2},$$

$$\frac{\partial\sigma_{rz}^n}{\partial r}+\sigma_{rr}^0(r_n)\frac{\partial^2 u_z^n}{\partial r^2}+\frac{1}{r}\sigma_{rz}^n+\sigma_{rr}^0(r_n)\frac{1}{r}\frac{\partial u_z^n}{\partial r}+\frac{\partial\sigma_{zz}^n}{\partial z}+\sigma_{zz}^0(r_n)\frac{\partial^2 u_z^n}{\partial z^2}=\rho\frac{\partial^2 u_z^n}{\partial t^2}. \quad (2.13)$$

In this way, the solution to the systems of equations with variable coefficients is reduced to the solution of the series of equations with constant coefficients that can be solved analytically. To solve this system of equations we use the classical Lame decomposition, which for axisymmetric problems can be presented as follows:

$$u_r^n=\frac{\partial\Phi^n}{\partial r}+\frac{\partial^2\Psi^n}{\partial r\partial z},u_z^n=\frac{\partial\Phi^n}{\partial z}-\frac{\partial^2\Psi^n}{\partial r^2}-\frac{\partial\Psi^n}{r\partial r}. \quad (2.14)$$

Performing some awkward mathematical manipulations, we find that the potentials $\Phi^{(n)}$ and $\Psi^{(n)}$ must satisfy the following equations:

$$(1+\frac{\sigma_{rr}^0(r_n)}{\lambda+2\mu})\frac{\partial^2\Phi^n}{\partial r^2}+(1+\frac{\sigma_{\theta\theta}^0(r_n)}{\lambda+2\mu})\frac{\partial\Phi^n}{r\partial r}+(1+\frac{\sigma_{zz}^0(r_n)}{\lambda+2\mu})\frac{\partial^2\Phi^{n_i}}{\partial z^2}=\frac{1}{(c_1)^2}\frac{\partial^2\Phi^n}{\partial t^2},$$

$$(1+\frac{\sigma_{rr}^0(r_n)}{\mu})\frac{\partial^2\Psi^n}{\partial r^2}+(1+\frac{\sigma_{\theta\theta}^0(r_n)}{\mu})\frac{\partial\Psi^n}{r\partial r}+(1+\frac{\sigma_{zz}^0(r_n)}{\mu})\frac{\partial^2\Psi^n}{\partial z^2}=\frac{1}{(c_2)^2}\frac{\partial^2\Psi^n}{\partial t^2}, \quad (2.15)$$

where $c_1=\sqrt{(\lambda+2\mu)/\rho}$ and $c_2=\sqrt{\mu/\rho}$ are the speeds of the dilatation and distortion wave propagation velocities, respectively, for the cylinder's material.

Representing the functions Φ^n, u_r^n, ρ', $\sigma_{\theta\theta}^n$ with the multiplying $\sin(kz-\omega t)$ and the functions Ψ^n, u_z^n and σ_{rz}^n with the multiplying $\cos(kz-\omega t)$ and denoting the amplitudes of these quantities with the same symbols, we obtain the following equations for the amplitudes of the potentials Φ^n and Ψ^n:

$$\frac{d^2\Phi^n}{d(r_2)^2}+\frac{\alpha_1(r_n)}{r_2}\frac{d\Phi^n}{dr_2}+\Phi^n=0,\frac{d^2\Psi^n}{d(r_1)^2}+\frac{\alpha(r_n)}{r_1}\frac{d\Psi^n}{dr_1}+\Psi^n=0. \quad (2.16)$$

where

$$\alpha(r_n)=\frac{1+\sigma_{\theta\theta}^0(r_n)/\mu}{1+\sigma_{rr}^0(r_n)/\mu},\beta(r_n)=\frac{1+\sigma_{zz}^0(r_n)/\mu}{1+\sigma_{rr}^0(r_n)/\mu},c=\omega/\kappa,$$

$$r_1^n=kr\sqrt{\frac{c^2}{(c_2)^2(1+\sigma_{rr}^0(r_n)/\mu)}-(\beta(r_n))^2},\alpha_1(r_n)=\frac{1+\sigma_{\theta\theta}^0(r_n)/(\lambda+2\mu)}{1+\sigma_{rr}^0(r_n)/(\lambda+2\mu)},$$

$$\beta_1(r_n)=\frac{1+\sigma_{zz}^0(r_n)/(\lambda+2\mu)}{1+\sigma_{rr}^0(r_n)/(\lambda+2\mu)},$$

$$r_2^n=kr\sqrt{\frac{c^2}{(c_1)^2(1+\sigma_{rr}^0(r_n)/(\lambda+2\mu))}-(\beta_1(r_n))^2}. \quad (2.17)$$

According to [2, 3], we use the following presentations.

$$\Phi^n(r_2)=(r_2)^{(1-\alpha_1(r_n))/2}\Phi_1^n(r_2),\ \Psi^n(r_1)=(r_1)^{(1-\alpha(r_n))/2}\Psi_1^n(r_1), \quad (2.18)$$

Substituting expressions in (2.18) in equations in (2.16) we obtain the following equations with respect to the unknown functions $\Phi_1^n(r_2)$ and $\Psi_1^n(r_1)$:

$$\frac{d^2\Phi_1^{(i)n_i}}{d(r_2^{(i)})^2}+\frac{1}{r_2^{(i)}}\frac{d\Phi_1^{(i)n_i}}{dr_2^{(i)}}+(1-\frac{(1-(\alpha_1^{(i)}(r_{n_i})))^2/4}{(r_2^{(i)})^2})\Phi_1^{(i)n_i}=0,$$

$$\frac{d^2\mathit{\Psi}_1^{(i)n_i}}{d(r_1^{(i)})^2}+\frac{1}{r_1^{(i)}}\frac{d\mathit{\Psi}_1^{(i)n_i}}{dr_1^{(i)}}+(1-\frac{(1-(\alpha^{(i)}(r_{n_i})))^2/4}{(r_1^{(i)})^2})\,\mathit{\Psi}_1^{(i)n_i}=0, \tag{2.19}$$

Thus, from (2.18) and (2.19), we obtain the following expressions for the potentials $\mathit{\Phi}^n(r_2)$ and $\mathit{\Psi}^n(r_1)$:

$$\mathit{\Phi}^n=A_1^n E_{\gamma_1(r_n)}(r_2^{n_i})(r_2)^{\gamma_1(r_n)}+A_2^n F_{\gamma_1(r_n)}(r_2^n)(r_2)^{\gamma_1(r_n)},$$

$$\mathit{\Psi}^n=B_1^n E_{\gamma(r_n)}(r_1^n)(r_1)^{\gamma(r_n)}+B_2^n F_{\gamma(r_n)}(r_1^n)(r_1)^{\gamma(r_n)}, \tag{2.20}$$

where

$$E_{\gamma_1(r_n)}(r_2^n)=\begin{cases} J_{\gamma_1(r_n)}(r_2^n) \text{ if } (r_2^n)^2/r^2>0 \\ I_{\gamma_1(r_n)}(r_2^n) \text{ if } (r_2^n)^2/r^2<0 \end{cases},\gamma_1(r_n)=(1-\alpha_1(r_n))/2$$

$$F_{\gamma_1(r_n)}(r_2^n)=\begin{cases} Y_{\gamma_1(r_n)}(r_2^n) \text{ if } (r_2^n)^2/r^2>0 \\ K_{\gamma_1(r_n)}(r_2^n) \text{ if } (r_2^n)^2/r^2<0 \end{cases},\gamma(r_n)=(1-\alpha(r_n))/2$$

$$E_{\gamma(r_n)}(r_1^n)=\begin{cases} J_{\gamma(r_n)}(r_1^n) \text{ if } (r_1^n)^2/r^2>0 \\ I_{\gamma(r_n)}(r_1^n) \text{ if } (r_1^n)^2/r^2<0 \end{cases}$$

$$F_{\gamma(r_n)}(r_1^{n_i})=\begin{cases} Y_{\gamma(r_{n_i})}(r_1^n) \text{ if } (r_1^n)^2/r^2>0 \\ K_{\gamma(r_n)}(r_1^n) \text{ if } (r_1^{n_i})^2/r^2<0 \end{cases}. \tag{2.21}$$

In (2.21), $J_\delta(x)$ and $I_\delta(x)$ are the Bessel and modified Bessel functions of the first kind, $Y_\delta(x)$ and $K_\delta(x)$ are also the Bessel and modified Bessel functions of the second kind, A_1^n, A_2^n, B_1^n,and $B_2^n$are unknown constants.

Thus, substituting the expressions in (2.21) and (2.20) into the presentations in (2.14) and using the relations (2.4) and (2.5) we determine all the sought values related to each sublayer of the cylinder.

For finding the solutions to the field equations (2.6) -8), according to [4], we employ the following presentations:

$$\rho'=-a_0^{-2}\rho_0\frac{\partial}{\partial t}\mathit{\Phi}_f, p'=-\rho_0\frac{\partial}{\partial t}\mathit{\Phi}_f, V_r=\frac{\partial}{\partial r}\mathit{\Phi}_f, V_z=\frac{\partial}{\partial z}\mathit{\Phi}_f, \tag{2.22}$$

where the function $\mathit{\Phi}_f$ satisfies the equation

$$\left[\mathit{\Delta}-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\mathit{\Phi}_f=0, \mathit{\Delta}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}. \tag{2.23}$$

Representing the function $\mathit{\Phi}_f$as $\mathit{\Phi}_f=\Phi_{f1}(r)\cos(kz-\omega t)$ we obtain from (2.23) the following expression for the function $\mathit{\Phi}_{f1}(r)$:

$$\mathit{\Phi}_{f1}(r)=\begin{cases} FY_0(r_3)\ if\ r_3^2>0 \\ FK_0(r_3)\ if\ r_3^2<0 \end{cases} \tag{2.24}$$

where $Y_0(r_3)$ ($K_0(r_3)$) is the first kind of Bessel (modified Bessel) function of the zeroth order and F is an unknown constant. Substituting this expression in (2.22) we determine the following expressions for the sought values related to the fluid flow.

$$p'=\rho_0\omega\sin(kz-\omega t)\begin{cases} FJ_0(r_3)\ if\ r_3^2>0 \\ FI_0(r_3)\ if\ r_3^2<0 \end{cases},$$

$$\rho'=a_0^{-2}\rho_0\omega\sin(kz-\omega t)\begin{cases} FJ_0(r_3)\ if\ r_3^2>0 \\ FI_0(r_3)\ if\ r_3^2<0 \end{cases},$$

$$V_r = k\frac{dr_3}{dr}\cos(kz-\omega t)\begin{cases} -FJ_1(r_3)\ if\ r_3^2>0 \\ FI_1(r_3)\ if\ r_3^2<0 \end{cases}$$

$$,V_z = -k\sin(kz-\omega t)\begin{cases} FJ_0(r_3)\ if\ r_3^2>0 \\ FI_0(r_3)\ if\ r_3^2<0 \end{cases}. \tag{2.25}$$

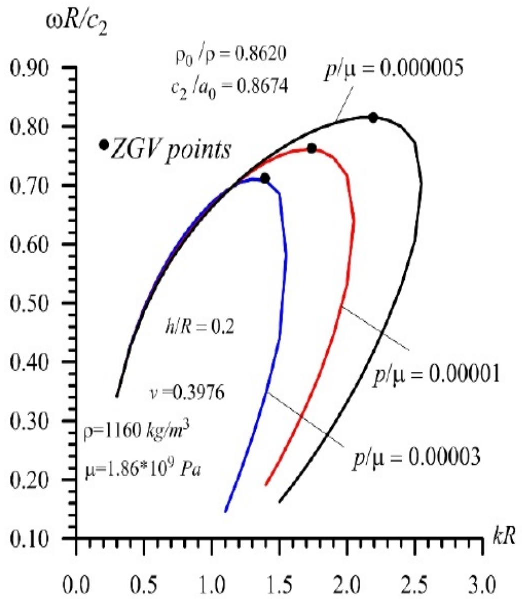
Thus, after determination the analytical expressions for the sought values which contain the unknown constants mentioned above. From the contact, compatibility, boundary conditions the system of homogeneous linear algebraic equations with respect to these unknowns are obtained. If we set the determinant of the coefficient matrix equal to zero, we obtain the dispersion equation.

By solving the dispersion equation mentioned above, we construct the dispersion diagrams or dispersion curves. By analyzing these diagrams, we determine the ZGV modes and the points on these diagrams at which the group velocity of the propagating waves is zero.

3 On the numerical results

Consider the case where the material of the cylinder is Lucite and the fluid is water and assume that $h/R = 0.2$. Here we consider the dispersion curves related to the lowest mode and generated for different values of the p/μ ratio. Note that the dispersion diagrams of the first lowest mode have two branches, conditionally called the left and right branches. Here we consider the left and right branches of this mode, constructed for different values of p/μ and shown in Fig. 1. Note that in Fig. 1, the graphs grouped with the letter a refer to the left branches, while the graphs of the right branches are grouped with the letter b. The ZGV points resulting from the condition $d(\omega R/c_2)/d(kR) = 0$ are also indicated in these graphs.

From the analysis of these diagrams, it can be seen that an increase in the values of the p/μ ratio leads to a decrease (increase) in the wave frequency and the dimensionless wave number in relation to the ZVG points on the dispersion diagrams related to the left (right) branches of the first lowest mode.



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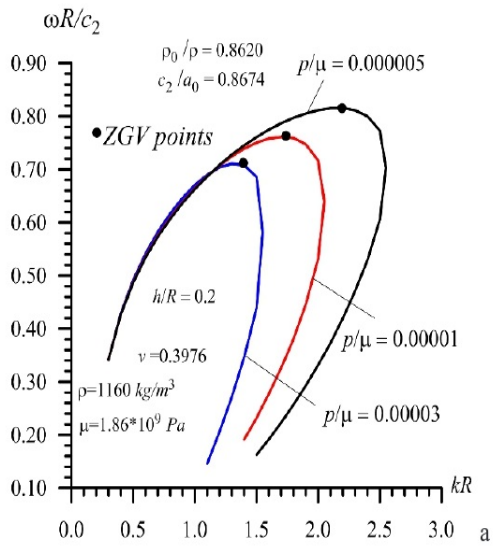

Fig.1. ZVG points on the dispersion diagrams related to the left (a) and right (b) branches of the dispersion diagrams of the first lowest mode

4 CONCLUSIONS

Thus, in the present work, the influence of the magnitude of the initial inhomogeneous stresses in the cylinder immersed in the fluid on the frequency and the dimensionless wave number of the ZGV points in the dispersion diagrams of the first lowest mode was investigated. The investigations are carried out using the so-called linearized 3D equations and relations of the theory of elastic waves in bodies with initial stresses to describe the motion of the cylinder and using the linearized Euler equations to describe the flow of the compressible barotropic inviscid fluid. The corresponding mathematical problem is solved by applying the discrete analytical method and the corresponding dispersion equation is obtained, which is solved numerically. As a result of this solution, the dispersion diagrams of the first lowest mode are obtained for different values of hydrostatic pressure causing the initial stresses in the cylinder. The specific numerical results are obtained for the case when the cylinder material is Lucite and the fluid is water. As a result of analyzing these results, it is found that the ZGV points appear on the dispersion diagrams and these diagrams have two branches: a left and a right branch. It is also found that when the hydrostatic pressure values are increased, the frequency and the dimensionless wave number with related to the ZGV points on the left (right) branches of the dispersion diagrams decrease (increase) monotonically

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