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Modelling of rheologically complex fluid flow in hard-to-recover fractal reservoirs

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Abstract. *One of the most important topical issues of hard-to-recover and shale oil production is the problem of compensating for the decline in oil production at the developed conventional oil fields. The proposed filtration model of flow of rheologically complex fluids in hard-to-recover fractal reservoirs, including shales, consisting mainly of micro-nanopores and microcracks allows to control, regulate and optimize hydrocarbon flows in reservoirs, as well as to increase reservoir productivity.*

Keywords. micro-nanopores · fractal · reservoirs · fractal-scaling approach · Laplace transformation

Mathematics Subject Classification (2010): 76S05

1 Introduction

In the last years, along with the latest advances in the field of oil and gas production, particularly in the late stage of development, new methods of development in the field of technology of production of hard-to-recover reserves, including shale oil reservoirs, are becoming more and more widely used.

Frequently, many fields with major oil reserves correspond to reservoirs with deteriorated collector properties, which can be classified as hard-to-recover and shale oil deposits [2, 6, 7, 13]. One of the most important actual issues of hard-to-recover and shale oil production is the task of compensating for the decline in oil production at the developed conventional oil fields.

It should be noted that in spite of some differences between the reserves of hard-to-recover and shale oils, there is a certain analogy between them. Numerous studies and analyses of various oil properties and reservoir conditions have shown that the main indicators characterising hard-to-recover and shale rocks are reservoirs containing, as a rule, anomalous oil in complex geological and geophysical conditions [5, 12, 14].

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The main problems in the recovery of hard-to-recover oils can be considered the content of various paraffin, asphalt-resin compounds, mechanical and other impurities in the oils, complicating the physical and mechanical properties of the fluid, as well as low porosity and poor permeability of rocks in the conditions of occurrence [12].

The characteristics of shale oils deposited in low-permeability reservoirs and adjacent rocks are diverse and can vary greatly from the parametric properties of both the reservoirs and the oils themselves, depending on the location in the field [14].

At the same time, a comparative analysis of conventional hard-to-recover and shale oils shows their certain similar dependence on quite a large number of geological and physical, geological and field reservoir characteristics, physical and chemical parameters of oil and conditions of their occurrence [12, 14].

However, a wide range of known studies shows that there are no unified definitions and quantitative boundary values of reservoir parameters and recoverable oil and, as a result, there is no single generally accepted approach to modelling of filtration processes of oil flow in conditions of their occurrence in low-permeability complex reservoirs.

Therefore, these parameters can be the most important in identifying classification features for oil samples and reservoirs when creating filtration models of fluid flow in difficult to recover and shale oil fields.

2 Methodology

1. Improving the oil recovery efficiency of large hard-to-recover and shale oil reservoirs is inextricably linked to horizontal wells for the purpose of multistage hydraulic fracturing (HF). As a result of high-pressure impact of fluid with various chemical reagents and nano-inclusions in the formation, various fractures are formed, which lead to an increase in reservoir permeability and, as a consequence, enhanced oil recovery [4, 9, 15].

Fractures appearing and existing in hard-to-recover and shale reservoirs, in contrast to conventional fractured and fractured-porous reservoirs, due to increased density, hydrophobicity and low permeability of reservoirs, have a somewhat different and more complex geophysical and geometric character [16].

Reliable modelling of filtration processes in hard-to-recover and shale formations is associated with the creation and development of a new concept of representative elementary volume REV (Representative Elementary Volume), which takes into account the specifics of multiporous, multi-permeable and multi-scale fractured and fracture-porous reservoirs.

Considering the specific geophysical and geometric character of hard-to-recover and shale formations, multiscale fractures (including hydrofracturing) in an anisotropic multipore medium are presented in Fig. 1, according to [8].

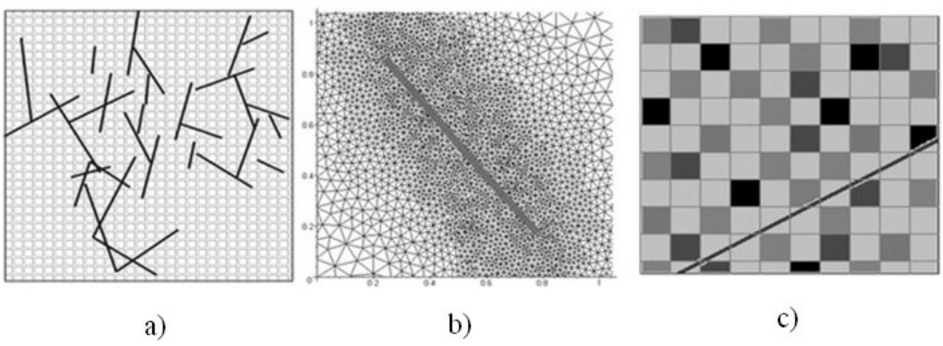

Fig. 1. Schematic diagram of multi-scale fractures (including cracks hydrofracturing) in hard-to-recover and shale formations:

a) micro-scale: a representative elementary volume consisting mainly of a continuum of porous blocks that includes a set of cracks as structural heterogeneities (”cobweb model of cracks”);

b) micro-scale: each crack is an internal or external boundary of the computational domain;

c) meso-scale: the crack scale is comparable to the REV and the crack is a structural element of the REV

Despite the different schemes of multiporous, multi-porous, multi-permeable and multi-scale characterization of tight and shale reservoirs, all of them can be considered from the position of the fractured-porous medium model, since the latter consist of different types of microporous and low-permeable blocks and are randomly distributed by existing and post hydrofracturing fractures.

As well as unfavourable geological and geophysical-geometrical conditions of hard-to-recover and shale formations, the presence of various rheologically complex oils in these reservoirs considerably complicates oil production processes. Such indicators as viscosity, density, tar and paraffin content are the key classification features in classifying oil samples as hard-to-recover.

2. Due to specific features of fractured porous medium mainly consisting of micro-nanopores and microcracks saturated with various kinds of anomalous oil, which leads to violation of traditional flow laws in hard-to-recover and shale reservoirs.

Deviation of filtration flow laws in reservoirs is primarily due to unusually low volumes of micropores in blocks, specifically distributed microcracks and rheological characteristics of the fluid. These factors can be reflected in delayed filtration velocities and relaxation processes of pressures or pressure gradients, and these phenomena can manifest differently depending on the geological-physical and geological-geometric characteristics of the reservoir, as well as the physicochemical and rheological properties of the oil and the conditions of its occurrence.

Present and post hydrofracturing micro-nanofractures in hard-to-recover and shale reservoirs reveal fractal properties, and the blocks usually remain low-permeability micro-nanoporous media. For such media with non-integer fractal dimensionality, the use of the classical physical Darcy’s law is not possible, because in this case in fractal media there is a need to transform the basic physical concepts of space and time in accordance with the fractal-fractional derivative.

At the same time, the micro and small-permeable blocks and anomalous fluid properties may lead to relaxation and retardation processes in hard-to-recover and shale formations. The above allows us to consider that for the description of filtration processes in micro-nanoporous and microfractured reservoirs of rheologically complex fluids, a generalized fractally differential Darcy model can be presented, which takes into account both pressure gradient relaxation and filtration rate lag functions, in accordance with the fractal-scaling approach [1, 10, 11].

$$\lambda^{\vartheta}\frac{d^{\vartheta}_{C_1}V_i}{dt^{\vartheta}}+V_i=-\frac{k_{ij}}{\mu}\left(\frac{\partial P}{\partial x_j}+\theta^{\nu}\frac{d^{\nu}_{C}}{dt^{\nu}}\frac{\partial P}{\partial x_j}\right). \tag{2.1}$$

Following [1, 11], the operation of fractional index differentiation or more commonly called fractional differentiation is introduced below.

The fractional order derivative v of a piecewise continuous function $f(t)$ is defined by the expression

$$\frac{d^{\nu}}{dt^{\nu}}=D^{\nu}_{!}f\left(t\right)=\frac{1}{\left(1-\nu\right)}\frac{d}{dt}\int_{!}^{t}\left(t-t'\right)^{-\nu}f\left(t'\right)dt',--8<\nu<1, \tag{2.2}$$

where C is an arbitrary real number that can have any value in the interval$-\infty < C < +\infty$. In the special case when $C = 0$, the index in equation (2.2) is omitted and simplified notations $D_0^\nu \equiv D^\nu$ and $\frac{d_0^\nu}{dt^\nu} \equiv \frac{d^\nu}{dt^\nu}$ are introduced.

The generalised fractal differential equation (2.1), taking into account (2.2), can be written in the following form

$$\frac{\lambda^\vartheta}{(1-\vartheta)}\frac{d}{dt}\int_{!_1}^{t}\left(t-t''\right)^{-\vartheta}V_i\left(t''\right)dt''+V_i=$$

$$=-\frac{k_{ij}}{\mu}\left[\frac{dP}{dx_j}+\frac{\theta^\nu}{(1-\nu)}\frac{d}{dt}\int_C^t\left(t-t'\right)^{-\nu}\frac{dP\left(t'\right)}{dx_j}dt'\right], \qquad (2.3)$$

where:

V_i - components of the fluid filtration velocity vector;

P - pressure at fluid filtration;

k_{ij} - fracture permeability tensor;

μ - fluid viscosity;

$(t-t')^{-v}, (t-t'')^{\vartheta}$- heredity kernels represented by proportional degree laws with negative fractional exponents - v, ϑ;

θ - relaxation time;

λ - retardation time;

$(1-v), (1-\vartheta)$ - Gamma-functions;

v, ϑ - fractality parameters;

x_j - coordinate axes;

t - time;

C_1 is an arbitrary real number, which can have any value in the interval $-\infty < C_1 < +\infty$.

According to [1, 3], a characteristic feature of fractured-porous medium is the fact that the main fluid reserves are contained in porous blocks, while the fluid flow is taking place in microcracks.

Then the continuity equation for the fluid filtration flow in the fractured region has the following form

$$\frac{\partial\left(m_1\rho\right)}{\partial t}+div\left(\rho\vec{V}\right)-q=0, \qquad (2.4)$$

where m_1 is the porosity of microcracks per reservoir volume unit; q is the amount of fluid flowing per unit time from blocks to microcracks per reservoir volume unit.

Ignoring filtration flows in the blocks, the continuity equation can be written as follows

$$\frac{\partial\left(m_2\rho\right)}{\partial t}+q=0, \qquad (2.5)$$

m_2 is the porosity of blocks in the unit volume of the reservoir.

Assuming permeability k_2, density ρ and viscosity μ in blocks of size l are constant, the expression for inertia-free flow q is written in the following form

$$q=\alpha\frac{\rho k_2}{\mu}\frac{P_2-P_1}{l^2}, \qquad (2.6)$$

where α is a dimensionless constant characterizing the geometry of the reservoir; P_1 is the pressure in the fractures; P_2 is the pressure in the blocks.

In fractured-porous medium, the porosity of cracks m_1 is usually small and can be neglected, while the porosity of blocks m_2 is a function of pressures in cracks P_1 and in blocks P_2.

In linear approximation, the dependence of block porosity on pressures can be written in the form

$$\frac{\partial m_2}{\partial t} = m_{20}\left(\beta_{21}\frac{\partial P_1}{\partial x} + \beta_{22}\frac{\partial P_2}{\partial x}\right), \tag{2.7}$$

where β_{21}, β_{22} and m_{20} are assumed constant.

For a weakly compressible fluid, the density function can be assumed to depend linearly on the pressure

$$\rho = \rho_0\left[1 + \beta_*\left(P - P_0\right)\right], \tag{2.8}$$

where $P = P_1, P_2$ - when considering pressure in microcracks or in blocks; P_0 - initial pressure in the fractured-porous formation.

Using the generalized fractal differential Darcy model (2.1), the dependencies (2.6), (2.7) and (2.8) in equations (2.4) and (2.5), while assuming $m_1 = 0$, the following system of equations can be written:

$$-\frac{\partial}{\partial x_j}\left[k_{ij}\left(1 + \theta^{\nu}\frac{d_C^{\nu}}{dt^{\nu}}\right)\frac{\partial P_1}{\partial x_j}\right] - \frac{\alpha k_2}{l^2}\left(1 + \lambda^{\vartheta}\frac{d_{C_1}^{\vartheta}}{dt^{\vartheta}}\right)\left(P_2 - P_1\right) = 0, \tag{2.9}$$

$$m_0\left[-\beta_{21}\frac{\partial P_1}{\partial t} + \left(\beta_{22} + \beta_*\right)\frac{\partial P_2}{\partial t}\right] + \frac{\alpha k_2}{\mu l^2}\left(P_2 - P_1\right) = 0. \tag{2.10}$$

Assuming that the medium is homogeneous and isotropic, then the permeability can be expressed by the ball tensor $k_{ij} = k_1\delta_{ij}$, and the system of equations (2.9) and (2.10) is shown in the following simplified way:

$$\chi\left(1 + \theta^{\nu}\frac{d_C^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2} + A\left(1 + \lambda^{\vartheta}\frac{d_{C_1}^{\vartheta}}{dt^{\vartheta}}\right)\left(P_2 - P_1\right) = 0, \tag{2.11}$$

$$\frac{\partial P_2}{\partial t} - \beta\frac{\partial P_1}{\partial t} + A\left(P_2 - P_1\right) = 0, \tag{2.12}$$

where,

$$A = \frac{\alpha k_2}{\mu m_0 l^2\left(\beta_{22} + \beta_*\right)}, \chi = \frac{k_1}{\mu m_0\left(\beta_{22} + \beta_*\right)}, \beta = \frac{\beta_{21}}{\beta_{22} + \beta_*}.$$

The system of equations (2.11) and (2.12) can be written with respect to any one of the pressures. Solving this system with respect to the pressure P_1, the following equation can be obtained

$$\left(1 + \lambda^{\vartheta}\frac{d_{C_1}^{\vartheta}}{dt^{\vartheta}}\right)\frac{\partial P_1}{\partial t} - \eta\frac{\partial}{\partial t}\left(1 + \theta^{\nu}\frac{d_C^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2} = \frac{\chi}{1 - \beta}\left(1 + \theta^{\nu}\frac{d_C^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2}, \tag{2.13}$$

where,

$$\eta = \frac{\chi}{A\left(1 - \beta\right)} = \frac{k_1 l^2}{\alpha k_2\left(1 - \beta\right)}.$$

When $C_1 = 0$, $C = 0$, equations (2.13) can be presented as follows:

$$\left(1 + \lambda^{\vartheta}\frac{d^{\vartheta}}{dt^{\vartheta}}\right)\frac{\partial P_1}{\partial t} - \eta\frac{\partial}{\partial t}\left(1 + \theta^{\nu}\frac{d^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2} = \frac{\chi}{1 - \beta}\left(1 + \theta^{\nu}\frac{d^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2}. \tag{2.14}$$

Assuming that there are no relaxation phenomena of the pressure gradient in the filtration process, i.e. $\theta = 0$, equation (2.14) is written in a somewhat simplified form

$$\left(1+\lambda^{\vartheta}\frac{d^{\vartheta}_{C_1}}{dt^{\vartheta}}\right)\frac{\partial P_1}{\partial t}-\eta\frac{\partial^3 P_1}{\partial t\partial x_j^2}=\frac{\chi}{1-\beta}\frac{\partial^2 P_1}{\partial x_j^2}. \tag{2.15}$$

In the case of filtration without delay processes $\lambda = 0$, equation (2.14) will take the form

$$\frac{\partial P_1}{\partial t}-\eta\frac{\partial}{\partial t}\left(1+\theta^{\nu}\frac{d^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2}=\frac{\chi}{1-\beta}\left(1+\theta^{\nu}\frac{d^{\nu}}{dt^{\nu}}\right)\frac{\partial^2 P_1}{\partial x_j^2}. \tag{2.16}$$

In the case of Darcy law fluid flow in conventional fractured-porous reservoirs, equation (2.14) is transformed into the well-known differential equation obtained in [3]. In the limiting case when $\lambda = 0, \theta = 0$ and $\eta \to 0$, equation (2.14) transforms into the classical elastic equation with the piezoconductivity coefficient $\frac{\chi}{1-\beta}$, which corresponds to the permeability of the fracture system and porosity, as well as to the compressibility of the blocks.

3. If we assume that fractal filtration of a rheologically complex fluid takes place in a semi-infinite linear micronanoporous and microfractured reservoir, then equation (2.16), with some simplifications, can be written in the following form

$$\left(1+\lambda^{\vartheta}\frac{\partial^{\vartheta}}{\partial t^{\vartheta}}\right)\frac{\partial P_1}{\partial t}-\eta\frac{\partial}{\partial t}\left(1+\theta^{\nu}\frac{\partial^{\nu}}{\partial t^{\nu}}\right)\frac{\partial^2 P_1}{\partial x^2}=\frac{\chi}{1-\beta}\left(1+\theta^{\nu}\frac{\partial^{\nu}}{\partial t^{\nu}}\right)\frac{\partial^2 P_1}{\partial x^2}. \tag{2.17}$$

It is also assumed that the fluid is at rest at the initial moment of time in a semi-infinite reservoir with constant reservoir pressure P_r. At some moment of time the pressure in section $x = 0$ starts to change according to the law $P_1 = P_0(t)$, in particular, the pressure can take a constant value equal to P_{00}.

In this case, the filtration of a rheologically complex fluid in a semi-infinite linear micronanoporous and microfractured reservoir is described by the fractional differential equation (2.17), with the following initial and boundary conditions:

$$P_1(0,x)=P_r \tag{2.18}$$

$$P_1(t,0)=P_0(t), P_1(t,\infty)=P_r \tag{2.19}$$

Introducing the function $\tilde{P}_1(t,x) = P_r - P_1(t,x)$, the fractional differential equation (2.17), with initial (2.18) and boundary conditions (2.19), can be written as follows:

$$\left(1+\lambda^{\vartheta}\frac{\partial^{\vartheta}}{\partial t^{\vartheta}}\right)\frac{\partial \tilde{P}_1}{\partial t}-\eta\frac{\partial}{\partial t}\left(1+\theta^{\nu}\frac{\partial^{\nu}}{\partial t^{\nu}}\right)\frac{\partial^2 \tilde{P}_1}{\partial x^2}=\frac{\chi}{1-\beta}\left(1+\theta^{\nu}\frac{\partial^{\nu}}{\partial t^{\nu}}\right)\frac{\partial^2 \tilde{P}_1}{\partial x^2}, \tag{2.20}$$

$$\tilde{P}_1(0,x)=0, \tag{2.21}$$

$$\tilde{P}_1(t,0)=P_r-P_1(t,0)=P_r-P_0(t)=\tilde{P}_0(t), \tilde{P}_1(t,\infty)=0. \tag{2.22}$$

Using the Laplace transformation, the fractional differential equation (2.20) and boundary conditions (2.22), taking into account the initial (2.21), can be written as follows

$$\frac{\partial^2 \hat{P}}{\partial x^2}-\beta_0^2\hat{P}=0, \tag{2.23}$$

$$\hat{P}(s,0)=\hat{P}_0(s), \hat{P}(s,\infty)=0, \tag{2.24}$$

$$\beta_0^2 = \frac{A\,(1-\beta)}{\chi}\frac{s+\lambda^{\vartheta}s^{1+\vartheta}}{(1+\theta^{\nu}s^{\nu})\,(A+s)},$$

where $\hat{P}\,(s,x) = \int_0^{\infty}\exp\,(-st)\,\tilde{P}_1\,(t,x)\,dt$, $\hat{P}_0\,(s) = \int_0^{\infty}\exp\,(-st)\,\tilde{P}_0\,(t)\,dt$.

The solution of the differential equation (2.23) under boundary conditions (2.24) has the form

$$\hat{P}\,(s,x) = \hat{P}_0\,(s)\exp\,(-\beta_0 x)\,. \tag{2.25}$$

To determine the filtering speed in images, we can use the Laplace transformation to the filtering equation (2.1), as a result, the last equation, after some transformations, will be written as follows

$$\hat{V}\,(s,x) = -\frac{k}{\mu}\frac{1+\theta^{\nu}s^{\nu}}{1+\lambda^{\vartheta}s^{\vartheta}}\frac{\partial\hat{P}\,(s,x)}{\partial x}, \tag{2.26}$$

where $\hat{V}\,(s,x) = \int_0^{\infty}\exp\,(-st)\,V\,(t,x)\,dt$.

Then, considering dependence (2.26), the image of filtration velocity in the initial section of the gallery $x=0$ is determined from relation (2.25) by the following formula

$$\hat{V}\,(s,0) = \frac{k}{\mu}\sqrt{\frac{A\,(1-\beta)}{\chi}}\sqrt{\frac{s\,(1+\theta^{\nu}s^{\nu})}{(A+s)\,(1+\lambda^{\vartheta}s^{\vartheta})}}\hat{P}_0\,(s)\,. \tag{2.27}$$

The asymptotic solutions in the original for $0<\vartheta; v>\vartheta$ at $t\to 0$ and t ? 8 are obtained from (2.27) and are, respectively, of the form:

$$V\,(t) \approx \frac{k}{\mu}\sqrt{\frac{A\,(1-\beta)}{\chi}\frac{\theta^{\nu}}{\lambda^{\vartheta}}}\left(\frac{d}{dt}\right)^{\frac{\nu-\vartheta}{2}}\tilde{P}_0\,(t) = \frac{k}{\mu}\sqrt{\frac{A\,(1-\beta)}{\chi}\frac{\theta^{\nu}}{\lambda^{\vartheta}}}\left(\frac{d}{dt}\right)^{\frac{\nu-\vartheta}{2}}[P_r - P_0\,(t)]\,, \tag{2.28}$$

$$\begin{aligned} V\,(t) &\approx \tfrac{k}{\mu}\sqrt{\tfrac{(1-\beta)}{\chi}}\left(\left(\tfrac{d}{dt}\right)^{\frac{1}{2}} - \tfrac{1}{2}\lambda^{\vartheta}\left(\tfrac{d}{dt}\right)^{\frac{1}{2}+\vartheta} + \tfrac{1}{2}\theta^{\nu}\left(\tfrac{d}{dt}\right)^{\frac{1}{2}+\nu}\right)\tilde{P}_0\,(t) = \\ &= \tfrac{k}{\mu}\sqrt{\tfrac{(1-\beta)}{\chi}}\left(\left(\tfrac{d}{dt}\right)^{\frac{1}{2}} - \tfrac{1}{2}\lambda^{\vartheta}\left(\tfrac{d}{dt}\right)^{\frac{1}{2}+\vartheta} + \tfrac{1}{2}\theta^{\nu}\left(\tfrac{d}{dt}\right)^{\frac{1}{2}+\nu}\right)[P_r - P_0\,(t)]\,. \end{aligned} \tag{2.29}$$

In the case when $P_0(t) = P_{00} = Const$, then the solutions of (2.28) and (2.29) in the original, respectively, will be written:

$$V\,(t) \approx \frac{k}{\mu}\frac{[P_r - P_{00}]}{\left(\frac{\nu-\vartheta}{2}\right)}\sqrt{\frac{A\,(1-\beta)}{\chi}\frac{\theta^{\nu}}{\lambda^{\vartheta}}}t^{-\frac{\nu-\vartheta}{2}}, \tag{2.30}$$

$$V\,(t) \approx \frac{k}{\mu}\,[P_r - P_{00}]\sqrt{\frac{(1-\beta)}{\chi}}\left[\frac{t^{-\frac{1}{2}}}{\left(\frac{1}{2}\right)} - \frac{1}{2}\lambda^{\vartheta}\frac{t^{-\left(\frac{1}{2}+\vartheta\right)}}{\left(\frac{1}{2}+\vartheta\right)} + \frac{1}{2}\theta^{\nu}\frac{t^{-\left(\frac{1}{2}+\nu\right)}}{\left(\frac{1}{2}+\nu\right)}\right]. \tag{2.31}$$

From the analysis of (2.30) and (2.31) we can see that at small times of the filtration process in a semi-infinite linear reservoir, the flow rate depends on relaxation times λ, θ and fractality parameters ϑ, νmultiplicatively, and at large times - additively. At the same time, as the retardation time and fractality parameter in the retardation processes increase, both the value of the filtration rate itself decreases and the rate of its decrease accelerates. While relaxation time and fractality parameter in the relaxation process slightly increase the value of filtration rate and slow down the rate of its change.

4. Now we consider unsteady fractal filtration of a rheologically complex fluid, described by the fractional differential equation (2.17) or (2.20), in a semi-infinite linear reservoir when the pressure at the initial cross section is a harmonic function of time with a given frequency:

$$\tilde{P}(t,0) = P_r - P_1(t,0) = \tilde{P}_0(t) = P_{00}\exp(i\omega t), \tilde{P}(t,\infty) = 0. \tag{2.32}$$

Assuming that after sufficiently distant from the initial moment of time, the influence of initial conditions practically does not affect the pressure distribution in the formation, the solution of the fractional differential equation (2.20) under boundary conditions (2.32) can be found in the form of

$$\tilde{P}(t,x) = P_{00}\exp(i\omega t + \delta x). \tag{2.33}$$

Substituting the solution (2.33) into the fractional differential equation (2.20), taking into account the boundary conditions (2.32), we obtain the following relation for determining δ

$$i\omega + \lambda^{\vartheta} i^{1+\vartheta}\omega^{1+\vartheta} - \eta i\omega\delta^2(1+\theta^{\nu} i^{\nu}\omega^{\nu}) = \frac{\chi}{1-\beta}\delta^2(1+\theta^{\nu} i^{\nu}\omega^{\nu}).$$

From the last relation δ is determined

$$\delta = \sqrt{\frac{i\omega A(1-\beta)}{\chi(1+A)}}\sqrt{\frac{1+\lambda^{\vartheta} i^{\vartheta}\omega^{\vartheta}}{1+\theta^{\nu} i^{\nu}\omega^{\nu}}}. \tag{2.34}$$

Assuming that the parameters $\lambda,\ \theta,\ \omega$ are sufficiently small, the relation (2.34) can be written approximatingly as follows.

$$\delta = -\sqrt{\tfrac{\omega A(1-\beta)}{2\chi(1+A)}}\left[1 + \tfrac{1}{2}\lambda^{\vartheta}\omega^{\vartheta}\left(\cos\tfrac{\pi}{2}\vartheta - \sin\tfrac{\pi}{2}\vartheta\right) - \tfrac{1}{2}\theta^{\nu}\omega^{\nu}\left(\cos\tfrac{\pi}{2}\nu - \sin\tfrac{\pi}{2}\nu\right)\right] -$$
$$-i\sqrt{\tfrac{\omega A(1-\beta)}{2\chi(1+A)}}\left[1 + \tfrac{1}{2}\lambda^{\vartheta}\omega^{\vartheta}\left(\cos\tfrac{\pi}{2}\vartheta + \sin\tfrac{\pi}{2}\vartheta\right) - \tfrac{1}{2}\theta^{\nu}\omega^{\nu}\left(\cos\tfrac{\pi}{2}\nu + \sin\tfrac{\pi}{2}\nu\right)\right]. \tag{2.35}$$

The analysis of dependence (2.35) shows that both relaxation times θ and retardation λ and fractality parameters ϑ, ν and depending on the values of ϑ, ν parameters θ, λ can both decrease and increase the process of filtration damping in comparison with traditional dependences of filtration processes.

At that, the attenuation coefficient at fractal filtration is determined proportionally to

$$\sqrt{\frac{\omega A(1-\beta)}{2\chi(1+A)}}\left[1 + \frac{1}{2}\lambda^{\vartheta}\omega^{\vartheta}\left(\cos\frac{\pi}{2}\vartheta - \sin\frac{\pi}{2}\vartheta\right) - \frac{1}{2}\theta^{\nu}\omega^{\nu}\left(\cos\frac{\pi}{2}\nu - \sin\frac{\pi}{2}\nu\right)\right].$$

3 Conclusions.

It should be noted that on the basis of the above-mentioned fractal models of anomalous fluid filtration in hard-to-recover and shale reservoirs, it is possible to significantly reveal new phenomena, increase the efficiency of diagnostics and regulation of properties and characteristics of the systems under consideration. In addition, the proposed filtration model of flow of rheologically complex fluids in hard-to-recover fractal reservoirs, including shales, consisting mainly of micro-nanopores and microcracks can allow to control, regulate and optimize hydrocarbon flows in reservoirs, as well as increase the productive capacity of reservoirs.

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