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Modelling of colmatant mass transfer during in situ gas generation process

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Abstract. *A model of accumulation during mass transfer in the near-wall flow is proposed, which takes into account not only the impact of hydrodynamic flows on the processes, but also the reversed impact of growing colmatant on the flow pattern. The numerical study of the model has shown that hydrodynamic flows can have a significant influence on the processes of colmatant formation. In particular, the colmatant availability can also lead to the destruction of concentration fronts of reactants. As a result of their evolution, accumulations of complex structure are formed. It is shown that the process of colmatant formation depends on the ratio of characteristic values of the displacement rate in the porous medium.*

Keywords. colmatant · flow · porous medium · mass transfer · carbon dioxide · brine solution.

Mathematics Subject Classification (2010): 76D55

1 Introduction

Problems of substance transfer and fluid flow in inhomogeneous porous media are actual tasks in many engineering and technology fields. Along with experimental and field studies, mathematical modelling of substance transport and fluid flow in such media makes it possible to effectively investigate the main characteristics of the process. Adsorption during substance transfer in porous media significantly affects the transfer characteristics. A number of models have been proposed to describe adsorption of chemical substances on rocks [18, 6, 20]. In [11], the problem of pollutant transport in a porous medium consisting of two zones (with moving and still water) is considered, taking into account the phenomena of convective transport, hydrodynamic dispersion, double adsorption and internal diffusive mass transfer between the two zones. Linear and nonlinear kinetic equations [11, 30, 29, 24] were used to describe the mass transfer between the zones.

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This paper considers the process of substance (sodium chloride, NaCl) transport in porous media adsorbed from salt solution, taking into account the non-equilibrium nature of adsorption resulting in the effect of colmatation.

The saturated salt solution in porous media is used as an agent controlling the filtration profile and formation medium coverage by injection of gas-generating solutions.

The chemical reaction of interaction of gas-generating mixtures - hydrochloric acid and sodium carbonate is shown in the following form:

$$\mathrm{Na_2CO_3{+}2HCl = 2NaCl + H_2O + CO_2}.$$

The resulting sodium chloride (NaCl) is a water-soluble salt. Carbon dioxide (CO2) under reservoir pressure is also characterized by good solubility in water and oil.

The filtration process in a porous medium involves the water, oil and gas phases, with the liquid phase being a gas-liquid mixture and the dissolved salt formed by a chemical reaction.

These conglomerates formed deep along the strike of the reservoir on the walls of pore channels and in the free space of pores lead to a local increase in flow resistance due to narrowing (retardation) and partial or complete blocking of individual pore channels, which in turn leads to a change in the direction of filtration flow and to an increase in reservoir sweep efficiency [9]. It should also be noted that some of the formations interact with the flowing fluid.

The paper provides some information on mathematical modelling of substance transport in inhomogeneous media, fluid flow in inhomogeneous porous media and basic characteristics of the process. Substance transport on the basis of kinetic diffusion approach to the occurring phenomena is considered.

Numerical methods for solving the problem of substance transport in a heterogeneous porous medium are analyzed. The formation of aggregates in a porous medium on the walls of a pore channel and in the free space of pores leads to a local increase in flow resistance due to complete blocking of individual pore channels, which in turn leads to a change in the direction of filtration flow.

2 Experimental studies

A series of laboratory studies were performed to investigate the climatizing properties of salt solution sediments due to the process of substance transfer in porous channels. By experimental studies the adsorption intensity in artificial samples of water-saturated porous medium with a diameter of 0.032 m and a length of 3.0 m was estimated.

In the course of laboratory experiments, 12% aqueous solution of Na_2CO_3 and 9% aqueous solution of hydrochloric acid HCl were pumped into the porous medium inlet (reservoir model) sequentially, in several cycles, for four hours. The technological effect before and after injection of solutions at different liquid flow rates at constant inlet pressures was estimated. As can be seen from the given figure in the case of using the method there is an oscillating decrease in flow rate in time (Fig. 1).

However, the change of its flow rate in time can only qualitatively characterize the colmatation process, as the attenuation rate depends not only on the solids concentration in the suspension, but also on such parameters as filtration area, initial flow rate, absolute permeability of the porous medium. Therefore, the most general character is the dependence of the relative flow rate of the suspension on the number of solid particles deposited on the unit filtration surface of the core. This parameter is calculated by the formula:

$$G = \frac{C \cdot 10^{-6} Q}{F}\,, mg/cm^2$$

The experiments show that the rate of filtration damping mainly depends on the number of solid particles deposited per unit of filtration surface, provided that such parameters as permeability of porous medium, injection pressure, as well as physical and chemical properties of the pumped suspension do not change significantly from experiment to experiment. Here we mean dispersibility and persistence of the suspension. To some extent, the shape of the solids can also influence the colmatation process. In addition, the presence of electrolytes in the solution in the specified amounts also favours the coagulation process.

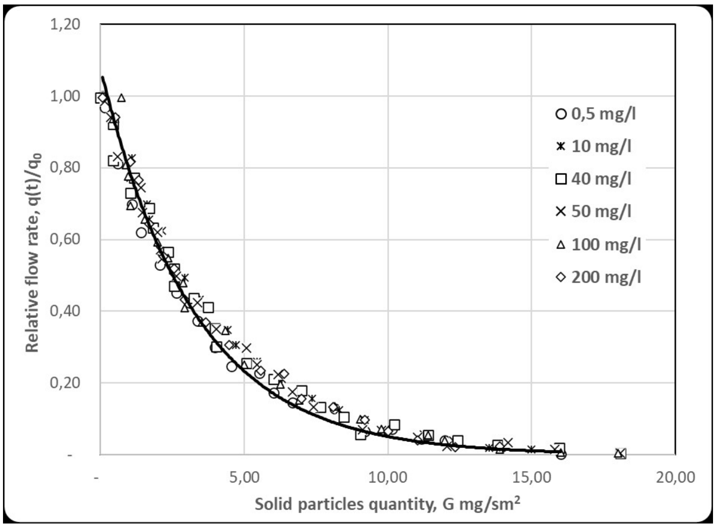

Fig. 1. Dependence of the relative flow rate of suspension on the number of solid particles.

Suspension leads to enlargement of individual aggregates and to a significant reduction of internal colmatation. Also, in the process of laboratory experiments the conductivity dynamics was evaluated from the pressure recovery curves before and after filtration (Fig. 2).

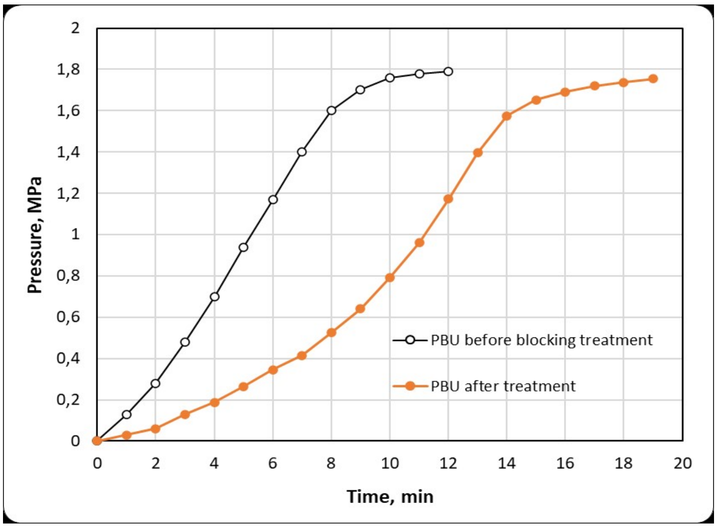

Fig. 2. Pressure build-up curves before and after porous media treatment

Thus, the experimental results indicate the blocking ability of the proposed method. It is revealed that salt deposition and pore blocking have a significant effect on the change of absolute permeability (Fig. 3a) and porosity (Fig. 3b), both in magnitude and along the strike.

Deposition of rock particles and clogging of pores by adsorbed salt leads to a decrease in the front advance velocity. Deterioration of filtration characteristics leads to an increase in the filtration resistance of the medium, which in the conditions of the considered injection mode leads to an increase in pressure.

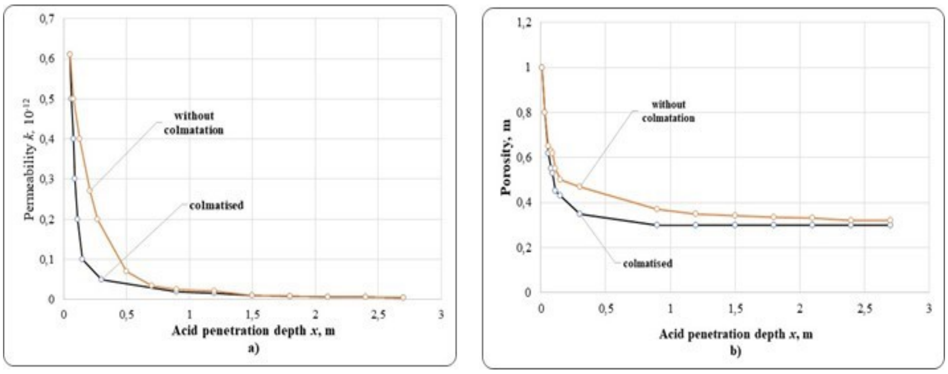

Fig. 3 Distribution of permeability (a), porosity (b) at injection of gas-forming solutions; 1 - without taking into account rock colmatation, 2 - with taking into account colmatation

Thus, the experimental results indicate the ”blocking” ability of the proposed method.

3 Mathematical model.

Most of the studies related to the study of ”structure formation” in models are based on investigations of multicomponent equations of the ”reaction-diffusion” type [28, 19, 23, 7, 15, 12].

Usually, in the considered models only random particle movements and sensitivity to gradients of the substances involved in the processes are taken into account [31]. Such an approach is applicable to modelling the processes of structure formation in a fixed medium: the kinetic part of the system and the geometry of the area under consideration [28, 19, 23, 7, 15, 12, 31, 22, 16] and is related to the properties of a nonlinear medium.

The study of the loss of stability of a homogeneous steady state under the action of convective flows was considered, for example, in [14]. Although the growing colmatant is a structure of NaCl formed in the solid phase, it is still carried away by the liquid due to the large pore size. Therefore, in a number of problems, the presence of shear stresses in the flow, which essentially change the picture of structure formation, is a fundamental point. In particular, consideration of shear stresses is fundamentally important when analysing the aggregation of colmatants [27]. Some aspects of the impact of inhomogeneity of the velocity field on structure formation processes are considered in the review [5]. The description of structure formation in a porous medium also belongs to such problems.

The main objective is to evaluate the effect of filtration on the formation of the colmatant structure. The experience of numerical modelling of complex multicomponent problems shows that fundamentally many physical effects in such systems appear at the stage of two-dimensional calculation, and taking into account the real geometry leads to changes in quantitative characteristics. For the same reason, it is possible to refuse a detailed description of chemical processes in the colmatant and limit ourselves to the well-proven simple phenomenological model [2, 3, 1].

The model of accumulation (structure formation) taking into account convective flows and diffusion of reagents can be represented by the equations:

$$\begin{aligned} \frac{\partial\theta}{\partial t} &= D_1\Delta\theta + \frac{\alpha\theta^2}{\theta+\theta_0} - \gamma\theta\phi - \chi_1\theta - div\,(V\theta) \\ \frac{\partial\theta}{\partial t} &= D_2\Delta\phi + \beta\theta\left(1-\frac{\phi}{C}\right)\left(1+\frac{\phi^2}{\phi_0^2}\right) - \chi_2\phi - div\,(V\phi) \end{aligned} \tag{3.1}$$

where $\theta,\ \varphi$ - concentrations of impurity (in carbonate and salt solutions) at the point at time t; D_1 and D_2 - their diffusion coefficients; $\alpha, \beta, \gamma, \chi_1, \chi_2, \theta_0, \phi_0, C$ - kinetic parameters of the model; V - flow velocity.

The equation describing the evolution of the colmatant is of the form:

$$\frac{\partial \psi}{\partial t} = 0, \tag{3.2}$$

where ψ - is the unmeasured concentration of impurities. In addition, it was assumed that the formed accumulations do not affect the diffusion of impurity and colmatant, i.e., the diffusion coefficients do not depend on the impurity concentration ψ.

Equations (3.1) to (3.2) are written in a moving frame of reference bound by the center of the colmatant tangle. Here, the used aqueous solution is a viscous incompressible fluid and the continuity condition is fulfilled $divV = 0$.

In the convective flow model, it was assumed that the accumulation in the system did not directly affect the solution flow rate. This means that the change in flow was only due to changes in the shape and size of the colmatant.

The problem of the evolution of a colmatant broken away from the wall of a porous medium in the near-wall layer of a viscous incompressible fluid in the two-dimensional planar case was considered. When the system (3.1) - (3.2) is de-dimensioned, the following values are chosen as the characteristic scales of the problem: u is the maximum flow velocity (in a stationary coordinate system), L is the characteristic size determined on the basis of studies of the system (3.1) - (3.2) without considering convective flows $L \sim 100 \cdot \sqrt{D_1/\alpha}$ [10]. In this work it was considered that $L << d-$ the capillary diameter of the porous medium.

It should be noted that the Reynolds numbers for various porous media are estimated for their diameters in [8]; therefore, it is necessary to recalculate their values for numerical calculations. Since the Reynolds numbers for this problem are small [20], the linear shear flow approximation is applicable. Consequently, to determine the velocity field, it is necessary to solve the problem of flowing a porous particle by a linear shear flow. In this case, the velocity fields inside and outside the particle were assumed to be stationary at each moment of time, i.e., transients after a sharp, as shown in [1, 3, 10], increase in the accumulation radius were neglected. Such an assumption is justified by the fact that the characteristic relaxation time τ of perturbations in a viscous fluid is much shorter, than the time interval t^* between two consecutive increases of the accumulation radius. For τ validity of the estimation: $\tau \sim \rho L^2/\mu$, where μ - the dynamic viscosity of the aqueous solution of the composition, ρ - its density. In the coordinate system associated with the accumulation center, the motion of a viscous incompressible fluid outside the cylindrical particle is described by the Stokes equations:

$$r > a \ : \ \mu \cdot \mathit{\Delta} v = \nabla p, \ \ divv = 0,$$

where a is the current accumulation radius. Darcy's law is used for filtration flow inside the accumulation:

$$r < a \ : \ V = -\frac{k}{\mu} \nabla P, \ \ divV = 0,$$

where is k - the accumulation permeability coefficient.

On the surface of a porous cylinder, the external normal stress is equal to the internal pressure, and the normal component of velocity is continuous. In addition, at the boundary "fluid - porous medium" the tangential component of velocity suffers a discontinuity proportional to the value of its first derivative along the external normal [21, 32, 13]. Thus, we have the following conditions (in polar coordinates) on the cylinder surface:

$$p - 2\mu \frac{\partial v_r}{\partial r} = P, \ \ v_r = V_r, \qquad \lambda \sqrt{k} \frac{\partial v_\phi}{\partial r} = v_\phi - V_\phi,$$

where λ - dimensionless value determined empirically for porous materials. The limits of variation for different samples are from 0.1 to 10. At distances much larger than the radius of the ball, the perturbation of the linear shear flow should tend to zero. So, the boundary condition at infinity can be written as $r >> a$:

$$v_y = 0, \qquad v_x = \varepsilon y,$$

where ε is the linear shear flow parameter. If we consider the flow in the vessel as Poiseuille flow, then we can express through u - velocity in the center of the vessel, d - its diameter and h - distance from the center of the colmatant to the nearest wall of the porous medium:

$$\varepsilon = \frac{4u}{d}\left(1 - 2\frac{h}{d}\right).$$

In such a formulation, the problem for determining the velocity field has an analytical solution obtained in [32]:

$$\begin{aligned} &r < a: \ V = 8kK_3E \cdot r \\ &r \geq a: \ v = \left(1 - K_2\tfrac{a^4}{r^4}\right)E \cdot r - 2\tfrac{a^2}{r^4}\left(K_1 - K_2\tfrac{a^2}{r^2}\right)(r, E \cdot r)\, r - \left(1 - \delta\tfrac{a^2}{r^2}\right)\Omega r, \\ &E = \begin{pmatrix} 0 & \varepsilon/2 \\ \varepsilon/2 & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 0 & \varepsilon/2 \\ -\varepsilon/2 & 0 \end{pmatrix} \end{aligned} \tag{3.3}$$

where K_1, K_2, K_3 and δ are dimensionless quantities depending on k/a^2 and λ. Expressions for them are given in [32, 13, 25, 26].

Using analytical expressions for the velocity field (3.3), it is possible to determine changes in the distributions of reactant concentrations with time by solving the system (3.1) - (3.2) with a known velocity field. The proposed model allows us to draw a number of important conclusions:

- hydrodynamic flows can have a significant influence on the transport processes in solution by salt inclusions.

- saturated solution with salt forms colmatant aggregates of large size and complex shape. They can cause blockage of highly permeable formation pores. In addition, they can initiate the process of blockage far from the place of accumulation detachment and lead to the development of in-situ blocking formations.

Along with this, it has been found that the difference between upstream and downstream pressures varies due to the colmatation process in the porous medium. If we take pressure as a function of time:

$$dP = -q_0\mu\frac{dx}{k} \tag{3.4}$$

Integrating this expression

$$P(x,t) = P(x_0,t) - q_0\mu\int_{x_0}^{x}\frac{dx}{k} \tag{3.5}$$

This is where porosity changes. If this porosity is the amount (weight) of accumulations deposited in the filtrate (depending on the saturation of the solution), then

$$k = k_0\left(1 - \sqrt{\xi}\right)^3, \xi = \xi(x,t) \tag{3.6}$$

If the length of the filtered model is equal and we assume pressure in the colmatised area, then

$$P(0,t) = P(L) + \frac{q_0\mu}{k_0}\int_0^L \frac{dx}{\left(1 - \sqrt{\xi}\right)^3} \tag{3.7}$$

4 Conclusions

Taking into account the obtained results, a graph of pressure changes in time at different moments of time at the outlet is plotted and colmatisation at this length is calculated.

The results obtained can provide a basis for the development of a method of increasing the displacement coverage to enhance oil recovery and control the water cut in porous media in the reservoirs flooding process.

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