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Parametric vibrations of a viscous-elastic medium-contacting, damaged, orthotropic cylindrical shell stiffened with inhomogeneous rods

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Abstract. *In the paper we consider a problem on parametric vibrations of a viscous-elastic medium-contacting cylindrical shell subjected to the external* $p = p_0 + p_1 \sin \omega_1 t$ *influence (here,* p_0 *is a mean or main force* p_1 *is the change amplitude of the force,* ω_1 *is the change frequency of the variable part of the force) whose material was stiffened with inhomogeneous rods along the generate. It is accepted that the material of the cylindrical shell is orthotropic, the material of rods is inhomogeneous. The medium was modelled in viscous-elastic form. For finding critical force by means of contact condition, a frequency equation was built and was studied depending on mechanical and geometrical parameters characterizing the system. The hereditary type damage theory was used for taking into account the damages created at the expense of vibrations in the structure of a cylindrical shell subjected to the action of external force.*

Keywords. viscous-elastic medium · inhomogeneous rod · orthotropic cylindrical shell · parametric vibrations · damage.

Mathematics Subject Classification (2010): 74K10, 74K25

1 Introduction

The paper [10] was devoted to nonlinear parametric vibrations of a viscous liquid-filled, longitudinally stiffened orthotropic cylindrical shell. The motion of the liquid was described by the linearized Navier-Stocks equations. The motion equation of a viscous liquid-filled, longitudinally stiffened cylindrical shell was obtained by using the Ostrogradsky-Hamilton variation principle.

The dependence of the ratio of nonlinear frequency to linear frequency on the curvature of the shell was determined for various number of rods.

Calculation of liquid-contacting structures are widely used in designing hydrotechnical units, underwater structures.

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At the same time, these calculations can be found in some fields of engineering, for example in the calculation of elastically connected structural elements. Parametric vibrations can just be formed in such elements. In [5 - 9], parametric materials vibrations of damaged structural elements made of smooth isotropic materials modelled as damaged rod and shell are considered in a medium. Parametric vibrations of stiffened and damaged structural elements made of isotropic material are studied in [3].

In [4, 12 - 14], a problem on parametric vibrations of a viscous-elastic medium-contacting cylindrical shell affected by external force and stiffened with ribs was considered. Three cases of location of ribs on the surface of the cylindrical shell are considered: 1) the ribs were located on the surface of the cylindrical shell along its generatrix; 2) The cylindrical shell was stiffened by means of annular ribs; 3) The ribs form an orthogonal network on the surface of the cylindrical shell.

The cylindrical shell was accepted as orthotropic. The medium was modeled in a viscous-elastic form and elastic effect was studied by the Lame equation system in displacements.

For finding parametric vibrations frequencies of the system by means of control conditions, a frequency equation was built and was asymptotically studied depending on mechanical and geometrical parameters characterizing the system. An optimization parameter was included, an optimal variant of the number of ribs was found.

2 Mathematical formulation of the problem and its solution

For studying the problem on parametric vibrations of a damaged, viscous-elastic medium-contacting orthotropic cylindrical shell affected by external medium and stiffened with inhomogeneous rods, we will use the Hamilton-Ostrograsky variation principle (Fig. 1).

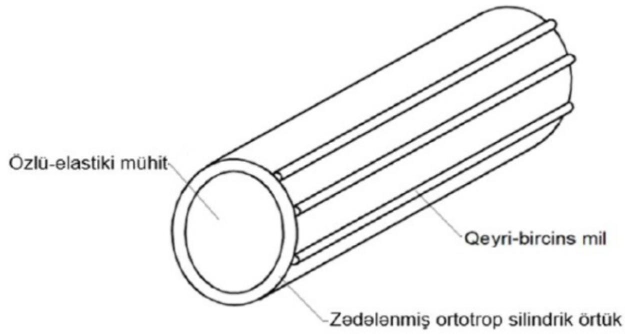

Fig. 1. Damaged, viscous-elastic medium-contacting orthotropic cylinder stiffened with inhomogeneous rods.

The total energy of the studied system can be written as:

$$W = J + A_0 + A_1 + J_i \tag{2.1}$$

Here J is the total energy of a cylindrical shell with damages taken into account and stiffened with rods; J_i is the total energy of inhomogeneous rods fixed in the direction of the generatrix of the cylindrical shell, A_0 is the work performed by the force as viewed from the medium in the displacements of the cylindrical shell, A_1 is the work performed by the force p acting on the surface of the cylindrical shell in the displacements of the points of the shell.

We wrote expressions of quantities involved in the expression (2.1). The hereditary type damage theory was used to take into account the damages. According to this theory, deformation components are determined in a homogeneous body as follows [11]:

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + M^* \cdot \sigma_{ij} \tag{2.2}$$

Here M^* is a hereditary type integral operator that describes the damage process and is in the following form:

$$M^* \cdot \sigma_{ij} = \sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot \sigma_{ij}(\tau) d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot \sigma_{ij}(\tau) d\tau \tag{2.3}$$

In the expression (2.3) $M(\overline{x}, t - \tau)$ is a damage kernel, $\left(t_k^-; t_k^+\right)$ is a time interval affected by active stress that provides the increase in damage, $f(t_k^+)$ is a defect recovery function dependent on the volume of damages accumulated in a cycle. The value $f(t_k^+) = 0$ of this function, corresponds to full restoration of damages accumulated in one cycle, the value $f(t_k^+) = 1$ to the absence of the damage restoration process. The values between zero and a unit express partial restoration of damages. To determine the interval $\left(t_k^-; t_k^+\right)$ we need some special condition. This condition consists of specific features of the construction, its operation condition and the loading types.

Taking into account the expression (2.3), for the total energy of a cylindrical shell with damages taken into account, we can write:

$$\begin{aligned}
J = \frac{1}{2} R^2 \int_{x_1}^{x_2} \int_{y_1}^{y_2} \{ & N_{11}\varepsilon_{11} + N_{22}\varepsilon_{22} + N_{12}\varepsilon_{12} - M_{11}\chi_{11} - M_{22}\chi_{22} - M_{12}\chi_{12} + \\
& + N_{11} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot N_{11} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot N_{11} d\tau \right) + \\
& + N_{22} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot N_{22} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot N_{22} d\tau \right) + \\
& + N_{12} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot N_{12} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot N_{12} d\tau \right) - \\
& - M_{11} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot M_{11} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot M_{11} d\tau \right) \\
& - M_{22} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot M_{22} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot M_{22} d\tau \right) - \\
& - M_{12} \left(\sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\overline{x}, t_k^+ - \tau) \cdot M_{12} d\tau + \int_{t_{n+1}^-}^{t} M(\overline{x}, t - \tau) \cdot M_{12} d\tau \right) \Bigg\} dxdy + \\
& + \rho_0 h \int_{x_1}^{x_2} \int_{y_1}^{y_2} \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] dxdy
\end{aligned} \tag{2.4}$$

Internal forces and moments involved in the expression (2.4) will be taken as follows [2]:

$$N_{ij} = \int\limits_{-h/2}^{h/2} (\sigma_{ij} + zw_{ij})\, dz;\; M_{ij} = -\int\limits_{-h/2}^{h/2} (\sigma_{ij} + zw_{ij})\, zdz \tag{2.5}$$

$$w_{11} = b_{11}\chi_{11} + b_{12}\chi_{22}; w_{22} = b_{12}\chi_{11} + b_{22}\chi_{22}; w_{21} = w_{12} = b_{66}\chi_{12}.$$

The stress σ_{ij} and strain ε_{ij} in the relations (2.5) in the middle surface are determined in the following form:

$$\sigma_{11} = b_{11}\varepsilon_{11} + b_{12}\varepsilon_{22}; \quad \sigma_{22} = b_{12}\varepsilon_{11} + b_{22}\varepsilon_{22}; \quad \sigma_{12} = b_{66} \tag{2.6}$$

$$\varepsilon_{11} = \frac{\partial u}{\partial x}; \quad \varepsilon_{22} = \frac{\partial \vartheta}{\partial y} + w;\, \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x};\, \chi_{11} = \frac{\partial^2 w}{\partial x^2}; \chi_{22} = \frac{\partial^2 w}{\partial y^2}; \chi_{12} = -2\frac{\partial^2 w}{\partial x \partial y}$$

$$b_{11} = \frac{E_1}{1-\nu_1\nu_2}; b_{22} = \frac{E_2}{1-\nu_1\nu_2}; b_{12} = \frac{\nu_2 E_1}{1-\nu_1\nu_2} = \frac{\nu_1 E_2}{1-\nu_1\nu_2}$$

For taking into account the inhomogeneity of the rods, we will consider that the elasticity modulus and density is a function of the coordinate x. In this case, for the total energy of the rods along the generatrix of cylindric shell we can write [1]:

$$J_i = \frac{1}{2}\sum_{i=1}^{k_1} \int_{x_1}^{x_2} \left[E_i(x)F_i \left(\frac{\partial u_i}{\partial x}\right)^2 + E_i(x)J_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + \right.$$

$$\left. + E_i(x)J_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + G_i(x)J_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2 \right] dx +$$

$$+\sum_{i=1}^{k_1} \rho_i(x)F_i \int_{x_1}^{x_2} \left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \frac{J_{kpi}}{F_i}\left(\frac{\partial \varphi_{kpi}}{\partial t}\right)^2 \right] dx \tag{2.7}$$

In the expressions (2.6), (2.7) E_1, E_2 are the main elasticity module of the material of the orthotropic cylindrical shell, u, υ, w are the displacements of the middle surface of the cylindrical shell, F_i, is the area of the cross section of the $i{-}th$rod fastened in the direction of the generatrix of the cylindrical shell, J_{yi}, J_{zi}, are inertia moments with respect to the axis parallel to the axis of the section passing from the gravity center, J_{kpi} is inertia moment at torsion, E_i, G_i are elasticity module of the $i{-}th$ rod fastened in the direction of the generatrix of the cylindrical shell at stretching and shift t is time, ρ_i is the density of the material of the $i - th$ rod fastened in the direction of the generatrix to the cylindrical shell, u_i, ϑ_i, w_i are the displacements of the points of the $i{-}\mathrm{th}$rod, ρ_0 is the density of the material of the cylindrical shell.

It is considered that the rods were rigidly built in to the cylindrical shell. This time the following contact conditions are satisfied:

$$u_i(x) = u(x, y_i)\,, \;; \vartheta_i(x) = \vartheta(x, y_i)\, w_i(x) = w(x, y_i) \tag{2.8}$$

The work A_0 done at the displacements of the shell by the force affecting the cylindrical shell as viewed from the medium, the work A_1 done at the displacements of the points of the shell by the force p acting in the surface of the cylindrical shell is as follows

$$A_0 = -\int_{x_1}^{x_2}\int_0^{2\pi} q_z w dx dy \tag{2.9}$$

$$A_1 = -4\int_{x_1}^{x_2}\int_0^{\pi/4} pwdxdy \tag{2.10}$$

The force affecting the cylindrical shell as viewed from the medium and involved in expression (2.9) is determined as follows:

$$q_z = k_v w - k_p\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) - \int_0^t (t-\tau)w(\tau)d\tau \tag{2.11}$$

Here k_ϑ is Winkler's coefficient, k_{p^-} is Pasternak coefficient and is found by experience, t-is time $\Gamma\,(t-\tau)$ is a viscosity kernel.

3 Solution methods

We will look for the shell displacements for $\xi = 0$ and $\xi = \xi_1$ writhing the boundary conditions $\vartheta = w = T_1 = M_1 = 0$ $(\xi = \frac{x}{R}; \xi_1 = \frac{l}{R})$:

$$\begin{aligned} u &= u_0 \cos n\varphi \cos\frac{m\pi}{\xi_1}\xi\ \sin\omega t;\\ \vartheta &= \vartheta_0 \sin n\varphi \sin\frac{m\pi}{\xi_1}\xi\ \sin\omega t;\\ w &= w_0 \cos n\varphi \sin\frac{m\pi}{\xi_1}\xi\ \sin\omega t; \end{aligned} \tag{3.1}$$

Here $u_0,\ \vartheta_0,\ w_0$ are unknown constants. Using the solutions (3.1), the expressions (2.9), (2.10) and (2.11), we can calculate the work done in the displacements by the force p acting in the surface of the shell and by the medium:

$$A_0 = -2\pi R\left[\frac{\omega l}{4\left(\omega^2+\psi^2\right)}\left(sin\omega t - e^{\psi t}sin^2\omega t + \frac{\psi}{\omega}e^{\psi t}sin^2\omega t\right)+\right.$$
$$\left. + \frac{k_\vartheta l sin^2\omega t}{4} + k_p\left(\frac{m^2\pi^2}{l^2}+\frac{k^2}{4R^2}\right)\frac{l sin^2\omega t}{4}\right]w_0^2 \tag{3.2}$$

$$A_1 = -\frac{4}{nk}\left(coskL-1\right)sin\frac{n\pi}{4}\left(p_0 sin\omega t + p_1 sin\omega_1 t sin\omega t\right)w_0 \tag{3.3}$$

By means of the approximation (3.1) we can determine the active loading period involved in the damage operator from the decreasing condition of the function:

$$\left[\left(\frac{\pi}{2}+2\pi k\right)/\omega;\ \left(\frac{3\pi}{2}+2\pi k\right)/\omega;\ \right].$$

The characteristic T, time is determined as the greatest one from the times t_n^+ . Taking into account the expressions (3.2), (3.3), substituting (3.1) in (2.1) integrating from $x_1 = 0$ to $x_2 = l$ from $y_1 = 0$ to $y_2 = 2\pi$from to $t_0 = 0$ to $t_1 = T$, for the Hamilton action $W_c = \int_{t_o}^{t_1} W dt$ we obtain $M(\overline{x}, t-\tau) = \gamma = const$):

$$W_A = \frac{\pi L h R^2}{4}\left\{\left[\left(k^2 b_{11} + \frac{n^2}{R^2}b_{66}\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right) - \frac{\gamma h}{\omega}F\left(T\right)T_1 + \frac{\gamma h^3}{16\omega}F\left(T\right)T_1\right]u_0^2 +\right.$$
$$+\left[\left(\frac{n^2}{R^2}b_{22} + k^2 b_{66}\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right) - \frac{\gamma h}{\omega}F\left(T\right)T_2 + \frac{\gamma h^3}{16\omega}F\left(T\right)T_2\right]\vartheta_0^2 +$$

$$+\left[\left(b_{22}-\frac{hk^2b_{12}}{4}-\frac{hn^2b_{22}}{4R^2}-\frac{hk^2}{2}\left(\frac{hb_{11}k^2}{3}+\frac{hn^2b_{12}}{3R^2}-b_{12}\right)-\right.\right.$$

$$\left.-\frac{h}{2}\left(\frac{hn^2k^2b_{12}}{3R^2}+\frac{hn^4b_{22}}{R^4}-\frac{n^2b_{22}}{R^2}\right)-\frac{h^2n^2k^2b_{66}}{6R^2}\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right)-$$

$$\left.-\frac{\gamma hF\left(t\right)}{\omega}\left(T_3-T_4-\frac{h^3n^2k^2b_{66}}{4R^2}\right)+\frac{\gamma h^3}{16\omega}F\left(T\right)\left(T_3+T_4+\frac{4h^3n^2k^2b_{66}}{9R^2}\right)\right]w_0^2+$$

$$+\left[\frac{2nk}{R}\left(-b_{12}-b_{66}\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right)+\frac{2nk}{R}\frac{\gamma h^2}{\omega}F\left(T\right)\left(b_{11}b_{12}+b_{11}b_{22}\right)-\right.$$

$$\left.-\frac{\gamma h^4}{16\omega}F\left(T\right)\frac{2nk}{R}\ b_{11}b_{12}+b_{12}b_{22}+b_{66}^2\right]u_0\vartheta_0+$$

$$+\left[\left(-2kb_{12}+\frac{hk^3b_{11}}{4}+\frac{hn^2kb_{12}}{4R^2}-\frac{hn^2kb_{66}}{2R^2}-\frac{kh^2}{2\omega}\left(b_{11}k^2\ +\right.\right.\right.$$

$$\left.\left.+\frac{n^2b_{12}}{R^2}-\frac{2n^2b_{66}}{R^2}\right)\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right)+\frac{2k\gamma h^2}{\omega}F\left(T\right)\left(b_{11}T_3+b_{12}T_4+\frac{hn^2b_{66}^2}{2R^2}\right)$$

$$\left.-\frac{k\gamma h^4}{8\omega}F\left(T\right)\left(b_{11}T_5+b_{12}T_6+\frac{hn^2b_{66}^2}{2R^2}\right)\right]u_0w_0+$$

$$+\left[\left(\frac{hk^2nb_{66}}{2R}+\frac{hk^2nb_{12}}{4R}+\frac{h^2n^3}{4R^3}\ b_{22}-\frac{khn}{R}b_{66}+\frac{2b_{22}n}{R}\right)\left(\frac{T}{2}-\frac{sin2\omega T}{4\omega}\right)-\right.$$

$$-\frac{2n\gamma h^2}{R\omega}F\left(T\right)\left(b_{12}T_3+b_{22}T_4+\frac{hk^2b_{66}^2}{2R}\right)+$$

$$\left.\left.+\frac{n\gamma h^4}{8R\omega}F\left(T\right)\left(b_{12}T_5+b_{22}T_6\right)\right]u_0w_0\right\}+\rho_0h\frac{\pi\omega^2L}{2}\left(u_0^2+\vartheta_0^2+w_0^2\right)\left(\frac{T}{2}+\frac{sin2\omega T}{4\omega}\right)-$$

$$-\frac{4}{nk}\left(coskL-1\right)sin\frac{n\pi}{4}\left[-\frac{p_0}{\omega}\left(cos\omega T-1\right)+\frac{1}{2}p_1\left(\frac{2}{\omega-\omega_1}sin\left(\omega-\omega_1\right)T-\right.\right.$$

$$\left.\left.-\frac{2}{\omega+\omega_1}sin\left(\omega+\omega_1\right)T\right]w_0+\left\{\frac{m^2\pi^2}{2l^2}\sum_{i=1}^{k_1}\left[F_iI_1\ sin^2n\varphi_iu_0^2+(J_{xi}I_2+J_{kpi}I_3)\,cos^2n\varphi_i\vartheta_0^2+\right.\right.$$

$$\left.+(J_{zi}I_2+J_{kpi}\ I_3)sin^2n\varphi_iw_0^2+kJ_{kpi}\ \ I_3sin2n\varphi_i\vartheta_0w_0\right]+$$

$$+\omega^2\sum_{i=1}^{k_1}F_i\left[I_4\ sin^2knu_0^2+I_5\left(1+\frac{J_{kpi}}{F_iR^2}\right)cos^2n\varphi_i\vartheta_0^2+I_5\left(1+\frac{J_{kpi}k^2}{F_iR^2}\right)sin^2n\varphi_iw_0^2+\right.$$

$$\left.\left.+I_5\ \frac{J_{kpi}}{F_iR^2}sin2n\varphi_i\vartheta_0w_0\right]\right\}\ \left(\frac{T}{2}-\frac{sin2T}{4}\right)-\ 2\pi R\left\{\frac{\omega l}{4\left(\omega^2+\psi^2\right)}\left[\frac{1}{\omega}\left(1-cos\omega T\right)+\right.\right.$$

$$\left.+\frac{1}{\psi}(e^{\psi T}sin^2\omega T-\frac{\omega}{1+\psi^2}(\psi-\frac{1}{2\omega}\left(e^{\psi T}cos2\omega T-1\right)\right)\right\}w_0^2. \tag{3.4}$$

Here,

$$I_1=\int_o^l\widetilde{E_i}\left(x\right)cos^2\frac{m\pi x}{l}\ dx;I_2=\int_o^l\widetilde{E_i}\left(x\right)sin^2\frac{m\pi x}{l}\ dx;I_3=\int_o^l\widetilde{G_i}\left(x\right)sin^2\frac{m\pi x}{l}\ dx$$

$$I_4=\int_0^l\tilde{\rho}\left(x\right)cos^2\frac{\pi x}{l}dx;I_5=\int_0^l\tilde{\rho}\left(x\right)sin^2\frac{\pi x}{l}dx;F\left(T\right)=\ \frac{1}{2\omega}\left(sin^2\omega T+4R_tsin^2\frac{\omega T}{2}\right).$$

Using the Ostrogradski-Hamilton action's $\delta W = 0$ stationarity principle, with respect to the constants $u_i,\ \vartheta_i,\ w_i$ we obtain the following system of inhomogeneous equations:

$$\left\{ \begin{array}{l} \breve{\varphi}_{11}u_0 + \breve{\varphi}_{12}\vartheta_0 + \breve{\varphi}_{13}w_0 = 0 \\ \breve{\varphi}_{21}u_0 + \breve{\varphi}_{22}\vartheta_0 + \breve{\varphi}_{23}w_0 = 0 \\ \breve{\varphi}_{31}u_0 + \breve{\varphi}_{32}\vartheta_0 + \breve{\varphi}_{33}w_0 = \breve{\varphi}_* \end{array} \right\} \tag{3.5}$$

Here,

$$\breve{\varphi}_* = -\frac{4}{nk}\left(coskL - 1\right) sin\frac{n\pi}{4} \times \left[-\frac{p_0}{\omega}\left(cos\omega T - 1\right)\right.$$
$$\left. + \frac{1}{2}p_1\left(\frac{2}{\omega - \omega_1} sin\left(\omega - \omega_1\right)T - \frac{2}{\omega + \omega_1} sin\left(\omega + \omega_1\right)T\right)\right] \tag{3.6}$$

Having solved the system (3.5) we can determine the constants u_0, ϑ_0, w_0 :

$$u_0 = \frac{\Delta_1}{\Delta};\qquad \vartheta_0 = \frac{\Delta_2}{\Delta};\quad w_0 = \frac{\Delta_3}{\Delta} \tag{3.7}$$

In the expressions of (3.7) Δ is the principal determinant of the system (3.5), $\Delta_i\ (i = 1, 2, 3)$ are auxiliary determinants. Substituting the expressions of (3.7) in (2.11) for the displacements of the shell points we obtain:

$$u = \frac{\Delta_1}{\Delta}\cos n\varphi \cos\frac{m\pi}{\xi_1}\xi \sin\omega t;$$
$$\vartheta = \frac{\Delta_2}{\Delta}\sin n\varphi \sin\frac{m\pi}{\xi_1}\xi \sin\omega t; \tag{3.8}$$
$$w = \frac{\Delta_3}{\Delta}\cos n\varphi \sin\frac{m\pi}{\xi_1}\xi \sin\omega t$$

For determining the critical force, we will use the equality $\frac{\Delta_3}{\Delta} = w_0$. From this equality we obtain:

$$\breve{\varphi}_* = \frac{w_0\Delta}{\breve{\varphi}_{11}\breve{\varphi}_{22} - \breve{\varphi}_{21*}\breve{\varphi}_{12}} \tag{3.9}$$

Taking into account the expression (3.6) of $\underset{*}{\breve{\varphi}}$ in (3.9), we can find the force p_1:

$$p_1 = \left[\frac{w_0\Delta}{\alpha_{22}(\breve{\varphi}_{11}\breve{\varphi}_{22} - \breve{\varphi}_{21}\breve{\varphi}_{12})} - \frac{\alpha_{11}p_0}{\alpha_{22}}\right] \tag{3.10}$$

Here,

$$\alpha_{11} = \frac{4}{nk\omega}\left(coskL - 1\right) sin\frac{n\pi}{4}\left(cos\omega T - 1\right)$$

$$\propto_{22} = -\frac{2}{nk}\left(\frac{2}{\omega - \omega_1} sin\left(\omega - \omega_1\right)T - \frac{2}{\omega + \omega_1} sin\left(\omega + \omega_1\right)T\right)\left(coskL - 1\right) sin\frac{n\pi}{4}$$

By changing the wave numbers n and m we calculate p_1 and choosing $(p_1)_{min}$ from them, find the critical force p_{1b}. When there are no damages, in the expression $F\left(T\right) = \frac{1}{2\omega}\left(sin^2\omega T + 4R_t sin^2\frac{\omega T}{2}\right)$ taking $R_t = 0$, we can calculate the critical value of the force, p_{1b}.

4 Numerical calculations and conclusions

The expression (3.10) was numerically calculated for the following values of variables:

$$E_i = 6,67 \cdot 10^9 \frac{H}{m^2}; R = \ 160\,mm; h = 0,45mm; \nu_1 = 0,11; \ \nu_2 = 0,19; \ L = 800mm;$$

$$\beta = 0,05; \ \text{A} = 0,\ 1615; \rho_0 = \rho_i = 7,8q/sm^3; \omega_1 = 2\omega; k_1 = 10; \ n = 8;$$

$$\frac{I_{yi}}{2\pi R^3 h} = 0,8289 \cdot 10^{-6}; \quad \frac{F_i}{2\pi Rh} = 0,1591 \cdot 10^{-1}; \ \frac{I_{kp.i}}{2\pi R^3 h} = 0,5305 \cdot 10^{-6}.$$

$$h_i = 1,39mm; \ w_0 = 0,1\,mm, \omega = 100\,hs, \ k_\vartheta = 10^6 N/m^3, \quad k_p = 10^4 N/m.$$

The results of the calculations were given in the fig. 2 and fig.3 in the form of dependence of p_{1b}/E_1-on the number of ribs k_1 , dependence of the number of longitudinal waves m for different values of the ratio, $\frac{E_1}{E_2}$. In both figures, the dashed lines correspond to the values of critical forces of undamaged cylindrical shell stiffened with the ribs along the axis, the solid lines correspond to the values of critical forces of damaged cylindrical shell stiffened with ribs along the axis. As can be seen from Fig. 2, increasing the number of ribs $k_{1,}$ critical force at first increases and after certain value begins to decrease. This is explained by the fact that increasing the number of ribs their volume also increases and as a result the inertia affects to the vibration process are strengthened.

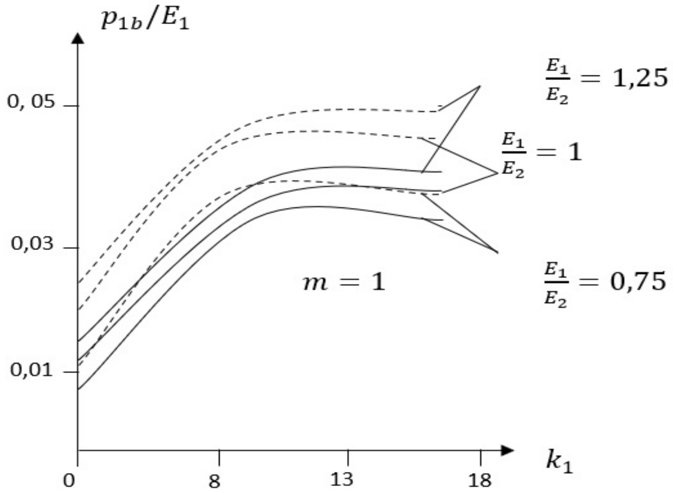

Fig. 2. Dependence of the critical force on the amount of longitudinal waves

At the same time as these properties become stronger, the value of the critical force increases. Fig. 3 shows that increasing the number m of waves in longitudinal direction, the critical force increases, obtains maximum value, again decreases and approaches the critical value corresponding to the unstiffened cylindrical shell.

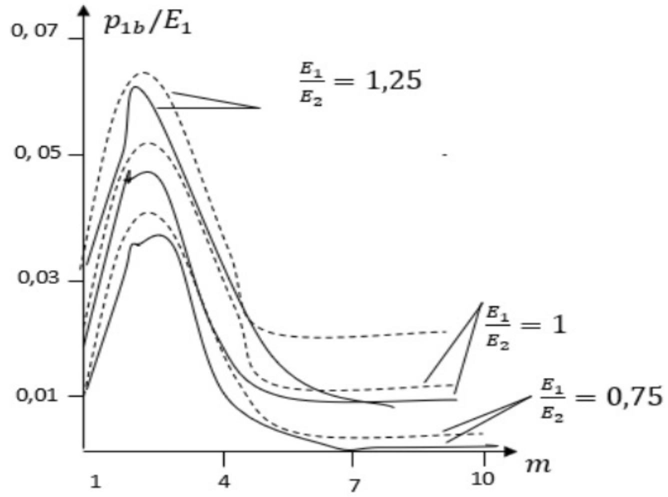

Fig. 3. Dependence of the critical force on the number m of longitudinal waves.

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