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Mixed-type variational principle for creep problems considering the aggressiveness of external fields

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Abstract. *A mixed type variational method for the three-dimensional theory of continuum mechanics has been developed. It is applied for the analysis of the stress-strain state (SSS) of inhomogeneous anisotropic elastoplastic bodies during creep under the action of neutron fluxes at finite deformations with consideration of damageability and diffusion. A modification of the formed and proved theorem for the case of composite material when different phase inclusions in heterogeneous medium are clearly expressed is given in the paper.*

Keywords. Creep, irradiation dose, SSS, damageability, corrosion, variational principle, generalized strengthening theory, composite.

Mathematics Subject Classification (2010): 74E10, 49S05

1 Introduction

Analysis of experimental data shows that irradiation accelerates deformation process and promotes destruction of structural elements. In published papers it has been shown that consideration of radiation effects on behavior of deformable structural materials and bodies is necessary for prediction of stress-strain state (SSS) and durability of a structure at simultaneous consideration of other effects [8, 13, 15].

Under the influence of irradiation the elastic modulus and Poisson's coefficient of a material change, radiation deformations arise. The influence of penetrating neutron irradiation on the mechanical properties of materials is significant and must be taken into account in the calculation and design of structural elements, devices, engines and reactors, since the supporting structures of nuclear reactors and radiation waste storage facilities must have sufficient strength and durability of materials and structures.

The data on effect of irradiation on elastic, plastic, strength properties of materials under constant and variable loads, on cracking resistance, on ability of materials to stress relaxation and energy dissipation under cyclic deformation are of interest for researchers [15].

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Penetrating neutron irradiation, as shown in many theoretical studies and accumulated experimental data sets, affects especially the creep characteristics [15, 19, 23]. Plastics, which are widely used in nuclear engineering as structural materials, are in most cases more sensitive to irradiation than metals [12]. The acceleration of the deformation and fracture process depends on the stress level, temperature, irradiation characteristics [8, 11]. At high temperatures the creep process in metals and alloys is followed by instantaneous elastic and plastic deformations [10, 15, 23].

If plastic deformation is associated with heating of the body or structure, only the linear deformations change by the same values as a result of heating and the components of the deformation deviator do not change during heating [6]. In elastic and elastic-plastic systems, unstable positions occur under certain values of external loads, i.e. very large displacements occur as a result of a minor change in the system parameters. And in creeping structural elements, the displacement rates can become unacceptably large under certain circumstances. Critical states of such types can occur after a certain period of residence of a structural element under load, therefore, we introduce the concept of critical time [8]. Researches on determine the effect of irradiation, damageability and corrosion on bearing capacity of structural elements in chemically aggressive media under creeping result in nonlinear boundary value problems of continuum mechanics [1, 9], analytical solution of which is impossible.

Therefore, the theoretical strength of an ideal crystal lattice of constructional metals, corresponding to the simultaneous breaking of all intermolecular bonds, is very high - only ten times less than the Young's modulus of elasticity of $2,1 \cdot 10^3$ GPa. It is known that the strength of real solids is several orders of magnitude lower than that of structural metals with a perfect structure. The strength of most structural polymers does not exceed 100-150 MPa and the Young's modulus is 3-4 GPa [22]. Further improvement of the mechanical properties of polymers through the design of the chemical structure of the molecules is not promising, so we have to look for other ways to improve the elastic strength characteristics of structural polymers, among which the transition from polymers to nanocomposites is currently considered most promising. Carbon nano-tubes (CNTs) are being used as a small strengthening additive in polymers because of their combination of extraordinary mechanical, electrical and thermal properties [7].

In this paper, a mixed type variational method for the three-dimensional theory of continuum mechanics (3D) is developed for determining the deflection of heterogeneous anisotropic elastoplastic bodies during creep under the action of neutron flows at finite deformations, taking into account the damageability and corrosion of the material. A modification of the formed variational theorem for the case of composite material when different phase inclusions are distinctly expressed in the heterogeneous medium is presented.

2 Laws of deformation

Assume that in the material there is instantaneous elastoplastic deformation $\varepsilon_{ij}^{(1)}$, creep deformation $\varepsilon_{ij}^{(1)}$ and deformation resulting from irradiation $\varepsilon_{ij}^{(2)}$, so that the covariant components of the total deformation will be

$$\varepsilon_{ij} = \varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)} + p_{ij}$$

In velocities

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{(1)} + \dot{\varepsilon}_{ij}^{(2)} + \dot{p}_{ij}, \tag{2.1}$$

where $\dot{\varepsilon}_{ij}$ is the deformation rate tensor, the dots above the letters denote the differentiation of the corresponding quantities by physical time.

Detailed consideration of the processes of plasticity, creep and damage accumulation without taking into account their mutual influence is characteristic of many theories used in calculations. However, the phenomenon of creep is accompanied by a process of damage accumulation of material and the generalized theory formulated by Yu.N. Rabotnov [15] in the form of the concept of mechanical state equation is an opportunity for their joint description if the damage parameters are introduced into the kinetic relationships of creep as structural parameters. To model processes of nonstationary corrosion, long-term strength and their interrelations for structurally stable material at given moment of time, we can write [8, 15, 19]

$$\dot{p}_{ij} = \dot{p}_{ij}(\sigma^{\alpha\beta}, T, q_1, ..., q_N)$$

During the deformation process, the structural parameters change according to the following non-integrable equations:

$$dq_i = a^i_{rs} dp_{rs} + b^i_{rs} d\sigma_{rs} + \varphi^i dt + d^i dT, (i = 1, 2, ..., n) \tag{2.2}$$

Here $a^i_{rs}, b^i_{rs}, \varphi^i, d^i$ the functions from $p_{ij}, \sigma^{rs}, T, t$ and also the parameters $q_1, q_2, ..., q_N$, i.e., $\varphi^i = \varphi^i(p_{ij}, \sigma^{\alpha\beta}, T, t, q_1, q_2, ..., q_N)$ etc. Structural parameters can be chosen differently: as creep deformation, damage, dissipated work, etc. In parallel with theoretical development of models of deformation and fracture of materials in conditions of influence of radiation environment, the system of experimental data on short-term and long-term mechanical characteristics for a definite material at various temperature and force influences has been accumulated [6, 15].

With known backgrounds of stress, deformation and durability changes under certain radiation active conditions, the refinement of experimental results for specific structural materials continues. The design methods of a structure in most cases are reduced to a calculation with variable and varying elastic characteristics under the influence of irradiation, considering the bulk radiation components of deformation by analogy with thermal deformations [6]. When building design models of deformation and fracture of materials and structures made of composites and metals, for aircraft objects, power plants and other engineering structures it is necessary to take into account all possible loads and the effects of the environment, especially chemical and radiation aggressiveness. Among many types of radiation ionizing radiation, only neutron flux leads to a change in the mechanical properties of materials and, as a result, to a change in the behaviour of structures as a whole [13]. Changes in the properties of structural materials begin to show up when irradiated with a fluence of neutrons above 1×10^{18} neutron/cm^2 [14]. The main phenomena determining the serviceability of structural materials in special-purpose power plants include high and low temperature embrittlement, radiation creep, reduction of resistance to corrosion failure and others. The effect of neutron irradiation on the durability properties of materials under constant and alternating loads is determined by both the mode of irradiation itself and the state of structure of the materials [4]. Irradiation modes are characterized by integral neutron dose, energy spectrum, temperature and irradiation medium.

Experimental data shows that irradiation significantly decreases the plasticity of steels and nickel alloys, but at high doses of irradiation (up to 10^{22} neutron/cm2) they preserve zones of plasticity in the range of normal and moderately elevated temperatures.

Since at high temperatures the effects of creep and slow (gradual) fracture of materials are found, consideration of the effect of irradiation on creep and long-term strength of materials is necessary. Comparison of creep curves corresponding to the same stress level shows that the kinetics of creep in irradiated steel is the same, but the deformation and fracture processes in irradiated material are much more intense than in unirradiated material. Irradiation causes more intense deformation and fracture processes and the acceleration of these processes depends on stress level, temperature and irradiation characteristics. Therefore, using the theory of structural parameters by Yu.N. Rabotnov some parameter integrally taking into account the effect of irradiation intensity, energy spectrum and irradiation dose on material structures is introduced into the phenomenological model. The body temperature is assumed to be the same everywhere and independent of time.

Based on the notion of [15], we have

$$\dot{p}_{ij} = \dot{p}_{ij}(\varepsilon_{\alpha\beta}, \sigma^{\alpha\beta}, \omega, c) \tag{2.3}$$

kinetic diffusion equation [10]

$$\dot{c} = div(D\nabla c) - kc \tag{2.4}$$

kinetic damage equation [3]

$$\dot{\omega} = \varphi(\sigma^{\alpha\beta}, \omega, c) \tag{2.5}$$

Here ω - damage parameter, c - parameter describing concentration level of chemically aggressive medium, $D = D(\sigma^{\alpha\beta}, \omega, c)$ - diffusion coefficient, ∇ - Hamilton differential operator, $k = const$ - characteristic speed of chemical reaction, kc - rate of decay of chemical bonds under the influence of aggressive chemical medium.

Among many effects caused by neutron irradiation, a special place is taken by the issues related to volumetric expansion and change of physical-mechanical properties of the body [6, 16]. The change of volumetric deformation under neutron irradiation proceeds rather slowly [23], therefore, the dynamic effects can be neglected in VAT estimation, and the duration of irradiation in time t can be considered as a parameter. The volume deformation (expansion) θ is a function of coordinates and irradiation dose d, i.e.

$$\theta = \theta(x^k, d).$$

Here, it is assumed that the change in the mechanical properties of the material at each point depends only on the irradiation dose at that point and on the temperature, and does not vary with changes at other points. The degree of change in mechanical properties depends on the irradiation dose, which is measured by the number of neutrons that enter the body through 1 cm^2 of the surface. If n (1/cm^3) is the number of neutrons in the unit of flux volume and the average flux speed, with constant irradiation intensity $d = nvt$ neutrons will penetrate into the body through 1cm^2 of the surface in 1 second. If the irradiation intensity remains unchanged, then the irradiation dose d can be taken as a parameter that describes the deformation process along with time. For this reason, in formula (**??**) and following, the point above the quantities will denote the differentiation by d. Changing over time, the dose level is distributed unevenly over the thickness of the structure and leads to a non-uniformity of the mechanical properties. If the flux rate is independent of time, then a total neutron flux $N = nvt \exp \mu(z - h/2)$ neutron/cm^2 will pass through a unit area of a plate with a thickness h over time [17]. In the case where deformation and displacement are constrained for some reason, internal forces and stresses arise in the deformed body [6] and the components of the strain tensor change in all [3].

$$\dot{\varepsilon}_{ij}^{(1)} = \left\{ C_{ijkm}(x^k, d)\sigma^{km} \right\}^{\cdot} (i, j, k, m = \overline{1, 3}),$$

$$\dot{\varepsilon}_{ij}^{(2)} = \dot{\theta}(x^k, d)\delta_{ij},$$

where δ_{ij} is the Kronecker tensor. Finally, for the components of the total strain rate tensor we have

$$\dot{\varepsilon}_{ij} = \left\{ C_{ijkm}\sigma^{km} \right\}^{\cdot} + \dot{p}_{ij} + \dot{\theta}\delta_{ij}. \tag{2.6}$$

Note that at $d = 0$, we have $\theta = 0$, and the covariant components of the fourth rank tensor C_{ijkm} for the anisotropic case are the physical and mechanical characteristics of the material of the unirradiated body C_{ijkm}. In metals and alloys, as well as in structures made of them, as a result of irradiation at high temperatures, the processes of creep and accumulation of damage occur, which depend on the type of stress state [6, 8].

3 Problem statement and solution method

Ignoring the dynamic effects, let us consider the equilibrium of a deformable solid body of volume V and bounded by a sufficiently smooth closed surface S. On some part of the surface S_u only the components of the displacement vector $\bar{u}_i$ are given and on the remaining part S_σ the load $\bar{T}^j$ is given. Since the variational theorem will be applied to solving problems of buckling of thin-walled structural elements, the finite-strain relations and nonlinear equilibrium equations are used here. Then the geometrically nonlinear theory of equilibrium of an elastic-plastic body in a chemically active medium in creep under the action of neutron flux will be described by means of the following boundary value problem

$$\begin{aligned}
\nabla_j \left\{ \sigma^{ij} (\nabla_i u^k + \delta_i^k) \right\} &= 0, (k = \overline{1,3}), \\
\dot{\varepsilon}_{ij} &= \left\{ C_{ijkm} \sigma^{km} \right\}^{\cdot} + \dot{p}_{ij} + \dot{\theta} \delta_{ij}, \\
2\varepsilon_{ij} &= \nabla_i u_j + \nabla_j u_i + \nabla_i u^k \nabla_j u_k, \\
T^i &= \bar{T}^i, x^k \in S_\sigma, \\
u_i &= \bar{u}_i, x^k \in S_u, \\
\dot{c} &= div(D\nabla c) - kc, \\
\dot{\omega} &= \varphi(\sigma^{\alpha\beta}, \omega, c).
\end{aligned} \tag{3.1}$$

where $T^k = \sigma^{ij} n_j \left(\nabla_i u^k + \delta_i^k \right)$

And $S = S_\sigma \bigcup S_u$, $\nabla_j -$a covariant differentiation operator is involved.

In some variational principles, based on varying tensors of stress and strain rates, a more general assumption is made about the elastic-plasticity law for the instantaneous strain.

$$\dot{\varepsilon}_{ij}^{(1)} = \left\{ C_{ijkl}(x^k, d) \sigma^{kl} \right\}^{\cdot}, x^k \in E_3$$

It is proved that the problem under consideration is equivalent to the problem formulated in the form of a variational principle. Note that in the theory of unsteady creep the possibilities of formulation of variational equations are wider than in the theory of steady-state creep and nonlinear elasticity theory, because in addition to tensors σ^{ij} and ε_{ij} in the controls appear tensors $\dot{\sigma}^{ij}$ and $\dot{\varepsilon}_{ij}$. In three-dimensional Euclidean space we will consider the creep process in an elastic-plastic anisotropic medium, which is subjected to irradiation by a neutron flux. In the case of a complex stress state, one must know the dependences of stress components on strain components in all stages of deformation. These dependencies are established in the theories of plasticity and creep.

This paper investigates the carrying capacity of thin-walled structure elements and solids under creep under the action of external physical fields and influences. A mixed-type variational principle in creep has been formulated for geometrically nonlinear problems of plastic bodies, taking into account within one functional the damage, diffusion process and irradiation with neutron flux.

$$J = \int_V \left\{ \dot{\sigma}^{ij} \dot{\varepsilon}_{ij} + \frac{1}{2} \sigma^{ij} \nabla_i \dot{u}^k \nabla_j \dot{u}_k - \frac{1}{2} C_{ijkm} \dot{\sigma}^{ij} \dot{\sigma}^{km} - \right.$$

$$-\dot{C}_{ijkm}\sigma^{km}\dot{\sigma}^{ij} - \left(\dot{\varepsilon}_{ij}^{(2)} + 2\dot{p}_{ij}\right)\dot{\sigma}^{ij} + \lambda_\omega\left(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi\right) +$$

$$+\lambda_c\left[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}\right]\Bigg\}dV - \int_{S_\sigma}\dot{\bar{T}}^i\dot{u}_i dS - \int_{S_u}\dot{T}^i(\dot{u}_i - \dot{\bar{u}}_i)dS \qquad (3.2)$$

In the functional the independent varying quantities are and , and the weight functions whose values are selected depending on the type of interpolation functions to refine the approximations. To overcome the difficulty of solving the variation problem by direct methods, we abandon the exact solution of the kinetic equations (2.4) and (2.5), replacing them by approximate integral relations

$$\int_V \lambda_\omega[\dot{\omega} - \varphi(\sigma^{\alpha\beta}, p_{ij}, \omega)]^2 dV \approx 0$$

$$\int_c \lambda_c[\dot{c} - div(D\nabla c) + kc]^2 dV \approx 0$$

where the damage and concentration level functions of the corrosive medium are searched for as a series

$$\omega = \sum_{k=1}^{p} a_k(t)\psi_k(x_j); a_k(0) = 0;$$

$$c = \sum_{k=1}^{m} b_k(t)\eta_k(x_j); \quad b_k(0) = 0;$$

$$c = \sum_{k=1}^{m} c_k(t)\eta_k(x_j); c_k(0) = 0;$$

$$c(t, x_j)\big|_{x_j=0} = c_0, \qquad \left.\frac{\partial c(t, x_j)}{\partial x_j}\right|_{x_j=0} = c^*(t)$$

and the corresponding boundary conditions for the concentration function

$$\frac{d\eta_k(0)}{dx_i} \text{ and } \eta_k(0).$$

Thus, this creep problem is reduced to a system of differential equations with a known systematics of procedures, i.e. with an algorithm that allows one to approach the mathematical solution of these problems.

Many different direct methods for constructing approximate solutions are possible, as well as direct methods for qualitative analysis on the existence and uniqueness of the solution or for deriving a priori evaluations for this problem.

The stationarity condition of the functional $\delta J = 0$ gives a system of equations, which is a mathematical model of the problem of determining the true stress-strain state of an elastoplastic continuous medium irradiated by a neutron flux in the process of creeping at finite deformations.

Functional (3.2) is derived on the basis of variation principles [2, 18, 19], formulated for the creep problems and variation method of the theory of plasticity of inhomogeneous bodies, proposed for problems with regard to irradiation in the geometrically linear formulation [3].

Let's find a variation of the functional (3.2) in the curvilinear coordinate system. Taking into account that the variation operator acts on velocities of quantities, we obtain from (3.2)

$$\delta J=\int_V\{\dot\varepsilon_{ij}\delta\dot\sigma^{ij}+\dot\sigma^{ij}\delta\dot\varepsilon_{ij}+\frac{1}{2}\sigma^{ij}\nabla_i\dot u^k\delta(\nabla_j\dot u_k)+\frac{1}{2}\sigma^{ij}\nabla_j\dot u_k\delta(\nabla_i\dot u^k)-$$

$$-\frac{1}{2}C_{ijkm}\dot\sigma^{km}\delta\dot\sigma^{ij}-\frac{1}{2}C_{ijkm}\dot\sigma^{ij}\delta\dot\sigma^{km}-\dot C_{ijkm}\sigma^{km}\delta\dot\sigma^{ij}-(\dot\varepsilon^{(2)}_{ij}+\dot p_{ij})\delta\dot\sigma^{ij}+\lambda_m(\dot\omega-\varphi)\delta\dot\omega+$$

$$+\lambda_c[\dot c-div(D\nabla c)-kc]\delta\dot c\}dV-\int_{S_\sigma}\dot{\bar T}^i\delta\dot u_i dS-\int_{S_u}[(\dot u_i-\dot{\bar u}_i)\delta\dot T^i]dS. \qquad (3.3)$$

It was taken into account that, by definition, the volumetric deformation is a function of coordinates and irradiation dose, and that the creep deformation rate generally depends on stress, temperature, time and structural parameters [15, 18, 23], therefore

$$\delta\dot\theta=0,\delta\dot p_{ij}=0,$$

and to satisfy the boundary conditions, the following equations $\delta\dot T^i=0$ are taken on $S_\sigma$and $\delta\dot u_i=0$ on S_u, respectively.
Since the tensor C_{ijkl} is velocity-independent, the relations are valid

$$\delta C_{ijkm}=0,\delta\dot C_{ijkm}=0,\delta\dot\varepsilon^{(1)}_{ij}=C_{ijkl}\delta\dot\sigma^{kl},$$

as well as equality

$$C_{ijkm}\dot\sigma^{km}\delta\dot\sigma^{ij}=C_{ijkm}\dot\sigma^{ij}\delta\dot\sigma^{km}$$

$$-\frac{1}{2}C_{ijkm}\dot\sigma^{km}\delta\dot\sigma^{ij}-\frac{1}{2}C_{ijkm}\dot\sigma^{ij}\delta\dot\sigma^{km}-\dot C_{ijkm}\sigma^{km}\delta\dot\sigma^{ij}-(\dot\varepsilon^{(2)}_{ij}+\dot p_{ij})\delta\dot\sigma^{ij}=$$

$$=-\left(C_{ijkm}\dot\sigma^{km}+\dot C_{ijkm}\sigma^{km}\right)\delta\dot\sigma^{ij}-(\dot\varepsilon^{(2)}_{ij}+\dot p_{ij})\delta\dot\sigma^{ij}=$$

$$=-\left[\left(C_{ijkm}\sigma^{km}\right)^{\cdot}+\dot\varepsilon^{(2)}_{ij}+\dot p_{ij}\right]\delta\dot\sigma^{ij} \qquad (3.4)$$

Consider separately the fourth term in expression (3.3). From the symmetry of the stress tensor $\sigma^{ij}=\sigma^{ji}$, we have

$$\frac{1}{2}\sigma^{ij}\nabla_j\dot u_k\delta(\nabla_i\dot u^k)=\frac{1}{2}\sigma^{ij}\nabla_i\dot u^k\delta(\nabla_j\dot u_k).$$

Hence, the equality of the third and fourth terms in (3.3) follows. This circumstance and formula (3.4) allow us to simplify expression (3.3) and write it down as follows.

$$\delta J=\int_V\left\{\dot\varepsilon_{ij}\delta\dot\sigma^{ij}+\dot\sigma^{ij}\delta\dot\varepsilon_{ij}+\sigma_{ij}\nabla_i\dot u^k\delta\left(\nabla_j\dot u_k\right)-\frac{1}{2}C_{ijkm}\delta\dot\sigma^{ij}\dot\sigma^{km}-\right.$$

$$-\frac{1}{2}C_{ijkm}\dot\sigma^{ij}\delta\dot\sigma^{km}-\dot C_{ijkm}\sigma^{km}\delta\dot\sigma^{ij}-$$

$$\left.-\left(\dot\varepsilon^{(2)}_{ij}+\dot p_{ij}\right)\delta\dot\sigma^{ij}+\lambda_\omega\left(\dot\omega-\varphi\right)\delta\dot\omega+\lambda_c\left[\dot c-div\left(D\nabla c\right)-kc\right]\delta\dot c\right\}dV-$$

$$-\int_{S_\sigma}\dot{\bar T}^i\delta\dot u_i dS-\int_{S_u}\left[\left(\dot u_i-\dot{\bar u}_i\right)\delta\dot T^i\right]dS. \qquad (3.5)$$

The time derivatives of the strain components are calculated using (3.1), namely

$$\dot\varepsilon_{ij}=\frac{1}{2}\left\{\nabla_i\dot u_j+\nabla_j\dot u_i+\nabla_i u^k\nabla_j\dot u_k+\nabla_j u_k\nabla_i\dot u^k\right\},$$

and its variation is written as

$$\delta\dot{\varepsilon}_{ij} = \frac{1}{2}\left\{\delta\left(\nabla_i \dot{u}_j\right) + \delta\left(\nabla_j \dot{u}_i\right) + \nabla_i u^k \delta\left(\nabla_j \dot{u}_k\right) + \nabla_j u_k \delta\left(\nabla_i \dot{u}^k\right)\right\}.$$

Now rewrite the second term in (3.5) in a form suitable for the following tabs

$$\dot{\sigma}^{ij}\delta\dot{\varepsilon}_{ij} = \frac{1}{2}\left\{\dot{\sigma}^{ij}\delta\nabla_i \dot{u}_j + \dot{\sigma}^{ij}\delta\nabla_j \dot{u}_i + \dot{\sigma}^{ij}\nabla_i u^k \delta\nabla_j \dot{u}_k + \dot{\sigma}^{ij}\nabla_j u_k \delta\nabla_i \dot{u}^k\right\}$$

Let us rearrange in this formula the indices i and j. Using the tensor symmetry property σ_{ij} and performing some transformations characteristic of tensor analysis, we will have from here

$$\dot{\sigma}^{ij}\delta\dot{\varepsilon}_{ij} = \dot{\sigma}^{ij}\left\{\delta_i^k + \nabla_i u^k\right\}\delta\nabla_j \dot{u}_k \tag{3.6}$$

Then the variational equation (3.5) is written by means of the equality

$$\delta J = \int_V \{\dot{\varepsilon}_{ij}\delta\dot{\sigma}^{ij} + \dot{\sigma}^{ij}\left[\delta_i^k + \nabla_i u^k\right]\delta\nabla_j \dot{u}_k + \sigma^{ij}\nabla_i \dot{u}^k \delta(\nabla_j \dot{u}_k) -$$
$$- C_{ijkm}\dot{\sigma}^{km}\delta\dot{\sigma}^{ij} - \dot{C}_{ijkm}\sigma^{km}\delta\dot{\sigma}^{ij} -$$
$$- \left(\dot{\varepsilon}_{ij}^{(2)} + \dot{p}_{ij}\right)\delta\dot{\sigma}^{ij} + \lambda_\omega(\dot{\omega} - \varphi)\delta\dot{\omega} + \lambda_c\left[\dot{c} - div(D\nabla c) - kc\right]\delta\dot{c}\}dV -$$
$$- \int_{S_\sigma} \dot{\bar{T}}^i \delta\dot{u}_i dS - \int_{S_u}\left[(\dot{u}_i - \dot{\bar{u}}_i)\delta\dot{T}^i\right]dS \tag{3.7}$$

Separately, consider the integral

$$\int_V \dot{\sigma}^{ij}\left\{\delta_i^k + \nabla_i u^k\right\}\delta(\nabla_j \dot{u}_k)dV$$

By converting it using the Gauss-Ostrogradsky formula, we get:

$$\int_V \dot{\sigma}^{ij}\left\{\delta_i^k + \nabla_i u^k\right\}\delta\left(\nabla_j \dot{u}_k\right)dV =$$
$$\int_S \left\{\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right)n_j \delta\dot{u}_k\right\}dS - \int_V \left\{\nabla_j\left[\dot{\sigma}_{ij}\left(\delta_i^k + \nabla_i u^k\right)\right]\delta\dot{u}_k\right\}dV \tag{3.8}$$

Similarly, calculate the third integral, for which we write

$$\int_V \sigma^{ij}\nabla_i \dot{u}^k \delta\left(\nabla_j \dot{u}_k\right)dV = \int_S \sigma^{ij}n_j \nabla_i \dot{u}^k \delta\dot{u}_k dS - \int_V \nabla_j(\sigma^{ij}\nabla_i \dot{u}^k)\delta\dot{u}_k dV \tag{3.9}$$

Dividing in (3.8) and (3.9) the surface integrals S by the sum of the integrals over $S_\sigma$and S_u, and considering that the surface integrals S_σ are nonzero only on the surface where the load $\delta T^k = 0$ is given S_u, and on these integrals are equal to zero separately when the displacement $\delta u_i = 0$ is given , these formulas can be rewritten as

$$\int_V \dot{\sigma}^{ij}\left\{\delta_i^k + \nabla_i u^k\right\}\delta\nabla_j \dot{u}_k dV =$$
$$= \int_{S_\sigma}\left\{\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right)n_j \delta\dot{u}_k\right\}dS - \int_V \left\{\nabla_j\left[\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right]\delta\dot{u}_k\right\}dV, \tag{3.10}$$

$$\int_V \sigma^{ij}\nabla_i \dot{u}^k \delta\nabla_j \dot{u}_k dV = \int_{S_\sigma}\sigma^{ij}n_j \nabla_i \dot{u}^k \delta\dot{u}_k dS - \int_V \nabla_j\left(\sigma^{ij}\nabla_i \dot{u}^k\right)\delta\dot{u}_k dV \tag{3.11}$$

By introducing the transformed values of integrals (3.10) and (3.11) into (3.7) and collecting the terms at the same independent variations, we find

$$\delta J = \int_V \{[\dot{\varepsilon}_{ij} - (\dot{\varepsilon}_{ij}^{(1)} + \dot{p}_{ij} + \dot{\theta}\delta_{ij})]\delta\dot{\sigma}^{ij} -$$

$$- \left[\nabla_j \left(\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right) + \nabla_j\left(\sigma^{ij}\nabla_i \dot{u}^k\right)\right]\delta\dot{u}_k +$$

$$+\lambda_\omega(\dot{\omega} - \varphi)\delta\dot{\omega} + \lambda_c[\dot{c} - div(D\nabla c) - kc]\delta\dot{c}\}dV +$$

$$+\int_{S_\sigma}\left\{\left[\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right) + \sigma^{ij}\nabla_i\dot{u}^k\right]n_j - \dot{\bar{T}}^k\delta\dot{u}_k\right\}dS - \int_{S_u}\left(\dot{u}_k - \dot{\bar{u}}_k\right)\delta\dot{T}^k dS$$

Having the obvious equalities

$$\nabla_j\left[\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right] + \nabla_j(\sigma^{ij}\nabla_i\dot{u}^k) = \left\{\nabla_j\left[\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right]\right\}^{\cdot}$$

$$\dot{\sigma}^{ij}\left(\delta_i^k + \nabla_i u^k\right) + \sigma^{ij}\nabla_i\dot{u}^k = \left\{\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right\}^{\cdot}$$

for the expression we find:

$$\delta J = \int_V\left\{\left[\varepsilon_{ij} - (\varepsilon_{ij}^{(1)} + p_{ij} + \theta\delta_{ij})\right]^{\cdot}\delta\dot{\sigma}^{ij} - \nabla_j\left[\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right]^{\cdot}\delta\dot{u}_k +\right.$$

$$+\lambda_\omega(\dot{\omega} - \varphi)\delta\dot{\omega} + \lambda_c\left[\dot{c} - div(D\nabla c) - kc\right]\delta\dot{c}\}\,dV +$$

$$+\int_{S_\sigma}\left[\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)n_j - \bar{T}^k\right]^{\cdot}\delta\dot{u}_k dS - \int_{S_u}\left(\dot{u}_k - \dot{\bar{u}}_k\right)\delta\dot{T}^k dS = 0$$

Given the basic lemma of calculus of variations and formula (3.1), from the condition of zero-turning δJ, as the Euler equations in the curvilinear coordinate system we obtain

$$\left\{\nabla_j\left[\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right]\right\}^{\cdot} = 0, \tag{3.12}$$

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^M + \dot{p}_{ij} + \dot{\theta}\delta_{ij}, \tag{3.13}$$

$$\dot{\omega} = \varphi, \tag{3.14}$$

$$\dot{c} = div(D\nabla c) - kc, \tag{3.15}$$

$$\left[\sigma^{ij}n_j\left(\nabla_i u^k + \delta_i^k\right)\right] = \bar{T}^k, x^k \in S_\sigma, \tag{3.16}$$

$$u_k = \bar{u}_k, x^k \in S_u, \tag{3.17}$$

After integrating in time t equations (3.12) and boundary conditions (3.16) and (3.17), we finally write in curvilinear coordinate system

$$\nabla_j\left[\sigma^{ij}\left(\delta_i^k + \nabla_i u^k\right)\right] = 0, \varepsilon_{ij} = (C_{ijkl}\sigma^{kl}) + p_{ij} + \theta\delta_{ij}, \dot{\omega} = \varphi,$$

$$\dot{c} = div(D\nabla c) - kc, \left[\sigma^{kj}n_j\left(\nabla_i u^k + \delta_i^k\right)\right] = \bar{T}^k, x^k \in S_\sigma, u_k = \bar{u}_k, x^k \in S_u.$$

If we make some simplifying assumptions regarding the dependence of various displacements and stresses on some coordinate, we obtain modifications from the formulated theorem for thin-walled structural elements subjected to neutron irradiation in a corrosive environment during creep.

4 Modified variational principle for a composite body

The strength of real solids and structural elements is several orders of magnitude lower than the theoretical strength of the ideal crystal lattice of structural metals, corresponding to the simultaneous breaking of all intermolecular bonds, which is usually due to the existence of lattice defects. The strength of most structural polymers does not exceed 100-150 MPa and the Young's modulus is 3-4 GPa [8]. Further improvement of the mechanical properties of polymers through the design of the chemical structure of the molecules is not promising, so we have to look for other ways to improve the elastic strength characteristics of structural polymers, among which the transition from polymers to nanocomposites is currently considered most promising. Carbon nano-tubes (CNTs) are being used as a small strengthening additive in polymers because of their combination of extraordinary mechanical, electrical and thermal properties [7]. CNT production processes have been significantly improved and some of them scaled up, as a result, high quality single and multi-layer CNTs have become available on an industrial scale. For this reason, studying the mechanical properties of polymer nano-composites with CNTs, as well as evaluating the prospects for practical applications, is relevant. However, there are currently no mathematical models that can quantitatively predict the mechanical characteristics of a nano-composite based on data on its composition and some (as yet unknown) characteristics of individual components [5]. Based on the introduced index of carbon nanotube (CNT) efficiency in nanocomposite as the ratio of the load carried by nanotubes at a given average deformation of the matrix to the maximum possible load that can be transferred to the nanotubes at this deformation, the paper [22] presents an analysis of the results. Analysis of the data given in the published references has shown that in polymers the upper limit of CNT efficiency is achieved if a mesh of interconnected nano-tubes is formed inside the polymer. This mesh can be formed by integration of nano-tubes into polymer matrix through covalent bonding of nano-tubes by molecular bridges or through physical entanglement of nano-tubes between each other. In thermoplastic crystallising polymers, the upper limit of CNT efficiency is also achieved by increasing the degree of crystallinity and improving the microstructure of the polymer, including the use of orientation stretching of nanocomposite [19] and functionally graded materials (FGM). Composite materials include all heterogeneous media consisting of two or more phases; they also include virtually all alloys used in the manufacture of structural elements subjected to stresses and irradiation. At present, nuclear installations are used not only in stationary power facilities, but also in ship- and aircraft-building, space technology, etc., whose structures are made of composite materials. In this connection, we will further present a modification of the functional (3.2) for the case of a composite body, proved for geometrically nonlinear problems. For infinitely small deformations and small displacements the corresponding variational theorem is proved in [3].

Let us introduce a composite body which in a three-dimensional Euclidean space with curvilinear coordinates x^α occupies a region V bounded by a closed surface S. Let us now proceed to the description of the composite material. In formulating the contact boundary problem, we assume that the body consists of K elements (Fig. 4.1). The element k with number occupies a volume V_k with surface S_k. We assume that $S_k = S_k^{(1)} \bigcup S_k^{(2)}$, where $S_k^{(1)}$ is the boundary of the volume $V_k^{(1)}$ having no common points with S, and $S_k^{(2)}$ is the boundary of the volume $V_k^{(2)}$ being a part of the common boundary of the body.

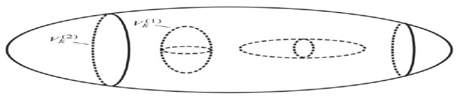

Fig. 4.1. Schematic of a composite body - $V_k^{(1)}$ is the volumes having no common points with the surface S

Apparently, the equality $S_k = S_k^{(1)}$ is true for the volumes $V_k^{(2)}$.

Let the surface $S_{k\sigma}^{(2)}$ be given the forces $T^{\alpha(0)}$, and the rest of the surface be given the displacements $u_\alpha^{(0)}$. Let us assume that the surfaces $S,\ S_k^{(1)}$ and $S_k^{(2)}$ are sufficiently smooth.

The theory of composite media used is based on the following assumptions:

- during the deformation process, the elements are in contact with each other along their entire common surface;

- the deformations are finite;

- full adhesion conditions are met on the contact surfaces.

Further we will assume that materials of different elements are different and their physical and mechanical properties are described according to the elastic-plastic law of the flow theory type (2.6).

For the k-th element we will introduce the following notations: $\sigma_{(k)}^{ij}$ and $\varepsilon_{ij(k)}$ - stress and strain tensor respectively; $u_{i(k)}$ - displacement vector; $n_{i(k)}$ - unit normal to the surface. Then let us consider the equilibrium of the volume V_k by applying to a part of the boundary $S_k^{(1)}$ the forces $T_{(k)}^{(00)}$ acting on it from the other volumes contacting with it, or considering the displacements $u_{i(k)}^{(00)}$. Then the geometrically nonlinear equilibrium theory is described by the following boundary problem:

$$\nabla_j \left\{ \sigma_{(k)}^{ij} \left(\delta_i^\alpha + \nabla_i u_{(k)}^\alpha \right) \right\} = 0, \left(\alpha = \overline{1,3} \right) \tag{4.1}$$

$$\dot{\varepsilon}_{ij(k)} = \left\{ C_{ijmn(k)} \sigma_{(k)}^{mn} \right\}^{\cdot} + \dot{p}_{ij(k)} + \dot{\theta}_{(k)} \delta_{ij}, \tag{4.2}$$

$$\dot{c} = div(D\nabla c) - kc,$$

$$\dot{\omega} = \varphi(\sigma^{\alpha\beta}, \omega, c).$$

$$2\varepsilon_{ij(k)} = \nabla_i u_{j(k)} + \nabla_j u_{i(k)} + \nabla_i u_{\alpha(k)} \nabla_j u_{(k)}^\alpha, \tag{4.3}$$

$$u_{i(k)} = \bar{u}_{i(k)}, S_{ku}, \tag{4.4}$$

$$T_{(k)}^\alpha = \bar{T}_{(k)}^\alpha, S_{k\sigma}, \tag{4.5}$$

where $T_{(k)}^\alpha = \sigma_{(k)}^{ij} n_j \left(\nabla_i u^\alpha + \delta_i^\alpha \right)$

It is important to note here that in the most general case, according to the general statement of the problem

$$\bar{u}_{i(k)} = \begin{cases} u_{i(k)}^{(0)} & \forall S \in S_{ku}^{(2)}, \\ u_{i(k)}^{(00)} & \forall S \in S_{ku}^{(1)}, \end{cases} \tag{4.6}$$

$$\bar{T}_{(k)}^\alpha = \begin{cases} T_{(k)}^{\alpha(0)} & \forall S \in S_{k\sigma}^{(2)}, \\ T_{(k)}^{\alpha(00)} & \forall S \in S_{k\sigma}^{(1)}, \end{cases} \tag{4.7}$$

To the above equations (4.1) - (4.5) the coupling conditions on $S_k^{(1)}$. At full coupling between neighbouring elements on the interface, we have continuity of displacements and forces

$$[u_i] = 0, \left[T^i \right] = 0.$$

where square brackets denote the jump in the corresponding value. In expanded form, the last equations take the form

$$u_{i(k)}^+ = u_{i(k)}^-, T_{(k)}^{i+} = T_{(k)}^{i-}. \tag{4.8}$$

Here ”+” and ”-” denote the function values at the coupling points when approaching them to the right and left of the contact line.

As before, let us again select an arbitrary element of volume V_k. Following (3.2), let us write out the corresponding functional for this volume:

$$J_k = \int_{V_k} \{\dot{\sigma}^{ij}_{(k)}\dot{\varepsilon}_{ij(k)} + \frac{1}{2}\sigma^{ij}_{(k)}\nabla_i \dot{u}^{\alpha}_{(k)}\nabla_j \dot{u}_{(k)\alpha} - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}_{(k)}\dot{\sigma}^{km}_{(k)} - \dot{C}_{ijkm}\sigma^{km}_{(k)}\dot{\sigma}^{ij}_{(k)} -$$

$$- \left(\dot{\varepsilon}^{(2)}_{ij(k)} + \dot{p}_{ij(k)}\right)\dot{\sigma}^{ij}_{(k)} + \lambda_\omega \left(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\phi\right) +$$

$$+\lambda_c \left[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}\right]\}dV - \int_{S_\sigma} \dot{\bar{T}}^i_{(k)}\dot{u}_{i(k)}dS - \int_{S_u} \dot{T}^i_{(k)}(\dot{u}_{i(k)} - \dot{\bar{u}}_{i(k)})dS \quad (4.9)$$

Here S_{ku} and $S_{k\sigma}$ are the boundary areas where displacements $\bar{u}_{i(k)}$ and forces $\bar{T}^i_{(k)}$ are assumed to be known or given by formulas (4.6) and (4.7).

Now, let us turn to generalisation of the functional for the whole volume when the body is composed of elements. In this case, the functional (4.9) needs to be modified so that the conjugation conditions (4.8) are taken into account. In this connection, let us first summarise (4.9) over all its components. Further, without prejudice to the rigor and compactness of the note, we will omit the index k of the quantities appearing under the integrals. Then we write

$$J = \sum_{k=1}^{K} J_k$$

or

$$J = \sum_{k=1}^{K} \int_{V_k} \left\{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i \dot{u}^{\alpha}\nabla_j \dot{u}_\alpha - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\dot{\sigma}^{km} -\right.$$

$$-\dot{C}_{ijkm}\dot{\sigma}^{ij}\sigma^{km} - (\dot{\varepsilon}^{(2)}_{ij} + \dot{p}_{ij})\dot{\sigma}^{ij} + \lambda_\omega \left(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi\right) +$$

$$+\lambda_c[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}]\}dV - \int_{S_\sigma} \dot{\bar{T}}^i \dot{u}_i dS - \int_{S_u} \dot{T}^i(\dot{u}_i - \dot{\bar{u}}_i)dS \qquad (4.10)$$

Then it becomes necessary to consider the contact conditions (4.9) of the problem. It is easy to see that in this case the surface integrals in (4.10) cancel each other and the functional (4.10) finally takes the form

$$J = \sum_{k=1}^{K} \int_{V_k} \left\{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i \dot{u}^{m}\nabla_j \dot{u}_m - \frac{1}{2}C_{ijkm}\dot{\sigma}^{ij}\dot{\sigma}^{km} -\right.$$

$$-\dot{C}_{ijkm}\sigma^{km}\dot{\sigma}^{ij} - (\dot{p}_{ij} + \dot{\theta}\delta_{ij})\dot{\sigma}^{ij} + \lambda_\omega \left(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi\right) +$$

$$\left.+\lambda_c \left[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}\right]\right\} dV - \int_{S_u} \dot{T}^i(\dot{u}_i - \dot{\bar{u}}_i)dS - \int_{S_\sigma} \dot{\bar{T}}^\alpha \dot{u}_\alpha dS. \qquad (4.11)$$

Thus, the derived functional (4.11) describes the equilibrium of the whole multicomponent body under neutron irradiation.

Let us consider a situation in which the materials of all volumes K_1 of the composite $V_k^{(1)}$, having a common surface with V, are described by the same physical and mechanical characteristics C^*_{ijmn}. Then we have a matrix of volume V_m, equal to

$$V_m = \sum_{k=1}^{K_1} V_k^{(1)},$$

with internal inclusions (phases) of volume $V_k^{(2.2)}$. In this special case we write the functional in the form

$$J = \int_{V_m} \{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i \dot{u}^k \nabla_j \dot{u}_k - (\dot{\varepsilon}^{(2)}_{ij} + \dot{p}_{ij})\dot{\sigma}^{ij} +$$

$$+\sum_{k=1}^{K_2} \int_{V_k} \left\{\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \frac{1}{2}\sigma^{ij}\nabla_i \dot{u}^k \nabla_j \dot{u}_k - (\dot{\varepsilon}^{(2)}_{ij} + \dot{p}_{ij})\dot{\sigma}^{ij}\right\} dV +$$

$$+\lambda_\omega(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi) + \lambda_c[\frac{1}{2}\dot{c}^2 - \dot{c}div(D\nabla c) - kc\dot{c}]\}dV - \int_{S_\sigma} \dot{\bar{T}}^i \dot{u}_i dS - \int_{S_u} \dot{T}^i(\dot{u}_i - \dot{\bar{u}}_i)dS.$$

Here K_2 is the total number of inner contact surfaces, and $K > K_2$. Finally, let us formulate the essence of the proposed variational methods more definitely. A characteristic feature of the constructed functionals is that they are written out in velocities. If similar Reisner-type functionals are constructed, then application of numerical methods, for example Ritz method, leads to solution of system of non-linear algebraic equations or system of transcendental equations which are not easily realizable on computer [2]. Application of a similar approximate method in this formulation leads to solution of a system of quasilinear ordinary differential equations with given initial conditions (Cauchy problem), numerical realization of which is much simpler [18].

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