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Investigation of non-stationary processes of an elastic half-space with a built-in elastic cylinder

Nazila B. Rassoulova · Tahmina M. Mahmudzade

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Abstract. *The process of propagation of non-stationary waves in an elastic half-space is studied, inside which a cylinder made of another linearly elastic material is embedded, on the end of which a normal impact is applied. A solution is found for the initial stages of the process, graphs of some characteristic quantities are plotted.*

Keywords. non-stationary waves · cylinder · half-space · Lame equation

Mathematics Subject Classification (2010): 74J05, 74B05

1 Introduction

The study of the dynamic resistance of bodies consisting of different materials is of great interest, but it is accompanied by great mathematical difficulties. The purpose of this work is to find methods for overcoming emerging complications and obtaining reliable analytical results.

Considering that the impact process occurs in a rather short period of time, the part that describes the initial moment of the process is extracted from the general solution. This part of the decisions can be called the main part because it not only meets all theoretical expectations but is also confirmed by the known data observed in practice [3].

The problem is solved using double integral transformations (Laplace and Fourier), which leads to rather complex solutions in transformations. To find the original, a method is used that was first used in [3]. Since this method has proven itself well for finding originals of this type, the solution obtained in the named work coincides exactly with the known data that are observed during tectonic processes in the earth's crust.

Graphs of some basic quantities are constructed, which are in good agreement with the theoretical assumptions of the process under study.

Nazila B. Rassoulova
Ministry of Science and Education Republic of Azerbaijan Institute of Mathematics and Mechanics, Baku, Azerbaijan
E-mail: rasulova@gmail.com

Tahmina M. Mahmudzade
Ministry of Science and Education Republic of Azerbaijan Institute of Mathematics and Mechanics, Baku, Azerbaijan
E-mail: tehminemahmudzade1996@gmail.com

In the literature [2] on this topic, the work is known, where only the stationary motion of the cylinder and half-space system is considered, and the dispersion characteristics of the process are studied.

2 Statement and method of solution

An elastic half-space is considered, which occupies a region $z \geq 0$, $a < r \leq \infty$ in a cylindrical coordinate system. An elastic cylinder made of another material is built into a part of the space $z \geq 0,\ \ 0 \leq r < a$. The blow in the moment is made only in the end area of the cylinder.

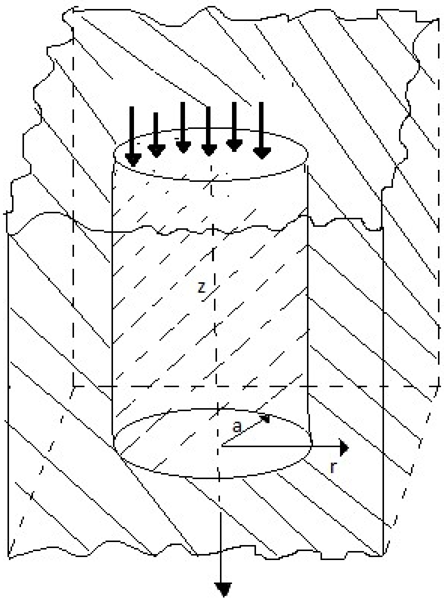


Fig. 2.1. Half space with built-in cylinder

The described process is controlled by the following initial-boundary value problem.

To solve the problem, the Lame equations are considered in the selected cylindrical coordinate system, the centre of which is located in the centre of the end area of the cylinder:

$$\begin{aligned} &\frac{\partial \sigma_{rr}^{(i)}}{\partial r} + \frac{\partial \sigma_{rz}^{(i)}}{\partial z} + \frac{\sigma_{rr}^{(i)} - \sigma_{\theta\theta}^{(i)}}{r} = \rho_{(i)} \frac{\partial^2 u_r^{(i)}}{\partial t^2} \\ &\frac{\partial \sigma_{rz}^{(i)}}{\partial r} + \frac{\partial \sigma_{zz}^{(i)}}{\partial z} + \frac{\partial \sigma_{rz}^{(i)}}{\partial z} = \rho_{(i)} \frac{\partial^2 u_z^{(i)}}{\partial t^2} \end{aligned} \tag{2.1}$$

$$\sigma_{rr}^{(i)} = 2\mu_{(i)}\varepsilon_{rr}^{(i)} + \lambda_{(i)}e^{(i)}\ ,\ \ \sigma_{\theta\theta}^{(i)} = 2\mu_{(i)}\varepsilon_{\theta\theta}^{(i)} + \lambda_{(i)}e^{(i)}\ ,\ \ \sigma_{zz}^{(i)} = 2\mu_{(i)}\varepsilon_{zz}^{(i)} + \lambda_{(i)}e^{(i)}$$

$$\sigma_{rz}^{(i)} = 2\mu_{(i)}\varepsilon_{rz}^{(i)}\ ,\ \ e^{(i)} = \varepsilon_{rr}^{(i)} + \varepsilon_{\theta\theta}^{(i)} + \varepsilon_{zz}^{(i)}$$

$$\varepsilon_{rr}^{(i)} = \frac{\partial u_r^{(i)}}{\partial r};\ \varepsilon_{\theta\theta}^{(i)} = \frac{u_r^{(i)}}{r}\ ;\ \ \varepsilon_{zz}^{(i)} = \frac{\partial u_z^{(i)}}{\partial z}$$

$$\varepsilon_{rz}^{(i)} = \frac{1}{2}\left(\frac{\partial u_r^{(i)}}{\partial z} + \frac{\partial u_z^{(i)}}{\partial r}\right)$$

$$\varepsilon_{z\theta}^{(i)} = \varepsilon_{r\theta}^{(i)} = 0 \tag{2.2}$$

Initial conditions are zero:

$$\left.\begin{array}{l} u_r^{(i)} = u_z^{(i)} = 0 \\ \frac{\partial u_r^{(i)}}{\partial t} = \frac{u_z^{(i)}}{\partial t} = 0 \end{array}\right\} t = 0 \;\; ; \;\; i = 1,2 \tag{2.3}$$

Here are $u_r^{(i)},\ u_z^{(i)}-$ the components of the displacement vector, $\varepsilon_{ij}^{(i)},\ \sigma_{ij}^{(i)}-$the components, respectively, of the strain and stress tensors in different media, density, λ_i and μ_i- Lame coefficients, $t-$time.

The value with index 1 refers to a cylinder, and the value with index 2 refers to a half-space.

On the end area of the cylinder, and around it, the boundary conditions are given in the following form:

$$\left.\begin{array}{l} \sigma_{zz}^{(1)} = \sigma_0(r) f(t) \\ u_r^{(1)} = 0 \end{array}\right\} z = 0,\ \ 0 \leq r \leq a$$

$$\left.\begin{array}{l} \sigma_{zz}^{(2)} = 0 \\ u_r^{(1)} = 0 \end{array}\right\} z = 0,\ \ a < r < \infty \tag{2.4}$$

Because the blow is applied only along the cross section of the cylinder.

In addition, there are obvious contact conditions along the surface.

$$\left.\begin{array}{l} u_r^{(1)} = u_r^{(2)} \\ u_z^{(1)} = u_z^{(2)} \\ \\ \sigma_{rr}^{(1)} = \sigma_{rr}^{(2)} \\ \sigma_{rz}^{(1)} = \sigma_{rz}^{(2)} \end{array}\right\} r = a,\ z > 0 \tag{2.5}$$

Applying the double integral Laplace and Fourier transform with respect to systems (2.1), considering (2.2) and (2.4), we obtain different systems of equations for different media [4]:

$$\left.\begin{array}{l} (\lambda_1 + 2\mu_1)(B_1\varphi_1) = q(B_2\psi_1) \\ q^2(B_2\psi_1) - \frac{1}{r}\frac{d}{dr}(B_2\psi_1) - \frac{d^2(B_2\psi_1)}{dr^2} = \frac{\sigma_0\tau(p)}{\mu_1} \end{array}\right\} 0 < r \leq a \tag{2.6}$$

$$\left.\begin{array}{l} (\lambda_2 + 2\mu_2)(B_1\varphi_2) = q(B_2\psi_2) \\ q^2(B_2\psi_2) - \frac{1}{r}\frac{d}{dr}(B_2\psi_2) - \frac{d^2(B_2\psi_2)}{dr^2} = 0 \end{array}\right\} a < r < \infty \tag{2.7}$$

Where $p$and $q-$ are the parameters of the Laplace and Fourier transforms, B_1, $B_2-$the Bessel operators:

$$B_k = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(\frac{p^2}{c_k^2} + q^2\right) \quad ; \quad k = 1,2$$

In formulas (2.6) and (2.7), the quantities $u_r^{s^{(i)}}, u_z^{c^{(i)}}$are, respectively, the sine and cosine of the Fourier transforms of the displacement components and $\varphi^{(i)}, \psi^{(i)}$ are associated with the following formulas [4]:

$$\bar{u}_r^{s^{(i)}} = \frac{\partial\varphi_i}{\partial r} - q\frac{\partial\psi_i}{\partial r}$$

$$\bar{u}_z^{c^{(i)}} = q\varphi_i - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_i}{\partial r}\right)$$

Considering the peculiarities of the form of domains, systems (2.6) and (2.7) can have obvious solutions:

$$\begin{aligned}
&\varphi_1 = -\frac{\sigma_0 f(p)}{q\upsilon_1^{(1)^2}(\lambda_1+2\mu_1)} + A_0 I_0\left(\upsilon_1^{(1)} r\right)\\
&\psi_1 = -\frac{\sigma_0 f(p)}{q^2\upsilon_2^{(1)^2}\mu_1} + C_0 I_0\left(\upsilon_2^{(1)} r\right)\\
&\varphi_2 = A_1 K_0\left(\upsilon_1^{(2)} r\right)\\
&\psi_2 = C_1 K_0\left(\upsilon_2^{(2)} r\right)
\end{aligned} \tag{2.8}$$

The coefficients A_0, C_0, A_1, C_1 must be determined from the contact compatibility conditions (2.5). In (2.8) $I_0\left(\upsilon_k^{(1)} r\right)$, $K_0\left(\upsilon_k^{(2)} r\right)$- modified Bessel and Hankel functions, which are linearly independent solutions of the modified zero-order Bessel equation.

Condition (2.5) is reduced to an inhomogeneous system of linear algebraic equations with respect to unknown coefficients A_0, C_0, A_1, C_1:

$$\left\{\begin{matrix} A_0 \\ C_0 \\ A_1 \\ C_1 \end{matrix}\right\} \cdot \{D\} = \left\{\begin{matrix} 0 \\ a_0 \\ b_0 \\ 0 \end{matrix}\right\} \tag{2.9}$$

$$\begin{aligned}
&a_0 = \frac{\sigma_0 f(p)}{\upsilon_1^{(1)^2}(\lambda_1+2\mu_1)}\\
&b_0 = -\frac{\lambda_1 q\sigma_0 f(p)}{\upsilon_1^{(1)^2}(\lambda_1+2\mu_1)}
\end{aligned}$$

Here $\upsilon_k^{(i)} = \sqrt{\frac{p^2}{c_k^2} + q^2}$, $(k, i = 1, 2)$, $\{D\}$- matrix of the fourth rank has the following components:

$$\begin{aligned}
&a_{11} = \upsilon_1^{(1)}\\
&a_{12} = -q\\
&a_{13} = \upsilon_1^{(2)}\\
&a_{14} = -q\upsilon_2^{(2)}
\end{aligned}$$

$$\begin{aligned}
&a_{21} = q\\
&a_{22} = -\upsilon_2^{(1)^2}\\
&a_{23} = -q\\
&a_{24} = \upsilon_2^{(2)^2}
\end{aligned}$$

$$\begin{aligned}
&a_{31} = \left[\left((\lambda_1 + 2\mu_1)\,\upsilon_1^{(1)^2} - \lambda_1 q^2\right) - \frac{2\mu_1\upsilon_1^{(1)}}{a}\right]\\
&a_{32} = \left[-2\mu_1 q\upsilon_2^{(1)^2} + \frac{2\mu_1 q\upsilon_2^{(1)}}{a}\right]\\
&a_{33} = -\left[\left((\lambda_2 + 2\mu_2)\,\upsilon_1^{(2)^2} - \lambda_2 q^2\right) + \frac{2\mu_2\upsilon_1^{(2)}}{a}\right]\\
&a_{34} = \left[2\mu_2 q\upsilon_2^{(2)^2} + \frac{2\mu_2 q\upsilon_2^{(2)}}{a}\right]
\end{aligned}$$

$$
\begin{aligned}
a_{41} &= 2\lambda_1 q v_1^{(1)} \\
a_{42} &= \left[-\frac{2\lambda_1 {v_2^{(1)}}^2}{a} + \frac{\lambda_1 v_2^{(1)} \left(4 + {v_2^{(1)}}^2 a^2 - q^2 a^2 \right)}{a^2} \right] \\
a_{43} &= 2\lambda_2 q v_1^{(2)} \\
a_{44} &= \left[\frac{2\lambda_2 {v_2^{(2)}}^2}{a} + \frac{\lambda_2 v_2^{(2)} \left(4 + {v_2^{(2)}}^2 a^2 + q^2 a^2 \right)}{a^2} \right]
\end{aligned}
\tag{2.10}
$$

$$
\begin{aligned}
A_0 &= \frac{|D_1|}{|D|} \\
C_0 &= \frac{|D_2|}{|D|} \\
A_1 &= \frac{|D_3|}{|D|} \\
C_1 &= \frac{|D_4|}{|D|}
\end{aligned}
\tag{2.11}
$$

Where $|D|$ −and $|D_k|$, $\left(k = \overline{1,4}\right)$ − are fourth-rank determinants compiled according to Cramer's rules with respect to equation (2.9).

Thus, the task in transformations is solved completely. But the complete solution requires the determination of the originals of these transformations. Judging by expression (2.11), this is a rather complicated problem.

In order to obtain inverse transformations of the solutions (2.8) found here, one should determine the behaviour of the expression $\frac{|D_k|}{|D|}$ for $p \to \infty$, because this is the main factor that will make it possible to find a solution for a short period of time during which the shock process occurs. It should be noted that these transformations are twofold and the transformation parameters p and q , are present everywhere in the formulas together in the form $\sqrt{p^2 + c_k^2 \cdot q^2}$, therefore, assuming $p \to \infty$, it is understood that the value $\sqrt{p^2 + c_k^2 \cdot q^2} \to \infty$. In this case, the resulting expansion will obviously turn out to be a Fourier transform as well.

For small values of time, one can be content with only the first term $\frac{|D_k|}{|D|}$ when it is expanded in powers of p . The fact is that in the work where solutions are represented through the 5th rank of determinants and the use of only the first term, gave fairly accurate results. These results quite accurately coincided with the results that were known from practice, for example, in the study of tectonic processes in the earth's crust.

This first term is easily determined by analysing the expressions of each matrix included in the formulas $\frac{|D_k|}{|D|}$, since the ratio of the terms of the greatest degree in the parameter p of these determinants, will give us the necessary part of the main solution [3].

These methods are used to obtain expressions for the constants appearing in the solutions (2.8) of the problem.

$$
\begin{aligned}
A_0 &= \frac{-\lambda_1 q [a_{13}a_{24}a_{42} - a_{22}a_{13}a_{44}] - [a_{12}a_{33}a_{44} + a_{13}a_{34}a_{42} - a_{14}a_{33}a_{42} + a_{13}a_{32}a_{44}]}{[a_{13}a_{31} - a_{11}a_{33}] \cdot [a_{24}a_{42} - a_{22}a_{44}]} \\
C_0 &= -\frac{a_{44}}{a_{24}a_{42} - a_{22}a_{44}} \\
A_1 &= \frac{-\lambda_1 q [a_{11}a_{22}a_{44} - a_{11}a_{24}a_{42}] - [a_{11}a_{32}a_{44} + a_{14}a_{31}a_{42} - a_{11}a_{34}a_{42} - a_{12}a_{31}a_{44}]}{[a_{13}a_{31} - a_{11}a_{33}] \cdot [a_{24}a_{42} - a_{22}a_{44}]} \\
C_1 &= \frac{a_{42}}{a_{24}a_{42} - a_{22}a_{44}}
\end{aligned}
\tag{2.12}
$$

Then we can determine the longitudinal velocity of the particles inside and outside the cylinder:

$$\bar{u}_z^{(1)} = -\frac{\sigma_0 f(p)}{(\lambda_1 + 2\mu_1)} \left(\frac{1}{{\upsilon_1^{(1)}}^2 -} \right.$$

$$-\frac{\lambda_1 q^2 [a_{13}a_{24}a_{42} - a_{22}a_{13}a_{44}] - [a_{12}a_{33}a_{44} + a_{13}a_{34}a_{42} - a_{14}a_{33}a_{42} + a_{13}a_{32}a_{44}]}{{\upsilon_1^{(1)}}^2 [a_{13}a_{31} - a_{11}a_{33}] \cdot [a_{24}a_{42} - a_{22}a_{44}]} \times$$

$$\left. \times \frac{I_0\left(\upsilon_1^{(1)} a\right)}{I_0\left(\upsilon_1^{(1)} r\right)} + \frac{{\upsilon_2^{(1)}}^2 a_{42}}{(a_{24}a_{42} - a_{22}a_{44})} \cdot \frac{I_0\left(\upsilon_2^{(1)} a\right)}{I_0\left(\upsilon_2^{(1)} r\right)} \right)$$

$$-\bar{u}_z^{(2)} = -\frac{\sigma_0}{{\upsilon_1^{(1)}}^2 (\lambda_1 + 2\mu_1)} \times$$

$$\times \left(\frac{q^2 [a_{11}a_{22}a_{44} - a_{11}a_{24}a_{42}] - [a_{11}a_{32}a_{44} + a_{14}a_{31}a_{42} - a_{11}a_{34}a_{42} - a_{12}a_{31}a_{44}]}{[a_{13}a_{31} - a_{11}a_{33}] \cdot [a_{24}a_{42} - a_{22}a_{44}]} \times \right.$$

$$\left. \times \frac{K_0\left(\upsilon_1^{(2)} a\right)}{K_0\left(\upsilon_1^{(2)} r\right)} - \frac{{\upsilon_2^{(2)}}^2 a_{42}}{a_{24}a_{42} - a_{22}a_{44}} \frac{K_0\left(\upsilon_2^{(2)} a\right)}{K_0\left(\upsilon_2^{(2)} r\right)} \right)$$

$$\hat{u}_z^{(1)} = -\frac{\sigma_0 f(p)}{(\lambda_1 + 2\mu_1)} \left(\frac{1}{{\upsilon_1^{(1)}}^2} - \right.$$

$$-\frac{q^2 \left\{ \upsilon_2^{(2)} \upsilon_1^{(2)} \lambda_1 \left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right] + \gamma_1^2 + 2\upsilon_2^{(2)} \upsilon_2^{(1)} \left[\lambda_1 \mu_2 \upsilon_2^{(1)} + \lambda_2 \mu_1 \upsilon_1^{(2)} \right] \right\}}{\upsilon_2^{(1)} {\upsilon_1^{(1)}}^3 \upsilon_2^{(2)} \left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right] \cdot \left[(\lambda_1 + 2\mu_1) \upsilon_1^{(1)} + (\lambda_2 + 2\mu_2) \upsilon_1^{(2)} \right]} \times$$

$$\left. \times \frac{I_0\left(\upsilon_1^{(1)} a\right)}{I_0\left(\upsilon_1^{(1)} r\right)} + \frac{\upsilon_2^{(2)} \lambda_2}{{\upsilon_1^{(1)}}^2 \left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right]} \cdot \frac{I_0\left(\upsilon_2^{(1)} a\right)}{I_0\left(\upsilon_2^{(1)} r\right)} \right);$$

$$\hat{u}_z^{(2)} = -\frac{\sigma_0}{{\upsilon_1^{(1)}}^2 (\lambda_1 + 2\mu_1)} \times$$

$$\times \left(\frac{q^2 \left\{ \upsilon_2^{(2)} \upsilon_2^{(1)} \lambda_1 \left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right] + \gamma_2^1 + 2\upsilon_2^{(2)} \upsilon_2^{(1)} \left[\lambda_1 \mu_2 \upsilon_2^{(1)} + \lambda_2 \mu_1 \upsilon_2^{(2)} \right] \right\}}{\upsilon_2^{(1)} \upsilon_1^{(2)} \upsilon_2^{(2)} \left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right] \cdot \left[(\lambda_1 + 2\mu_1) \upsilon_1^{(1)} + (\lambda_2 + 2\mu_2) \upsilon_1^{(2)} \right]} \times \right.$$

$$\left. \times \frac{K_0\left(\upsilon_1^{(2)} a\right)}{K_0\left(\upsilon_1^{(2)} r\right)} - \frac{\upsilon_2^{(1)} \lambda_1}{\left[\lambda_1 \upsilon_2^{(1)} + \lambda_2 \upsilon_2^{(2)} \right]} \cdot \frac{K_0\left(\upsilon_2^{(2)} a\right)}{K_0\left(\upsilon_2^{(2)} r\right)} \right). \tag{2.13}$$

Here:

$$\gamma_j^i = \upsilon_j^{(i)} (\lambda_i + 2\mu_i) \left(\lambda_i {\upsilon_2^{(i)}}^2 - \lambda_j {\upsilon_2^{(j)}}^2 \right).$$

For sufficiently large values, these formulas are converted to the following expressions:

$$u_z^{(1)} = -\frac{\sigma_0}{(\lambda_1 + 2\mu_1)} \left(\frac{1}{{\upsilon_1^{(1)}}^2} - \frac{q^2}{{\upsilon_1^{(1)}}^4} \times \right.$$

$$\times\frac{c_2^{(}1)\lambda_1\left[\lambda_1\frac{c_2^{(}1)}{c_2^{(2)}}+\lambda_2\right]+c_2^{(1)^2}\left[(\lambda_2+2\mu_2)\left(\lambda_2-\lambda_1\frac{c_2^{(2)^2}}{c_1^{(2)^2}}\right)\right]+2c_2^{(1)}c_1^{(2)}\left[\lambda_1\mu_2\frac{c_2^{(1)}}{c_2^{(2)}}+\lambda_2\mu_1\right]}{c_2^{(2)}c_1^{(2)}\left[\lambda_1+\lambda_2\frac{c_2^{(1)}}{c_2^{(2)}}\right]\cdot\left[(\lambda_1+2\mu_1)+(\lambda_2+2\mu_2)\frac{c_1^{(1)}}{c_1^{(2)}}\right]}\times$$

$$\left.\times\frac{I_0\left(\upsilon_1^{(1)}a\right)}{I_0\left(\upsilon_1^{(1)}r\right)}+\frac{1}{\upsilon_1^{(1)^2}}\cdot\frac{\lambda_2}{\left[\lambda_1\frac{c_2^{(2)}}{c_2^{(1)}}+\lambda_2\right]}\cdot\frac{I_0\left(\upsilon_2^{(1)}a\right)}{I_0\left(\upsilon_2^{(1)}r\right)}\right)$$

$$u_z^{(2)}=-\frac{\sigma_0}{\upsilon_1^{(1)^2}(\lambda_1+2\mu_1)}\left(\frac{q^2}{\upsilon_1^{(2)^2}}\times\right.$$

$$\times\frac{c_1^{(1)}\lambda_1\left[\lambda_1\frac{c_2^{(2)}}{c_2^{(1)}}+\lambda_2\right]+c_2^{(1)}\left[(\lambda_1+2\mu_1)\left(\lambda_1\frac{c_2^{(2)^2}}{c_2^{(1)^2}}-\lambda_2\right)\right]+2\left[\lambda_1\mu_2\frac{c_2^{(2)}}{c_2^{(1)}}+\lambda_2\mu_1\right]}{c_1^{(1)}\left[\lambda_1\frac{c_2^{(2)}}{c_2^{(1)}}+\lambda_2\right]\cdot\left[(\lambda_1+2\mu_1)\frac{c_1^{(2)}}{c_1^{(1)}}+(\lambda_2+2\mu_2)\right]}\times$$

$$\left.\times\frac{K_0\left(\upsilon_1^{(2)}a\right)}{K_0\left(\upsilon_1^{(2)}r\right)}-\frac{\lambda_1}{\left[\lambda_1+\lambda_2\frac{c_2^{(1)}}{c_2^{(2)}}\right]}\cdot\frac{K_0\left(\upsilon_2^{(2)}a\right)}{K_0\left(\upsilon_2^{(2)}r\right)}\right). \tag{2.14}$$

Where $c_1^{(i)}=\sqrt{\frac{E_i(1-\upsilon_i)}{\rho_i(1+\upsilon_i)(1-2\upsilon_i)}}$, $c_2^{(i)}=\sqrt{\frac{E_i}{2\rho_i(1+\upsilon_i)}}$; $i=1,2$.

Without going into details, we can give ready-made formulas for inverse transformations in formulas (2.14) for velocities. We only note that to derive these solutions, the second decomposition formula, the Efros theorem and the tables given in [1]. The behavior of Bessel functions at infinity is also taken into account.

$$u_z^{(1)}=-0,054\cdot\int_0^{500}(\cos(q\cdot z))\cdot\int_1^t[q\cdot\sin(q\cdot(t-\tau))]\times$$

$$\times\int_1^\tau\left[\frac{(\theta-1)^{-0.5}}{\Gamma(0.5)}\right]\cdot J_0\left(q\cdot\sqrt{\tau^2-\theta^2}\right)d\theta d\tau dq$$

$$u_z^{(1)}=-0.038\cdot\sqrt{\frac{1}{r}}\int_{(r-1)}^t-\frac{H[\tau-(r-1)]}{\sqrt{t^2-\tau^2-z^2}}+\frac{H\left[t^2-\tau^2\right]}{2\sqrt{t^2-\tau^2}}\cdot$$

$$\times\left[\frac{H\left[t^2-\tau^2-\left[\frac{z+0.25[\tau-(r-1)]}{\sqrt{t^2-\tau^2}}\right]^2\right]}{\sqrt{t^2-\tau^2-\left[\frac{z+0.25[\tau-(r-1)]}{\sqrt{t^2-\tau^2}}\right]^2}}+\frac{H\left[t^2-\tau^2-\left[\frac{z-0.25[\tau-(r-1)]}{\sqrt{t^2-\tau^2}}\right]^2\right]}{\sqrt{t^2-\tau^2-\left[\frac{z-0.25[\tau-(r-1)]}{\sqrt{t^2-\tau^2}}\right]^2}}\right]d\tau$$

$$u_z^{(2)}=0.00144\cdot\sqrt{\frac{1}{r}}\int_{2(r-1)}^t\frac{H\left[t^2-\tau^2\right]}{\sqrt{t^2-\tau^2}}\cdot\left[\frac{H\left[0.25\left(t^2-\tau^2\right)-\left[\frac{z+[\tau-2(r-1)]}{0.5\sqrt{t^2-\tau^2}}\right]^2\right]}{\sqrt{0.25\left(t^2-\tau^2\right)-\left[\frac{z+[\tau-2(r-1)]}{0.5\sqrt{t^2-\tau^2}}\right]^2}}-\right.$$

$$-\frac{H\left[0.25\left(t^2-\tau^2\right)-\left[\frac{z-[\tau-2(r-1)]}{0.5\sqrt{t^2-\tau^2}}\right]^2\right]}{\sqrt{0.25\left(t^2-\tau^2\right)-\left[\frac{z-[\tau-2(r-1)]}{0.5\sqrt{t^2-\tau^2}}\right]^2}}\Bigg]\,d\tau$$

It is assumed here that $f(t)=H(t)$, $H(t)$ –the Heaviside function.
Below are graphs of the dimensionless longitudinal velocity of particles of the central axis of the cylinder $\bar{u}_z^{(i)}=\dot{u}_z^{(i)}\Big/\frac{\sigma_0 c_1^{(1)}}{(\lambda_1+2\mu_1)}$.
Let us assume that Poisson's ratios $\upsilon_1=\upsilon_2=\frac{1}{3}$, and $\frac{E_1}{E_2}=1,5$; $\frac{\rho_1}{\rho_2}=1,2$.
The calculation is made for the following time values $\bar{t}=\frac{tc_1^{(1)}}{a}\cdot n$, $(n=2,\ 3,\ 5,\ 7,\ 10)$:

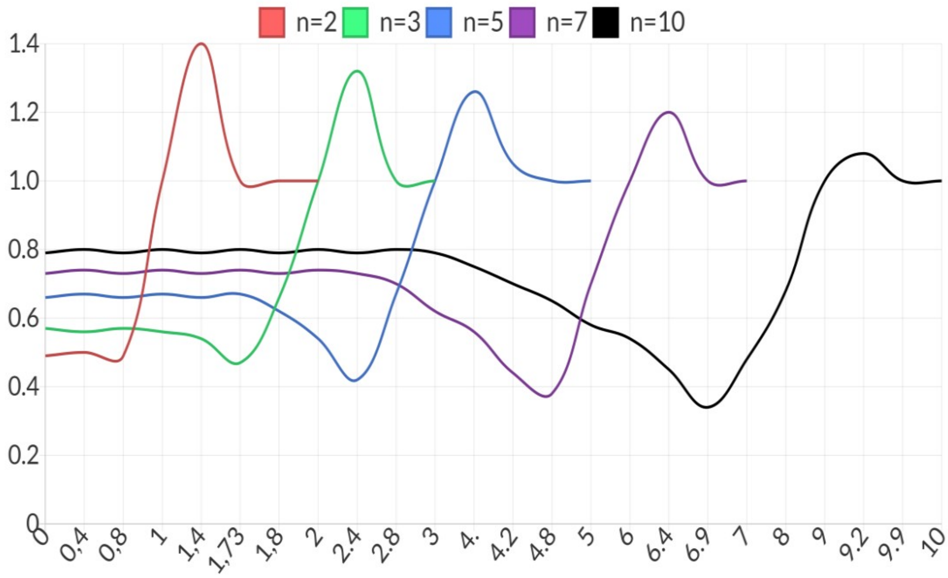
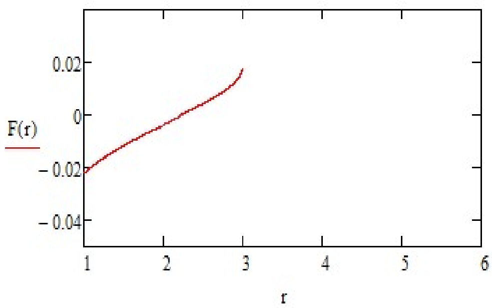
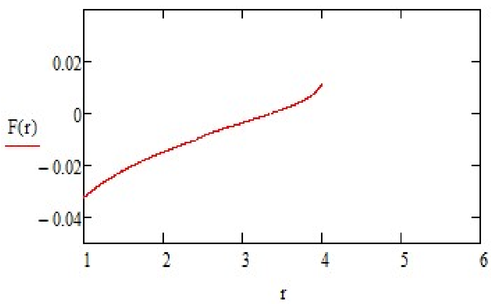
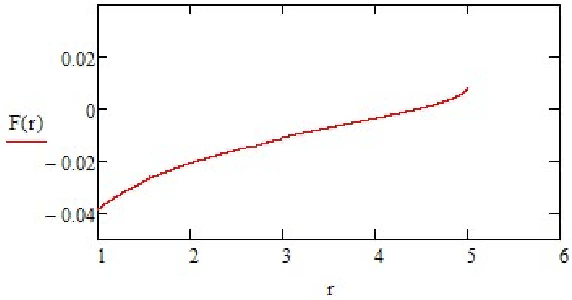


Fig. 2.2. Distribution of the longitudinal velocity of particles of the central axis of the cylinder for subsequent values of time

There are also graphs for the speed , on the free surface of the half-space around the end of the cylinder, but now for the following values of time $\bar{t}=\frac{tc_1^{(2)}}{a}\cdot n$, $(n=2,3,4)$:







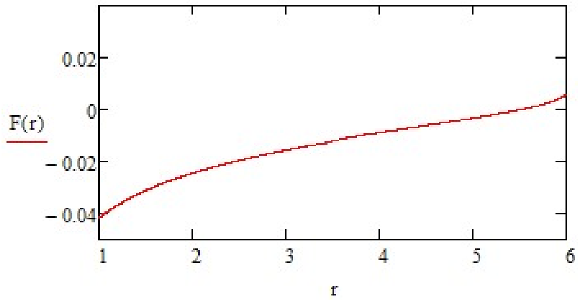


Fig. 2.3. Distribution of the normal velocity of particles of the free surface of the half-space for subsequent values of time

3 Conclusions

The unsteady motion of an elastic half-space, inside which a cylinder made of another linearly elastic material is embedded, is studied. Formulas for calculating almost all necessary quantities are obtained.

Graphical dependences of longitudinal velocities for each medium are given.

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