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Determination of the stress intensity factor during longitudinal displacement in a composite material reinforced with unidirectional fibers with linear cracks

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Abstract. *This article considers a double periodic cage with round plane holes filled with unstressed washers made of an isotropic elastic material, the surface of which is uniformly covered with a homogeneous film. Each fiber and medium (binder) is stretched by two biperiodic linear cracks. Each washer has a centrally located crack, the length of which is less than the diameter of the washer. The introduced stresses and their displacements are expressed by an analytical function. In the process of solving the problem, boundary conditions are set along the contour of round holes, as well as along straight cracks. Thus, on the basis of the boundary conditions, a system of linear algebraic equations is obtained. Their solution is solved by the Gauss method. In the latter case, the stress intensity factor at the crack tips is determined.*

Keywords. longitudinal shear · coating thickness · doubly periodic · lattice · fibers – coatings · coating – binder · holomorphic function

Mathematics Subject Classification (2010): 74A40 · 74S15

1 Introduction

The problem of fracture mechanics of the general regular structure of a linearly reinforced medium formed by cells containing an arbitrary finite number of fibers of various diameters is considered. The longitudinal shear of the plate, the cover of the holes, and the weakened by a bi-periodic system of rectilinear through cracks, the collinear axes of abscissas and ordinates are equal in length. The solution of the problem of equilibrium of a perforated

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body with a longitudinal shear with prefracture zones reduces to solving a single infinite algebraic system and a nonlinear singular integro-differential equation with a core such as a Cauchy kernel.

From the solution of these equations the forces in the crack zones are found. The crack condition is formulated taking into account the limit discontinuity criterion for material motions. Each singular integral equation reduces to a finite system of linear algebraic equations.

2 Formulation of the problem.

Let S_k^t and S_k^a be the regions of the cell structure occupied by the circular coating of the $k-$th fiber and the fiber itself, the center of which is located at the point $a_k + P$ $(P = m\omega_1 + nb\omega_2,\;\; m, n = 0, \pm 1, \pm 2, ..., \pm\infty)$, λ_k, and λ_k' are the external and internal dimensionless radii of the surfaces of this coating; a single cell contains n fibers. The Cartesian coordinate is combined with the axis of an arbitrary fiber, ω_1 and ω_2 basis vectors, equal in modulus to the lengths of the corresponding sides of the main cell, of which the macrostructure of the material is composed. For convenience, we should use the representation ω_2 in terms of complex quantities $\omega_2 = \omega_1 b e^{i\alpha}$, $\alpha \neq 0$, where $\omega_1 b$ is the length of the inclined side of the cell $0 < b < \infty$. Hereinafter, on the outer surface of the coating, $\tau = \lambda e^{iv}$, and on the inner $\tau_1 = \lambda_1 e^{iv} = (\lambda - h)\, e^{iv}$, where h is the thickness of the coating, α is the angle between the vectors ω_1 and ω_2.

Fibers over the entire macro section are arranged by doubly periodic continuation of the main cell [1]. The binder medium is weakened by a doubly periodic system of rectilinear cracks. Crack banks are free of external efforts. Lattices have average stresses $\tau_y = \tau_y^\infty$, $\tau_x = 0$ (shift at infinity).

The elastic moduli of the material are established through the expansion coefficients of the function $\phi_s(z)$ according to the considered method. For bodies having one plane of symmetry and belonging to the monoclinic system, Hooke's law for the case of a pure shift is expressed as follows:

$$\gamma_{31} = X_{44}\sigma_{31} + X_{45}\sigma_{21}, \quad \gamma_{12} = X_{45}\sigma_{31} + X_{55}\sigma_{21}.$$

If the so-called "technical" constants are introduced, then these relations have the form

$$\gamma_{31} = \frac{1}{G_{13}}\sigma_{31} + \frac{M_{31,21}}{G_{13}}\sigma_{21}, \quad \gamma_{12} = \frac{M_{12,31}}{G_{13}}\sigma_{31} + \frac{1}{G_{12}}\sigma_{21}.$$

where M_{31}, M_{21} and M_{12}, M_{31} are called Chentsov coefficients and characterize the shifts in the X_3X_1 plane caused by stresses (σ_{21}), or in the X_1X_2 plane under stresses (σ_{31}). Elastic displacements caused by uniform shifts in the considered planes are established by the dependence.

The increment of displacements in the reinforced medium in adjacent cells is expressed by the formula

$$\begin{gathered}\gamma_{12} = \tfrac{\omega_j + \overline{\omega_j}}{2} - i\gamma_{13}\tfrac{\omega_j - \overline{\omega_j}}{2} = \\ = \omega_j C_0 - \delta_j \sum_{k=1}^{n} \left(C_{k,2} + C_{k,1} a_k\right) + \overline{\omega_j}\overline{C_0} - \overline{\delta_j} \sum_{k=1}^{n} \left(\overline{C_{k,2}} + \overline{C_{k,1}}\overline{a_k}\right).\end{gathered} \tag{2.1}$$

Consistently setting $j = 1, 2$, the desired relations are found using this formula

$$\gamma_{12} = C_0 + \overline{C_0} - \tfrac{\delta_1}{\omega_1} \sum_{k=1}^{n} \left(C_{k,2} + C_{k,1} a_k\right) - \tfrac{\delta_1}{\omega_1} \sum_{k=1}^{n} \left(\overline{C_{k,2}} + \overline{C_{k,1}}\overline{a_k}\right),$$

$$\gamma_{12} \cos\alpha + \gamma_{13} \sin\alpha = e^{i\alpha} C_0 + e^{-i\alpha}\overline{C_0} - \tag{2.2}$$

$$-\tfrac{\delta_2}{\omega_1 b_1} \sum_{k=1}^{n} \left(C_{k,2} + C_{k,1} a_k\right) - \tfrac{\overline{\delta_2}}{\overline{\omega_1 b_1}} \sum_{k=1}^{n} \left(\overline{C_{k,2}} + \overline{C_{k,1}}\overline{a_k}\right).$$

If we take into account the relations $e^{iv} = -i\frac{dz}{ds}$, $e^{-iv} = i\frac{dz}{ds}$, s the arc of the circle of the fiber contour that follows from consideration, it is directly converted to the form

$$T = -iG \frac{d}{ds}\left[\phi(z) - \overline{\phi(z)}\right]. \tag{2.3}$$

The holomorphic function $\Phi_s(z)$ for the region under consideration should be chosen based on the general representation of the elliptic function, according to which

$$\Phi_s(z) = C_0 + \sum_{k=1}^{n} \sum_{s=1}^{\infty} \frac{(-1)^s}{(s-1)!} C_{k,s} \varsigma^{(s-1)} \; (z - a_k). \tag{2.4}$$

Here we introduce the Weierstrass sigma function, according to the definition of which

$$\frac{d}{dz} \ln \sigma(z) = \varsigma(z) \text{ and } \lim_{x \to 0} \frac{\sigma(z)}{z} = 1. \tag{2.5}$$

The solution of the problem of the shear of a reinforced medium with the structure under consideration is reduced to the construction of n functions $\Phi_k(z)$, holomorphic in the areas occupied by coatings and a function $\Phi_{k,a}(z)$ under the corresponding boundary conditions.

Unknown functions are constructed in the form of series:

$$\phi'_k(z) = \Phi_k(z) = \sum_{m=-\infty}^{\infty} b_{m,k} (z - a_k)^m \quad (k = 1, 2, ..., n),$$

for $z \in S_k^t$,

$$\phi'_{a,k}(z) = \Phi_{a,k}(z) = \sum_{n=1}^{\infty} d_n (z - a_k)^n \quad (k = 1, 2, ..., n),$$

for $z \in S_k^a$.

Satisfying the boundary conditions at the boundaries of the fiber – coating ω_{mn} and the coating – binder Ω_{mn}, where the indices $m, n = 0, \pm 1, \pm 2, ..., \pm\infty$ determine the condition on the contour of the $m-$th fiber. In the case of perfect contact, the displacements and stresses on the mating sites are equal to each other.

Boundary conditions are expressed through unknown functions as follows. From the conditions of equality of displacements and stresses on the surface of the junction of coating films with a binding medium in one cell at $z - a_k = \lambda_k e^{iv_k} = \tau_k$.

$$\left(1 + \frac{G_t}{G_s}\right) \phi_k(\tau_k) + \left(1 - \frac{G_t}{G_s}\right) \overline{\phi_k(\tau_k)} = 2 f_s(\tau_k) \quad (k = 1, 2, ..., n). \tag{2.6}$$

The second system of boundary equations follows from the one given when replacing functions with complex-stressed (conjugate) functions. On the inner surfaces of the fiber coatings for $z - a_k = \lambda_k' e^{iv_k} = t_k,\;\; n$ relations

$$\left(1 + \frac{G_a}{G_t}\right)\phi_{a,k}\left(t_k\right) + \left(1 - \frac{G_a}{G_t}\right)\overline{\phi_{a,k}\left(t_k\right)} = 2\phi_k\left(t_k\right) \;\; \left(k = 1, 2, ..., n\right). \qquad (2.7)$$

resulting from ideal contact conditions.

The boundary conditions on the banks of the cracks are:

$$f_s'\left(t\right) - \overline{f_s'\left(t\right)} = 0, \qquad (2.8)$$

$$f_s'\left(t_1\right) - \overline{f_s'\left(t_1\right)} = 0, \qquad (2.9)$$

$$f_b'\left(t\right) - \overline{f_b'\left(t\right)} = 0. \qquad (2.10)$$

We rewrite the boundary conditions (2.6), (2.7), (2.8), (2.9) and (2.10) as follows [2. 3].

$$\begin{gathered} f_s\left(z\right) = f_1\left(z\right) + f_2\left(z\right) + f_3\left(z\right), \\ f_b\left(z\right) = f_{1b}\left(z\right) + f_{2b}\left(z\right), \end{gathered} \qquad (2.11)$$

$$\varPhi_s\left(z\right) = C_0 + \sum_{k=1}^{n}\sum_{s=1}^{\infty}\frac{\left(-1\right)^s}{\left(s-1\right)!}C_{k,s}\varsigma^{\left(s-1\right)}\left(z - a_k\right),$$

$$\begin{gathered} f_2'\left(z\right) = \tfrac{1}{i\omega}\int_{L_1} g\left(t\right)\xi\left(t - z\right)dt + A, \\ f_3'\left(z\right) = \tfrac{1}{i\omega}\int_{L_2} g_1\left(t_1\right)\xi\left(t_1 - z\right)dt_1 + B, \end{gathered} \qquad (2.12)$$

$$f_{2b}\left(z\right) = \frac{1}{i\pi}\int_{-l}^{l}\frac{g\left(t\right)}{t - z}dt, \qquad (2.13)$$

where the integrals in (2.12), (2.13) are taken along the line $L_1 = \{[-l, -a] + [a, l]\}$, $L_2 = \{[-d, -b] + [b, d]\}$, $\gamma\left(z\right)$ and $\xi\left(z\right)$ are the Weierstrass functions [2], $g\left(t\right), g_1\left(t_1\right)$ are the desired function, A, B are constant, t and t_1 are the affix of the points of the crack faces directed along the abscissa axes and ordinates. Related to the coating, the fiber and the binder, hereinafter marked respectively by the indices k, a and s.

To the main concepts (2.12)–(2.13), additional conditions arising from the physical meaning of the problem are added

$$\begin{gathered} \textstyle\int_{-l}^{-a} g\left(t\right)dt = 0,\;\; \int_{a}^{l} g\left(t\right)dt = 0,\;\; \int_{-l}^{l} g\left(t\right)dt = 0, \\ \textstyle\int_{-d}^{-b} g_1\left(t_1\right)dt_1 = 0,\;\; \int_{b}^{d} g_1\left(t_1\right)dt_1 = 0, \end{gathered} \qquad (2.14)$$

3 The solution of the boundary value problem

The unknown function $g(x)$, $g_1(y)$ and the constants $b_{m,k}$, d_n, $c_{k,s}$ must be determined from the boundary conditions (2.6) – (2.7). To compose equations for unknown coefficients $c_{k,s}$, we represent the boundary condition (2.1) in the function $f_1'(z)$.

The constant decompositions of the displacement function is determined from an infinite system of algebraic equations that follows when the boundary conditions are satisfied.

To determine arbitrary constants, boundary conditions (2.6), (2.7) are considered where instead of the functions W_a, W_s, W_t and the corresponding stresses are introduced in the Fourier series. As a result of simple transformations, the following system of algebraic equations is obtained:

$$\lambda^{2k}a_{2k} = G_c(\lambda)\,G_a\left\{D_{2k+1}\left[F(g\lambda)\,M_{\frac{1}{2},2k+1}(g\lambda)\right]' + S_{2k+1}\left[F(g\lambda)\,M_{\frac{1}{2},2k+1}(g\lambda)\right]'\right\},$$

$$\frac{\lambda^{2k+1}}{2k+1}a_{2k} = F(g\lambda)\left[D_{2k+1}M_{\frac{1}{2},2k+1}(g\lambda) + S_{2k+1}W_{\frac{1}{2},2k+1}(g\lambda)\right]\quad (k=0,1,2,...,\infty),$$

$$\sum_{s=0}^{\infty} C_{2s+2}\lambda^{2s+1}\alpha_{k,s} - \frac{\overline{C_{2k+2}}\lambda^{2k+2}}{b^{4k+2}} = (2k+1)\,F(gb)\,b^{2k+1}\times$$

$$\times\left[D_{2k+1}M_{\frac{1}{2},2k+1}(gb) + S_{2k+1}W_{\frac{1}{2},2k+1}(gb)\right]\quad (k=0,1,2,...,\infty), \tag{3.1}$$

$$C_0 - \lambda^2 b^2\overline{C_2} + \sum_{k=1}^{\infty} C_{2s+2}\lambda^{2s+1}\alpha_{0,k} = \frac{F(gb)}{b}\left[D_1M_{\frac{1}{2},1}(gb) + S_1W_{\frac{1}{2},1}(gb)\right],$$

$$C_0\delta_{k0} + \frac{\lambda^{2k+1}}{b^{2k+2}\overline{C_{2k+1}}} - \sum_{s=0}^{\infty} C_{2s+2}\lambda^{2s+1}\alpha_{k,s}b^{2k} = D_{k+1}\left[F(gb)\,M_{\frac{1}{2},2k+1}(gb)\right]' +$$

$$+S_{2k+1}\left[F(gb)\,W_{\frac{1}{2},2k+1}(gb)\right]'\quad (k=0,1,2,...,\infty).$$

Marked here

$$F(x) = x^{-\frac{1}{2}}e^{\frac{x}{2}},\quad \left[F(g\lambda)\,M_{\frac{1}{2},n}(g\lambda)\right]' = \frac{d}{d\rho}\left[F(g\chi)\,M_{\frac{1}{2},n}(g\chi)\right],$$

$\chi = \lambda$, $\delta_{k,0}-$ Kronecker symbol. The remaining equations follow from the above given when replacing constants with complex conjugates. An additional algebraic relation that establishes a relation between the average voltages and arbitrary constants can be written in the form

$$\sigma_{12} - i\sigma_{13} = 2\xi G_a a_0 + 2\eta_s G_s C_0 - 2\eta_s G_s\sum_{k=1}^{\infty} C_{2k+2}\lambda^{2k+2}\alpha_{0,k} + D_1\beta_1 + S_1\beta_2, \tag{3.2}$$

where $\eta_0 = \frac{\pi b^2}{(\omega_1^2\sin\alpha)}$, $\eta_s = 1-\eta_0$,

$$D_1\beta_1 + S_1\beta_2 = G(\chi)\left(e^{-i\theta}\frac{\partial u_c}{\partial\chi} - ie^{-i\theta}\frac{1}{\chi}\cdot\frac{\partial u_c}{\partial v}\right),$$

$$\beta_1 = \frac{2\pi}{\omega_1^2\sin\alpha}\left[G_sF(gb)\,M_{\frac{1}{2},1}(gb) - G_0F(g\lambda)\,M_{\frac{1}{2},1}(g\lambda)\right] +$$

$$+ (1+2g\lambda)\int_{\lambda}^{b} d\chi G_s(\chi g)\, F(\chi g)\, M_{\frac{1}{2},1}(\chi g)\,.$$

The formula for β_2 is similar to the formula for β_1 you only need to replace $M_{\frac{1}{2},1}(x)$ with $W_{\frac{1}{2},1}(x)$.

The solution to system (3.2) is constructed as follows. Of the first two formulas, the constants D_{2k} and S_{2k} are expressed in terms α_{2k}, and from the third and fourth groups of equations (3.2), a connection is established between C_{2k} and α_{2k}, finally, the constants C_{2k} are found from the last equation taking into account dependence (3.2). The resolving equation has the form

$$C_{2p+2} = \sum_{k=1}^{\infty} A_{p,k} C_{2k+2} + B_{2p} \quad (p = 1, 2, ..., \infty)\,, \tag{3.3}$$

here, with increasing p and k, the following estimates hold:

$$\begin{aligned} A_{p,k} &\approx b^{4p+2}\lambda^{2k+2p}\alpha_{p,k}, \\ B_{p,k} &\approx b^{4p+4}\lambda^{-2k-2p}\alpha_{p,0}. \end{aligned} \tag{3.4}$$

Given estimates (3.4), it follows

$$|A_{p,k}| \le \frac{(2p+2k)!Nb^{4p+2}\lambda^{2k-2p}}{(2k+1)!\,(2p)!\omega^{2p+2k+2}}\; |B_{2p}| \le \frac{N}{\omega_1^{2p+2}}\, b^{4p+4}\lambda^{2p-2}.$$

Thus, solving the boundary value problem (2.6), (2.7), the definition of the desired coefficients $C_{k,s}$ is reduced to infinite algebraic equations, on the right side of which there are quantities depending on the sought functions $g(x)$ and $g_1(y)$ (y in the form of integrals). To determine the desired functions $g(x)\,,\, g_1(y)$, there are boundary conditions (2.8), (2.9), (2.10) on the crack faces.

$$\frac{1}{\pi}\int_{L_1} g(t)\,\xi(t-x)\,dt - Im\left[A + f_1'(x)\right] = 0 \;\text{ on }\; L_1, \tag{3.5}$$

$$\frac{1}{\pi}\int_{L_2} g_1(t_1)\,\xi(t_1-y)\,dt_1 - Im\left[A + f_1'(y)\right] = 0 \;\text{ on }\; L_2, \tag{3.6}$$

$$\frac{1}{\pi}\int_{-l}^{l} \frac{g(t)}{t-x}dt - Im\left[f_{1b}'(x)\right] = 0. \tag{3.7}$$

Each singular integral equation can be reduced to a standard form by changing variables. Further, applying the algebraization procedure instead of each integral equation, we obtain a finite system of linear algebraic equations [5, 6].

Depending on the type of bond and the strength of the boundary, the destruction of the composite can occur in different ways. If the crack propagating in the composite crosses the fibers, then the fracture toughness increases the more, the more the fibers exfoliate from the matrix. In this case, to increase the fracture toughness, a weak bond at the fiber–matrix interface is preferable. When a crack propagates parallel to the fibers, a strong bond at the fiber–matrix interface is preferable, which helps prevent fracture along the interface.

System (3.3), together with the singular equation (3.5), (3.6), and (3.7), are the main controls of the problem allowing one to determine $g(x)\,,\, g_1(y)$ and the coefficients α_{2k}, b_{2k}, c_{2k}. Recall that system (3.3) contains the coefficients C_{2k}, B_{2k} and depending on the desired function $g(x)\,,\, g_1(y)$. System (3.3) and equation (3.5), (3.6) and (3.7) turned out to be coupled, should be solved together.

Knowing the functions $\Phi_k(z), \Phi_{a,k}(z), \Phi_s(z)$, we can find the stress-strain state of the plate.

In practice, the thickness of the barrier coatings of metal fibers is usually in the order of one tenth of the diameter of the fiber $(f \approx 0,1)$. For the selected f, we determine the change in the shear stiffness of the linearly reinforced material depending on the volumetric fiber content $\xi_0 = f^2\xi$ and the given ratio $\frac{G_a}{G_t}$ and $\frac{G_s}{G_a}$.

a) the results of calculations are presented in relation to fiberglass plastic. Here, the change in the ratio $\frac{G_s}{G_a}$ is with an increase in ξ for fiberglass without coating $\frac{G_a}{G_s} = 25$, and for fiberglass with a coating in which $\frac{G_a}{G_t} = 50$.

b) for stiffer coatings, the macroscopic shear modulus changes very little. The dependence of the shear stiffness of coated materials on the shear modulus of the coating at $t = 0$ is more obvious. Using coatings, as follows from the results found, it is possible to vary the stiffness of the composition over a wide range.

In practice, they have been used as a filler along with continuous hollow fiberglass. The task of studying the stress-strain state of this material with a longitudinal shear is reduced to determining the harmonic functions $\Phi_k(z), \Phi_{a,k}(z)$ and $\Phi_s(z)$ under given boundary conditions on the areas of contact between the components.

In particular, for the stress intensity factor K_{III} at the crack tips we will have the formula

$$K_{III} = \mp \lim_{x \to c} \left\{ \sqrt{2\pi |x-c|} g(x) \right\}, \tag{3.8}$$

moreover, the upper sign is taken at $c = a$, the lower sign is taken at $c = l$.

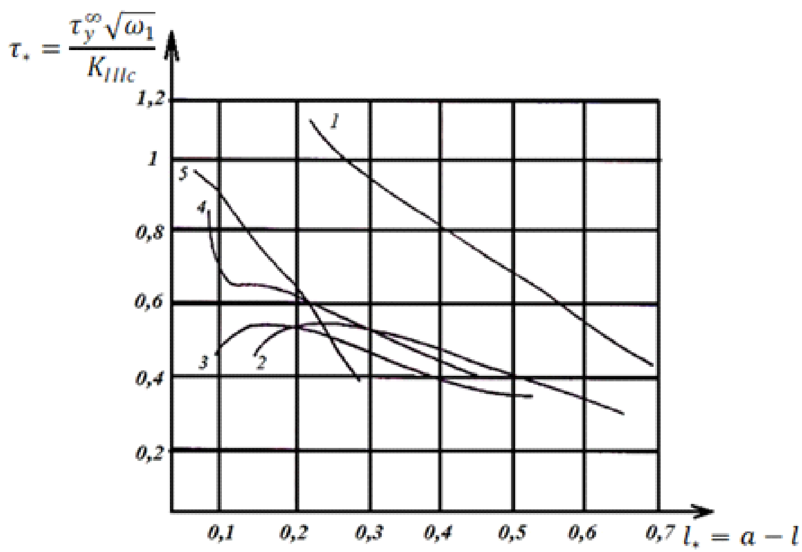

Fig. 3.1. The dependence of the critical external load $\tau_* = \tau_y^\pi \sqrt{\omega_1} / K_{IIIc}$ on the dimensionless length in cracks along the X axis of the binder material $l_* = a - l$ for some values of the fiber cross-section radius $\lambda = 0.2, 0.3, 0.4, 0.5, 0.6$ (curves 1–5)

System (3.5) – (3.7) is connected (closed) by infinite systems (3.3), in which relation (3.1) is substituted for C_{2k} and B_{2k}. The four systems noted completely determine the solution to the problem. After finding the value P_v^0, the stress intensity factor K_{III} is determined on the basis of relations (3.8):

$$
\begin{gathered}
K_{III}^{a}=\sqrt{\frac{\pi l\left(1-\lambda_1^2\right)}{\lambda_1}}\frac{1}{2\pi}\sum_{v=1}^{n}\left(-1\right)^{v+n}P_v^0 tg\frac{\theta_v}{2},\; K_{III}^{-a}=\sqrt{\pi l}\frac{1}{n}\sum_{k=1}^{n}\left(-1\right)^{k+n}P_k^0 tg\frac{\theta_k}{2},\\
K_{III}^{-l}=\sqrt{\pi l}\frac{1}{n}\sum_{k=1}^{n}\left(-1\right)^{k}P_k^0 ctg\frac{\theta_k}{2},\; K_{III}^{l}=\sqrt{\pi l\left(1-\lambda_1^2\right)}\frac{1}{2\pi}\sum_{v=1}^{n}\left(-1\right)^{v}P_v^0 tg\frac{\theta_v}{2},
\end{gathered}
\tag{3.9}
$$

4 Decision analysis

For numerical calculations, the case of the location of the hole at the apex of the triangular $\omega_1 = 2, \omega_2 = 2e^{\frac{1}{3}i\pi}$ and square $\omega_1 = 2, \omega_2 = 2i$ lattices was taken. The calculations were performed on an IBM computer using the MATLAB program. It was assumed that $n = 10$ and $n = 20$, which corresponds to dividing the interval into 10 and 20 Chebyshev nodes, respectively. The resulting systems were solved by the Gauss method with the choice of the main element.

Based on the results in Fig. 3.1, 4.1 and 4.2, we plotted the dependences of the critical (limiting) load $\tau_* = \tau_y^{\pi}\sqrt{\omega_1}/K_{IIIc}$ for both crack tips on its length $l_* = a - l$ for some values hole radius $\lambda = 0.2, 0.3, 0.4, 0.5, 0.6$ (curves 1–5).

The calculations were carried out for the following elastic parameters $\frac{G_a}{G_s} = 25$, $\frac{G_a}{G_t} = 50$.

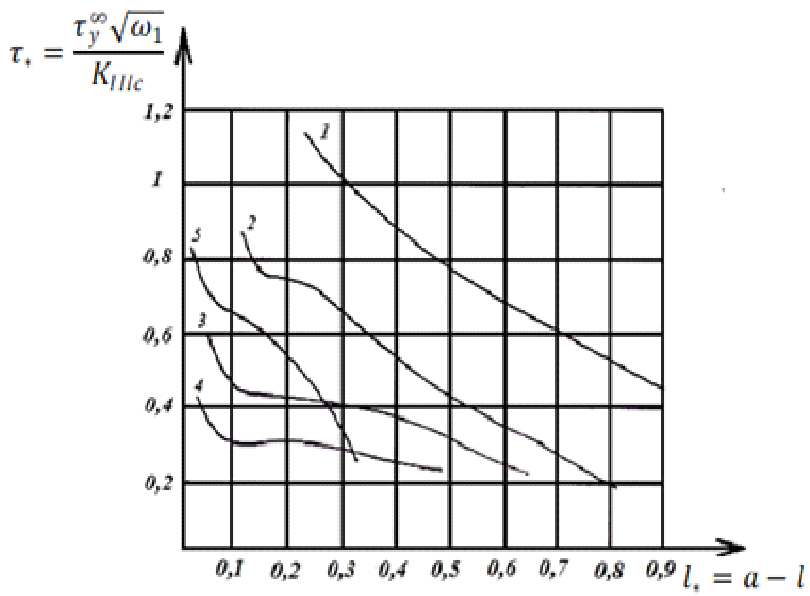

Fig. 4.1. The dependence of the critical external load $\tau_* = \tau_y^{\pi}\sqrt{\omega_1}/K_{IIIc}$ on the dimensionless length in cracks along the Y axis of the binder material $l_* = a - l$ for some values of the fiber cross-section radius $\lambda = 0.2, 0.3, 0.4, 0.5, 0.6$ (curves 1–5)

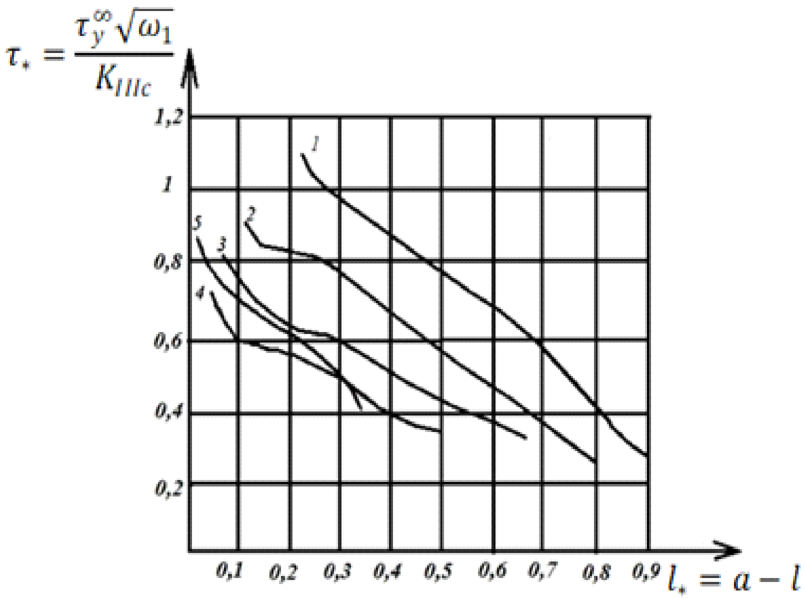

Fig. 4.2. Dependence of the critical external fiber load $\tau_* = \tau_y^\pi \sqrt{\omega_1}/K_{IIIc}$ **on the dimensionless crack length** $l_* = a - l$ **for some values of the fiber cross-section radius** $\lambda = 0.2, 0.3, 0.4, 0.5, 0.6$ **(curves 1–5)**

5 Conclusions

Analysis of the maximum equilibrium state of the composite with longitudinal shear. The fracture toughness viscosity equations of the fibrous composite stress coefficient are obtained depending on the nature of the internal structural defects. A mathematical description of the strength of the composite is carried out both at separation and at shear. As a result, the stress – strain state of the fibrous composite, weakened by periodic linear cracks, was determined.

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