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Development of a method for solving the problem of waves propagation in an elastic medium under the action of cylindrical inclusion

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Abstract. *In this paper, a technique is developed for solving the problem of elastic wave propagation in a medium with a cylindrical inclusion. On the basis of the obtained theoretical solution, numerical calculations were carried out. The analysis of constructed wave graphs describing wave propagation in various rocks and soils is given.*

Keywords. waves propagation, nonstationar waves, elliptic integral, elastic, cylindrical connection, polar coordinates, damping, boundary conditions

Mathematics Subject Classification (2010): 86A15

1 Introduction

Two-dimensional problems on wave propagation in an elastıc medıum are ınterestıng both from theretical and partical point of view [1-4]. Especially, a problem on seismic wave propagation can be considered two-dimensional taking into acoount their fast damping on depth. Below we consider the movement of a rigid cylinder in an unbounded elastic medium.

For applying the Duamel principle, the solution of the problem under the boundary conditions in the form of istant increase in the subsequent values on the rate of cylindrical inclusion, is of great interest. But the solution of such problems is related with analytic difficulties. Constructing analytic solution of a dynamics problem characterized in the form of impulses accompanied by instant increase parameters or subsequent damping in boundaries is not difficult, and this was succeeded in this paper.

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1.1. Problem statement and the basics of determining the ratios.

We consider a two-dimensional problem on propagation of two kind of waves in elastic medium with continuous breaks in the form of cylindrical connection

$$\begin{aligned} a^2\Delta\varphi - \tfrac{\partial^2\varphi}{\partial t^2} = 0 \\ b^2\Delta\psi - \tfrac{\partial^2\psi}{\partial t^2} = 0 \end{aligned} \tag{1.1}$$

Here φ is a potential function describing the waves that characterize the extension of rotation volume; the function ψ characterizes equivoluminal waves of rotation.

The quantities $\begin{matrix} a = \sqrt{\frac{\lambda+2\mu}{\rho}} \\ b = \sqrt{\frac{\mu}{\rho}} \end{matrix}$ determine the propagation rate of extensional and rotational waves, respectively;

λ and μ are Lame constants;
ρ is medium density;
Δ is a Laplace operator.
u and v displacements in the polar coordinate system are represented in the form of

$$\begin{aligned} u &= \frac{\partial\varphi}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial\theta} \\ v &= \frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta} \end{aligned} \tag{1.2}$$

Here r, θ are polar coordinates

$$u_{1t}/r = r_0 = H(t)V_0 \tag{1.3}$$

Taking into account that the ratios are determined in limit conditions on the cylindrical connection surface (1.1) we can find solutions of equations (1.2) in the polar coordinates r, θ.

Here, V_0 is constant rate of cylindrical connection;
r_0 is a radius;
$H(t)$ is a Heavisde function

$$H(t) = \begin{cases} 1 & t > 0 \\ 0, & t < 0 \end{cases}$$

1.2. The solution of the problem on propagation of nonstationary waves in an elastic medium interacting with cylindrical inclusion.

By menas of the Laplace-Carsont transformation we can represent the solution of wave equations (1.1) responding to the movement of the cylindrical connection in the following form.

$$\begin{aligned} \overline{\varphi_1} = CK_1\left(\tfrac{pr}{a}\right) \\ \overline{\psi_1} = DK_1\left(\tfrac{pr}{b}\right) \end{aligned} \tag{1.4}$$

Here:

$$\overline{\varphi_1} = \overline{\varphi}\,/\cos\theta$$

$$\overline{\psi_1} = \overline{\psi}/\sin\theta$$

K_1 is a first order McDonald function; p is a parameter of the Laplace-Carson transformation; C and D are determinants.

Without disconnecting during the movement of the medium, the boundary conditions

$$\frac{\partial\varphi_1}{\partial r}-\frac{\psi_1}{r}=-\frac{\partial\psi_1}{\partial r}+\frac{\varphi_1}{r} \tag{1.5}$$

for $r=r_0$ in (1.5) taking into account the solution (1.4)

$$\begin{aligned} C&=-f(p)\left[\tfrac{p}{b}K_0\left(\tfrac{pr_0}{b}\right)+\tfrac{2}{r_0}K_1\left(\tfrac{pr_0}{b}\right)\right]\\ D&=-f(p)\left[\tfrac{p}{a}K_0\left(\tfrac{pr_0}{a}\right)+\tfrac{2}{r_0}K_1\left(\tfrac{pr_0}{a}\right)\right]\end{aligned} \tag{1.6}$$

Here: $f(p)$ is a function determined from the boundary conditions.

Taking into account the expression (1.6) in (1.4) and then in (1.2), we obtain

$$\bar{u}_1=\bar{u}/\cos\theta=f(p)L \tag{1.7}$$

$$\begin{aligned} L=&\tfrac{p^2}{ab}K_0\left(\tfrac{pr_0}{b}\right)K_0\left(\tfrac{pr_0}{a}\right)+\tfrac{p}{br}K_0\left(\tfrac{pr_0}{b}\right)K_1\left(\tfrac{pr}{a}\right)+\\ &+\tfrac{2b}{ar_0}K_1\left(\tfrac{pr_0}{b}\right)K_0\left(\tfrac{pr_0}{a}\right)+\tfrac{2}{r_0r}K_1\left(\tfrac{pr_0}{b}\right)K_1\left(\tfrac{pr}{a}\right)-\\ &-\tfrac{p}{ar}K_0\left(\tfrac{pr_0}{a}\right)K_1\left(\tfrac{pr}{b}\right)-\tfrac{2}{r_0r}K_1\left(\tfrac{pr_0}{a}\right)K_1\left(\tfrac{pr}{b}\right)\end{aligned}$$

From (1.7) in the boundary $r=r_0$ we obtain

$$\begin{aligned}\bar{u}_1=&pf(p)\left(\tfrac{p}{ab}K_0\left(\tfrac{pr_0}{a}\right)K_0\left(\tfrac{pr_0}{b}\right)+\tfrac{1}{br_0}K_0\left(\tfrac{pr_0}{b}\right)K_1\left(\tfrac{pr_0}{a}\right)+\right.\\ &\left.+\tfrac{1}{ar_0}K_0\left(\tfrac{pr_0}{a}\right)K_0\left(\tfrac{pr_0}{b}\right)\right)\end{aligned} \tag{1.8}$$

Using the asymptotic approximation of the McDonald function in determining the function f from the boundary conditions, we can obtain an exact solution of the wave equations, and solutions in the field of wave propagation that respond to continuous attachment to the motion of the medium.

Taking the solution in (1.8)

$$\begin{aligned}&p\to\infty\\ &K_0(z)\approx\sqrt{\tfrac{\pi}{2z}}\cdot e^{-z}\\ &K_1(z)\approx\sqrt{\tfrac{\pi}{2z}}\cdot e^{-z}\end{aligned}$$

and taking into account in (1.3)

$$V_0=\frac{p^2}{ab}f(p)\sqrt{\frac{\pi b}{2pr_0}}\cdot\sqrt{\frac{\pi a}{2pr_0}}\left(p+\frac{a+b}{r_0}\right)e^{-\frac{pr_0}{a}}\cdot e^{-\frac{pr_0}{b}}$$

Here $f=\frac{2r_0\sqrt{ab}\,e^{\frac{pr_0}{a}}\cdot e^{\frac{pr_0}{b}}}{\pi p\left(p+\frac{a+b}{r_0}\right)}$ substituting in (1.7) we obtain $\bar{u}_{1t}=p\bar{u}_1$

$$\bar{u}_{1t}=\frac{2r_0\sqrt{ab}}{\pi\left(p+\frac{a+b}{r_0}\right)}V_0e^{-\frac{pr_0}{a}}\cdot e^{-\frac{pr_0}{b}}L \tag{1.9}$$

in the boundary $r = r_0$

$$\bar{u}_1 = pf(p)\left(\tfrac{p}{ab}K_0\left(\tfrac{pr_0}{a}\right)K_0\left(\tfrac{pr_0}{b}\right) + \tfrac{1}{br_0}K_0\left(\tfrac{pr_0}{b}\right)K_1\left(\tfrac{pr_0}{a}\right) + \right. \\ \left. + \tfrac{1}{ar_0}K_0\left(\tfrac{pr_0}{a}\right)K_0\left(\tfrac{pr_0}{b}\right)\right) \tag{1.10}$$

In the expression (1.9) we determine the originals

$$pK_0\left(\frac{pr}{c}\right) \longrightarrow \frac{H\left(t-\frac{r}{c}\right)}{\sqrt{t^2-\left(\frac{r}{c}\right)^2}}$$

$$K_1\left(\frac{pr}{c}\right) \longrightarrow \frac{r}{c}\sqrt{t^2-\frac{r}{c^2}^2}$$

$$pK_1\left(\frac{pr}{c}\right) \longrightarrow \frac{r}{c}\frac{t}{\sqrt{t^2-\frac{r}{c^2}^2}}$$

$$pK_0\left(\frac{pr_0}{b}\right)K_0\left(\frac{pr}{a}\right)e^{\frac{pr_0}{a}}\cdot e^{-\frac{pr_0}{b}} \longrightarrow A_1(a,b) =$$

$$= \int\limits_{\frac{r-r_0}{a}}^{t} \frac{d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)-\left(\frac{r_0}{b}\right)\right)\left(\left(\tau+\frac{r_0}{a}\right)-\left(\frac{r_0}{a}\right)\right)}} =$$

$$= \frac{2H\left(t-\frac{r-r_0}{a}\right)F(k(a,b))}{m(a,b)} \tag{1.11}$$

Taking into account the originals, where F is a first kind complete elliptic integral

$$K(a,b) = \sqrt{\frac{\left(t-\frac{r-r_0}{a}\right)\left(t+2\frac{r_0}{b}+\frac{r+r_0}{a}\right)}{\left(t+2\frac{r_0}{b}-\frac{r-r_0}{a}\right)\left(t+\frac{r+r_0}{a}\right)}}$$

$$m(a,b) = \sqrt{\left(t+2\frac{r_0}{b}-\frac{r-r_0}{a}\right)\left(t+\frac{r+r_0}{a}\right)}$$

$$K_0\left(\frac{pr_0}{b}\right)K_1\left(\frac{pr}{a}\right)e^{\frac{pr_0}{b}}\cdot e^{-\frac{pr_0}{a}} \longrightarrow A_2(a,b) =$$

$$= \frac{a}{r}\int\limits_{\frac{r-r_0}{a}}^{t} \frac{\left(\tau+\frac{r_0}{a}\right)d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} =$$

$$= \frac{r_0}{r}\int\limits_{\frac{r-r_0}{a}}^{t} \frac{d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} +$$

$$+ \frac{a}{r}\int\limits_{\frac{r-r_0}{a}}^{t} \frac{\tau d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} =$$

$$= \frac{r_0}{r}\frac{2HF(K(a,b))}{m(a,b)} + \frac{2Ha}{rm(a,b)}\left(\frac{2r}{a}\Pi(n(a,b),k(a,b) - \frac{r+r_0}{a}F(K(a,b))\right) =$$

$$= \frac{2H\left(t-\frac{r-r_0}{a}\right)(2\Pi(n(a,b)K(a,b)) - F(K(a,b)))}{m(a,b)}. \tag{1.12}$$

Here Π is a second kind complete elliptic integral $n(a,b) = \frac{t-\frac{r-r_0}{a}}{t+\frac{r+r_0}{a}}$

$$pK_1\left(\frac{pr_0}{b}\right)K_0\left(\frac{pr}{a}\right)e^{\frac{pr_0}{b}}\cdot e^{\frac{pr_0}{a}} \longrightarrow A_3(a,b) =$$

$$= \frac{b}{r_0}\int\limits_{\frac{r-r_0}{a}}^{t}\frac{\left(t-\tau+\frac{r_0}{b}\right)d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} =$$

$$= \left(\frac{bt}{r_0}+1\right)\int\limits_{\frac{r-r_0}{a}}^{t}\frac{d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} -$$

$$-\frac{b}{r_0}\int\limits_{\frac{r-r_0}{a}}^{t}\frac{\tau d\tau}{\sqrt{\left(\left(t-\tau+\frac{r_0}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right)\left(\left(\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)}} =$$

$$= \left(\frac{bt}{r_0}+1\right)\frac{2HF(K(a,b))}{m(a,b)} +$$

$$+\frac{2bH}{r_0 m(a,b)}\left(2\frac{r}{a}\Pi(n(a,b),k(a,b) - \frac{r+r_0}{a}F/K(a,b)\right) =$$

$$= \frac{2H\left(t-\frac{r-r_0}{a}\right)\left(\frac{bt}{r_0}+\frac{rb}{r_0a}+\frac{b}{a}+1\right)F\left(K(a,b)\right) - 2\frac{rb}{r_0a}\Pi(n(a,b),K(a,b))}{m(a_1b)} \tag{1.13}$$

$$\frac{1}{p}K_1\left(\frac{pr_0}{a}\right)K_1\left(\frac{pr}{b}\right)e^{\frac{pr_0}{a}}\cdot e^{\frac{pr_0}{b}} \longrightarrow A_4(a,b) =$$

$$= \frac{ab}{r_0 r}\int\limits_{\frac{r-r_0}{a}}^{t}\sqrt{\left(\left(t-\tau+\frac{r_0}{a}\right)^2-\left(\frac{r_0}{a}\right)^2\right)\left(\left(\tau+\frac{r_0}{b}\right)^2-\left(\frac{r}{b}\right)^2\right)} \tag{1.14}$$

Taking into account,

$$\frac{p}{p+\frac{a+b}{r_0}} \longrightarrow H(t)e^{-\frac{a+b}{r_0}t}$$

$$\frac{p}{p+\frac{a+b}{r_0}}\overline{A} \longrightarrow \int\limits_0^t e^{-\frac{a+b}{r_0}(t-\tau)}A(\tau)d\tau$$

$$\frac{p^2}{p+\frac{a+b}{r_0}}\overline{A} \longrightarrow A - \frac{a+b}{r_0}\int\limits_0^t e^{-\frac{a+b}{r_0}(t-\tau)}A(\tau)d\tau \tag{1.15}$$

fixing $e^{-\frac{a+b}{r_0}t} = \mu$, we obtain the resulting expression of the rate as follows:

$$u_t = \frac{2r_0V_0\sqrt{ab}}{\pi}\left(\frac{1}{ab}\left(A_1(a,b) - \frac{a+b}{r_0\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_1(a,b)\mu d\tau\right) + \right.$$

$$+\frac{1}{br\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_2(a,b)\mu d\tau) + \frac{2}{ar_0\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_3(a,b)\mu d\tau + \frac{2}{r_0 r}(A_4(a,b)-$$

$$-\frac{a+b}{r_0\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_4(a,b)\mu d\tau) - \frac{1}{ar\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_2(b,a)\mu d\tau -$$

$$\left. -\frac{2}{r_0 r}(A_4(a,b) - \frac{a+b}{r_0\mu}\int\limits_{\frac{r-r_0}{a}}^{t} A_4(b,a)\mu d\tau)\right) \quad (1.16)$$

It should be noted that the product in the denominator of the integrand function corresponding to the field $r > r_0$ vanishes at the value of the argument τ equal to the lower bound of the integral. In the values of the quatity τ equal to the upper bound of the integral the product corresponding to $r = r_0$ vanishes. In the integrals of (1.15) when the quantity τ equals the upper bound of the integral, the exponential argument vanishes.

So, the integration (1.11), (1.12), (1.13), (1.14) are fulfiled among special values of the subintegrals function.

For $r = r_0$, $F(a,b) = F(b,a)$

$$A_2^0(b,a) = \frac{2}{m}\left(2\Pi(n(b,a),k) - F(k)\right) = A_3^0(a,b) =$$
$$= \frac{2}{m}\left(\left(\frac{bt}{r_0} + 2\frac{b}{a} + 1\right)F(k) - 2\frac{b}{a}\Pi(a,b)\right) \quad (1.17)$$

Taking into account (1.17), thesolution (1.16) takes the form:

$$u_t = \frac{2r_0V_0\sqrt{ab}}{\pi}\left(\frac{1}{ab}A_1^0 - \frac{a+b}{r_0\mu}\int\limits_0^t A_1^0\mu d\tau\right) +$$
$$+\frac{1}{br_0\mu}\int\limits_0^t A_2^0(a,b)\mu d\tau + \frac{1}{ar_0\mu}\int\limits_0^t A_2^0(a,b)\mu d\tau \quad (1.18)$$

Here

$$A_1^0 = \frac{2F(k)}{m}$$

$$K = K(a,b) = K(b,a) = \sqrt{\frac{t\left(t + 2\frac{r_0}{b} + 2\frac{r_0}{a}\right)}{\left(t + 2\frac{r_0}{b}\right)\left(t + 2\frac{r_0}{a}\right)}}$$

$$m = m(a,b) = m(b,a) = \sqrt{\left(t + 2\frac{r_0}{b}\right)\left(t + 2\frac{r_0}{a}\right)}$$

$$n(a,b) = \frac{t}{t + 2\frac{r_0}{a}}$$

$$n(b,a) = \frac{t}{t + 2\frac{r_0}{b}}$$

In the calculations the values of the quatities k and n can be close to a unit and this leads to a large value of the elliptic integrals and make impossible to use tables.

Therefore, it is necsssary to use the following asymptotics formulas

$$\varphi = \frac{\pi}{2} n \to 1.$$

We must give asymptotic form to the third kind elliptic integral or as $k^2 \to 1$ this elliptic integral is divided. For that we divide the integral into two parts

$$\prod\left(\frac{\pi}{2}, n, k\right) = \int\limits_0^{\frac{\pi}{2}} \frac{d\varphi}{(1 - n\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}} =$$

$$= \int_0^{\frac{89}{180}\pi} \frac{d\varphi}{(1 - n\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}} +$$

$$+ \int_{\frac{89}{180}\pi}^{\frac{\pi}{2}} \frac{d\varphi}{(1 - n\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}} =$$

$$= \prod = (\frac{89}{180}\pi, n, k) + B$$

Here

$$B = \int_{\frac{89}{180}\pi}^{\frac{\pi}{2}} \frac{d\varphi}{(1 - n\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}} \tag{1.19}$$

Changing the parameter in the second integral and calling the new parameter as φ, we obtain

$$B = \int_0^{\frac{\pi}{180}} \frac{d\varphi}{(1 - n\cos^2\varphi)\sqrt{1 - k^2\cos^2\varphi}}$$

Changing $\cos^2\varphi \approx 1 - \varphi^2$

$$B = \int_0^{\frac{\pi}{180}} \frac{d\varphi}{(1 - n\ (1 - n\varphi^2))\ \sqrt{1 - k^2(1 - \varphi^2)}} =$$

$$= \frac{1}{n\sqrt{k^2}} \int_0^{\frac{\pi}{180}} \frac{d\varphi}{(\varphi^2 + \frac{1-n}{n})\sqrt{\varphi^2 + \frac{1-k^2}{k^2}}} = \Phi(n,k). \tag{1.20}$$

In our problem

$$k = k(a,b) = k(b,a) = \sqrt{\frac{t(t + \frac{2r_0}{a} + \frac{2r_0}{b})}{(t + \frac{2r_0}{a})(t + \frac{2r_0}{b})}}$$

$n = n(a,b)$ or $n = (b,a)$.

It can easily be seen that the such values of the quantities n and k

$$\frac{1-n}{n} > \frac{1-k^2}{k^2}$$

Then, according to the integration formulas we obtain

$$\Phi(n,k) = \frac{1}{\sqrt{1-n}\sqrt{k^2-n}} \ln \frac{\left|\varphi\sqrt{\frac{1-n}{n} - \frac{1-k^2}{k^2}} + \sqrt{\frac{1-n}{n}}\sqrt{\varphi^2 + \frac{1-k^2}{k^2}}\right|}{\sqrt{\varphi^2 + \frac{1-n}{n}}}\Bigg|_0^{\frac{\pi}{180}} =$$

$$= \frac{1}{\sqrt{1-n}\sqrt{k^2-n}} \times$$

$$\times \left[\ln \frac{\left|\frac{\pi}{180}\sqrt{\frac{k^2-n}{nk^2}} + \sqrt{\frac{1-n}{n}\left(\left(\frac{\pi}{180}\right)^2 + \frac{1-k^2}{k^2}\right)}\right|}{\sqrt{\left(\frac{\pi}{180}\right)^2 + \frac{1-n}{n}}} - \ln\sqrt{\frac{1-k^2}{k^2}}\right] \qquad (1.21)$$

Taking into account the expression (1.21) the asymptotic representation of (1.19) as $n \to 1$ or $k \to 1$ will be as follows:

$$\Pi\left(\frac{\pi}{2}, n, k\right) \approx \Pi\left(\frac{89}{180}\pi, n, k\right) + \Phi(n,k)$$

Thus, the integrals in the expressions (1.11), (1.12), (1.13) with the subintegral function properties were shaped as an elliptic integral.

For determining the rate ν_t it suffices to interchange the values of a and b. The constructed solution of the problem meets the implusive boundary conditions with further change in the rate with respect to time t.

1.3. Taking numerical calculations

The numerical calculations were made by means of the Turbo Paskal 7.0 software package.

In the below calculations, the estimations for propagation rate of elastic waves for various kind soil and mountain rocks indicated in Table 1 were used. Reference information on propagation rate and seismic density of elastic waves, the encountered sizes (volume mass) of density for various kinds of soil and mountain rocks was given.

Table 1.

Kind and name of the soil	Density ρ (volume mass) $1g/cm^3$	Rate of elastic waves V km/s				Seismic conditions	
		Iongitudinal Vp		Transverse Vs		Vp ρ	Vs ρ
			Average speed		Average speed		
I Mountain rocks			5,6				
Granits of depth zone, granits, bazalts, gabbro and other mountain rocks	2,8			3-5,5	3,2	18,2	9,4
a) in non-abrasive, natural moisture	2,5-3,8	2,0-7,0		1,0-4,8	2,8	5-28,6	4,5-15,2
b) abrasive, cracked ones, water proof ones	1,6-2,35	1,0-3,3	2,2-3,2	1,2-3,0 0,2-0,8	0,5	1,6-7,75	3,2-1,4
c) and also water permeable ones dense lines dense soils tight crackling dense sandstones, argillites	1,65-2,50 2,35-3,0 2,4-3,05 1,5-2,95	1,6-3,3 2,4-7,0 3,5-7,0 1,4-4,5	2,8-3,2 4-8,5 2,5-3,2	1,1-4 1,7-2 1,1-2	1,4-1,16 1,8-2 1,4-1,8	2,6-8,25 6-21 8,3-21 2,4-13	3,3-1,5 2,6-12,0 4,1-12,2 1,6-6
II. Semirock breeds							
Plasters (in natural moisture) margels (in natural moisture) clay shales	2,1-2,4 1,8-2,4 2,6-2,7	2,0-5,5 1,1-6,0 1,6-4,7	2,5-3,2 2,6-3,5 2,4-4,0	1-3 0,4-3,4 0,8-2,8	1,2-1,6 0,5-0,6 0,7-2	4,3-13 2-16 4,2-12	2,2-7,2 0,7-9,5 1,6-7,5

Using the numerical solution program of 1.2 and also the formulas (1.16) and (1.18) we make calculation for the following values of parameters:

$$a = 2000m/sec; \quad b = 1400m/sec;$$

$$r_0 = 10m; \; r = 100m; 1000m; 10000m.$$

In the considered medium, the nonstationary elastic wave that is in contact with connectiop and creates waves reflected by connection, also acts together with connection.

Taking into account fast damping of semismic waves in depth, we can consider the problem as a two dimensinal one. Obtaining numerical and analytic solution of the problem characterized by the parameters in the from of impluses accompanied with instant increase and further damping is of theretical and practical importance.

In numerical calculations the time changes between the interval $0 \leq t \leq 10$ sec $\Delta t = 0,01$ sec was accepted as a calulation step.

Time-dependence graphs were built based on the carried out numerical calculations and $r = 100; 1000; 10000$ m from the connection center.

In Fig. 1.1 the upper curve corresponds to the dependence of $u_t(t)$ on t (for $r = 100$ m), the middle curve corresponds to the dependence of $u_t(t)$ on t (for $r = 1000$ m); the lower curve corresponds to the dependense of $u_t(t)$ on t (for $r = 10000$ m).

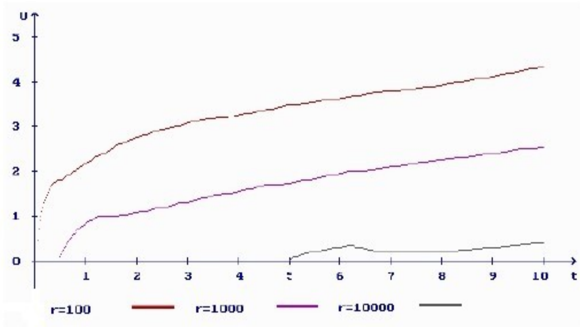

Fig. 1.1. The upper curve corresponds to the dependence $u_t(t)$ vs. t (at r = 100 m), the middle curve corresponds to the dependence $u_t(t)$ vs. t (at r = 1000 m); the lower curve corresponds to the dependence $u_t(t)$ vs. t (at r = 10000 m)

We can see from the figure that far away from the connection center, the damping of displacement waves is more noticable.

As can be seen, for $r = 100$ m and $t = 0.05$ sec, $u_t(t)$ = 0.318656 m/sec, $t = 0$; in the seconds 0.01; 0.02; 0.03; 0.04 the function $u_t(t)$ vanishes for $r = 1000$ m the time-delay is more observed. So, for $t = 0.50$ sec $u_t(t)$ is characterized by the value 0.092911 m/sec For $0 = t = 0.49$ sec the function $u_t(t)$ vanishes. At the distance $r = 10000$; m= 10 km for $t = 5.00$ sec the function becomes $U(t) = 0.029137$ m/sec and weak increase close to the constant value is observed.

It should be noted that the values of the rates of longitudinal and transverse waves used in calculations, correspond to propagation value of these elastic waves in mountain roks (granit, bazalt, gabbro and other rocks in open air and with natural moisture).

The calculations were carried out just in these media without changing (Fig. 1.2) the values of elastic rates. As can be seen from the figure, propagation of displacement waves delay at various values of the quantity r , as r beconme larger, the value incerases. Unlike Fig. 1.1, in creasing the radius of the cylindrical connection obtained based on the values $a = 2000$ m/sec; b = 1400 m/sec; r_0 = 90 m; r = 100 m; 1000 m; 10000 m and represented in Fig. 1.3 we observe weak increase in the value of displacement rate.

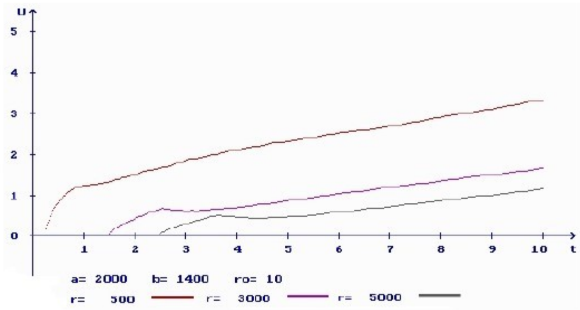

Fig. 1.2. Diagram of distribution of displacement wave velocities at different r values

It is seen from Fig. 1.4 that for r_0 = 10 m the curvature corresponding to the value $r = 100$ m unlike the curvature corresponding to the value $r = 100$ m for r_0 = 90 m takes its origin not from the point $t = 0$ (Fig. 1.3), but from the point where $t = 1$ sec and each value of t the value of the displacement rate (Fig. 1.1) is more than the value of the appropriate curvature in Fig. 3.

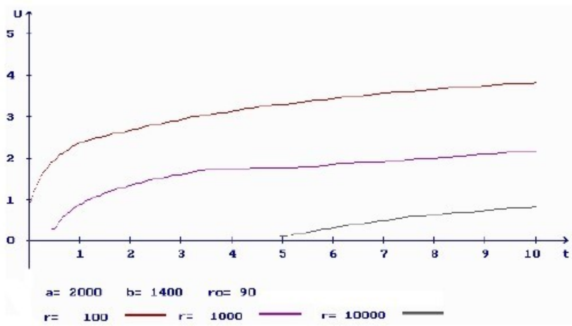

Fig. 1.3. Graph of a weak increase in the value of the displacement velocity by increasing the radius of the cylindrical coupling

Fig. 1.4 represents dependence curvature at various radius of cylindrical connection r_0 = 10 m; 50 m; 90 m during displacement. In these three cases in the carried out calculations the distance from the connection center was accepted as $r = 100$ m. Propagation rate of elastic waves are a = 2000 m/sec; b = 1400 m/sec. It is seen from the figure that the curvatures are close to each other and they intersect in some places. This is characterized by the propagation of the waves at a close distance in to the same soil.

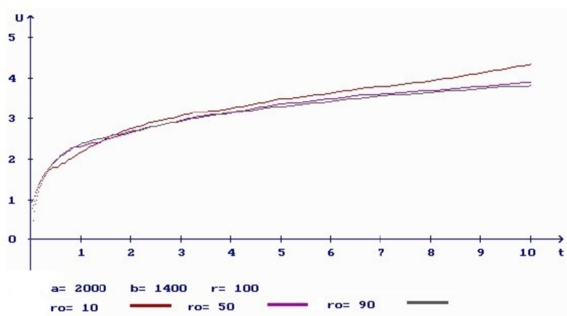

Fig. 1.4. Graphs of the dependence of the displacement time at different radii r_0=10 m; r_0=50 m; r_0=90 m

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