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Modeling of gas-oil displacement process in layered heterogeneous reservoirs

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Abstract. *This paper focuses on the oil recovery from a layered heterogeneous reservoir. Both reservoir layers are heterogeneous and the permeability coefficient in each layer varies linearly with radius. The permeability was assumed to decrease in one reservoir and increase in the other reservoir in the direction from the injection well to the production well, the oil filtration equation and the gas flow equation in the reservoir were written for each layer, and the problem was solved under boundary conditions. For each heterogeneous reservoir, analytical expressions for oil mass flow rate and displacement period were derived by constructing a model of the oil-gas displacement process.*

Keywords. dynamic viscosity coefficient · mass flow · conductivity · continuity condition.

Mathematics Subject Classification (2010): 76N25

1 Introduction

Waterflooding of an oil reservoir is one of the main secondary recovery methods in most fields. The efficiency of oil displacement also depends on the number of layers of heterogeneous formations. Research on oil displacement considering reservoir heterogeneity and dynamic interconnection of the reservoir-well system is of great practical and scientific importance. There exist many factors that influence on the gas-oil displacement process. Porosity of the reservoir, degree of saturation of the reservoir with oil, permeability of the reservoir, viscosity of oil and water in the reservoir, oil activity, etc. are among them [2 - 5, 8, 9]. If the residual oil content in the reservoir is 35%, then gas or air injection to this type of reservoir is not considered effective and not recommended. If the reservoir contains 35% water in addition to oil, gas can be used as a displacing agent [7]. To improve the process of oil recovery from the reservoir, a mathematical model of the process, with regard to main factors influencing on this process should be created and studied. This paper is concerned with solving and analysing the equations derived from a model of the oil-gas-displacement process in a heterogeneous reservoir.

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2 Problem statement

Let us consider the process of oil recovery by means of displacement when the reservoir is layered. Assume that the layer consists of two layers and each layer in its turn is heterogeneous (Fig. 1, Fig. 2). Assume that in the first approximation, the heterogeneity of the layer is linear (Fig. 2), the hydraulic connection of layers is weak, and we ignore it [6].

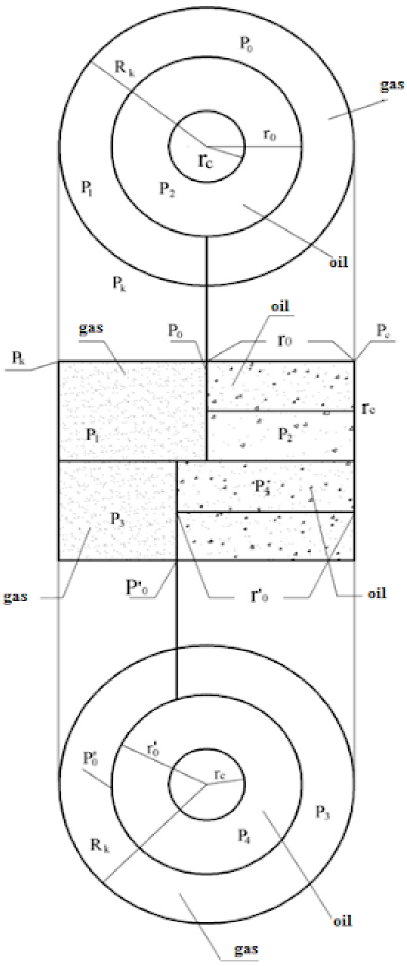

Fig. 1. The process of gas-oil displacement in a layered-heterogeneous reservoir

Let us assume that the linear change in the permeability coefficient as a function of radius in the direction from the injection well to the production well decreases in one reservoir layer and increases in the other (Fig. 2).

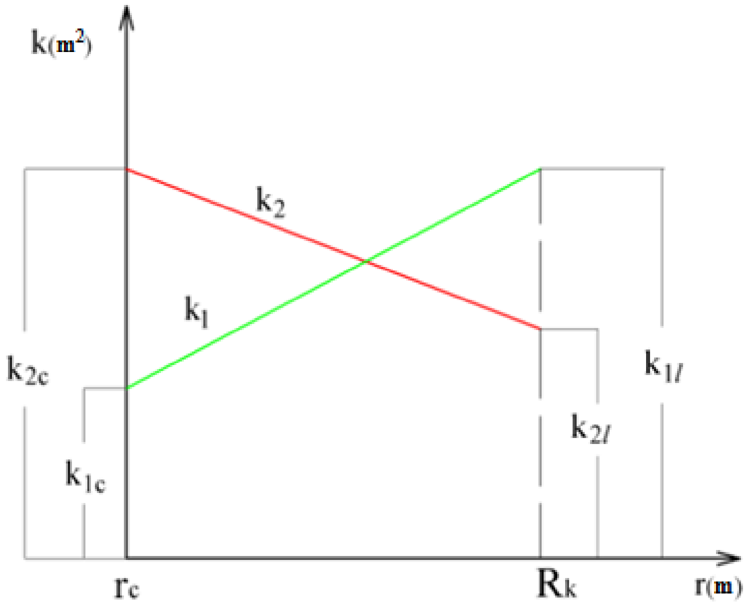

Fig. 2. Linear change in permeability of a layered-heterogeneous layer by radius

According to Fig.2, for the first and second layers we write:

$$k_1 = k_{1c} + \frac{k_{1l} - k_{1c}}{R_k - r_c}(r - r_c) \tag{2.1}$$

$$k_2 = k_{2c} - \frac{k_{2c} - k_{2l}}{(R_k - r_c)}(r - r_c) \tag{2.2}$$

Here k_1 and k_2 permeabilities of individual layers of the reservoir,k_{1l}, k_{2l} k_{1c}, k_{2c} are permeabilities of the contour and bottom hole zone, R_k is the radius of the contour, r is a coordinate.

3 Problem solution

Considering $r_c << R_k$, we can write the expressions (2.1) and (2.2) in the following way:

$$k_1 = k_{1c} + \frac{k_{1l} - k_{1c}}{R_k} r \tag{3.1}$$

$$k_2 = k_{2c} - \frac{k_{2c} - k_{2l}}{R_k} r \tag{3.2}$$

Since the displacement process occurs slowly, in the first approximation we will accept this process as stationary.

Then for the first layer, the equation of a gas flow in the reservoir will be as [9]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\bar{\chi}_1 r \frac{\partial P_1^2}{\partial r}\right) = 0 \tag{3.3}$$

where P_1 is a pressure at any point of gas-containing part of the reservoir, $\bar{\chi}_1 = \frac{k_1 P_k}{\mu_q m}$,μ_q is a coefficient of dynamic viscosity of gas, m is porosity, P_k is contour pressure.

The boundary conditions:

$$P_1^2\big|_{r=R_k} = P_k^2 \tag{3.4}$$

$$P_1^2\big|_{r=r_0} = P_0^2 \tag{3.5}$$

The filtration equation set in the first layer of oil, is written as follows:

$$\frac{d}{dr}\left(\frac{rk_1}{\mu_n}\frac{dP_2}{dr}\right) = 0 \tag{3.6}$$

The equation (3.6) is solved under the following boundary conditions:

$$P_2|_{r=r_0} = P_0 \tag{3.7}$$

$$P_2|_{r=r_c} = P_c \tag{3.8}$$

The gas flow equation for the second layer:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\bar{\chi}_2 r\frac{\partial P_3^2}{\partial r}\right) = 0 \tag{3.9}$$

$$\bar{\chi}_2 = \frac{k_2 P_k}{\mu_q m} \tag{3.10}$$

The boundary conditions:

$$P_3^2\big|_{r=R_k} = P_k^2 \tag{3.11}$$

$$P_3^2\big|_{r=r_0'} = P_0'^2 \tag{3.12}$$

In this layer, we write the oil filtration equation as follows:

$$\frac{d}{dr}\left(\frac{rk_2}{\mu_n}\frac{dP_4}{dr}\right) = 0 \tag{3.13}$$

The boundary conditions:

$$P_4|_{r=r_0'} = P_0' \tag{3.14}$$

$$P_4|_{r=r_c} = P_c \tag{3.15}$$

where P_1, P_3 are pressures at any point of the gas containing, reservoir part of the; P_2, P_4 are pressures at any point of the oil containing part of the reservoir, μ_q and μ_n are coefficients of dynamic viscosity of gas and oil, P_k is a contour pressure, $P_0^{'}$ pressure on the boundary $r_0^{'}$.

In the first layer of the reservoir the pressures P_1^2 and P_2 are distributed on the base of filtration equations (3.3) and (3.6) and boundary conditions (3.4), (3.5), (3.7), (3.8):

$$P_1^2 = A\left(\ln\left(\frac{r}{k_{1c} + \frac{k_{1l}-k_{1c}}{R_k}r}\right) - \ln\left(\frac{R_k}{k_{1l}}\right)\right) + P_k^2 \tag{3.16}$$

$$P_2 = B\left(\ln\left(\frac{r}{k_{1c} + \frac{k_{1l}-k_{1c}}{R_k}r}\right) - \ln\left(\frac{r_0}{k_{1c} + \frac{k_{1l}-k_{1c}}{R_k}r_0}\right)\right) + P_0 \tag{3.17}$$

where

$$A = \frac{P_k^2 - P_0^2}{\ln\left(\frac{R_k}{k_{1l}}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}{r_0}\right)}$$

$$B=\frac{P_0-P_c}{\ln\left(\frac{r_0}{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_c}{r_c}\right)}$$

Solving the problem, we get the following expression for the oil flow rate when oil is displaced by gas for the first layer of an inhomogeneous reservoir:

$$Q_n=\frac{2\pi r_c h k_n \rho_n}{\mu_n}\left[\frac{k_c}{r_c\left(k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_c\right)}\right]\times$$

$$\times\left(\frac{-\frac{1}{2}\frac{B_1\pm\sqrt{B_1^2+4A_1^2P_k^2+4A_1B_1P_c}}{A_1}}{\ln\left(\frac{r_0}{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_c}{r_c}\right)}-\frac{P_c}{\ln\left(\frac{r_0}{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_c}{r_c}\right)}\right) \quad (3.18)$$

The time to reach the water-oil contact to the well is determined by the following expression:

$$T=\frac{m\mu_n}{k_n}\int_{r_c}^{R_k}\frac{1}{\left.\frac{\partial P_2}{\partial r}\right|_{r=r_0}}dr_0 \quad (3.19)$$

where

$$A_1=\frac{\rho_{atm}k_q}{P_{atm}\mu_q}\frac{k_c}{\ln\left(\frac{R_k}{k_{1l}}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}{r_0}\right)}\frac{1}{r_0\left(k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0\right)}$$

$$B_1=\frac{2\rho_n k_n}{\mu_n}\frac{k_c}{\ln\left(\frac{r_0}{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0}\frac{k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_c}{r_c}\right)}\frac{1}{r_0\left(k_{1c}+\frac{k_{1l}-k_{1c}}{R_k}r_0\right)} \quad (3.20)$$

We solve the problem for the second layer in the same way. Having written the expressions (3.10) in (3.2) in the equation (3.9) and integrated with regard to the, boundary conditions (3.11) and (3.12), we obtain:

$$P_3^2=\frac{(P_k^2-P_0'^2)}{\ln\left(\frac{R_k}{k_{2l}}\frac{k_{2c}-\frac{k_{2c}-k_{2l}}{R_k}r_0'}{r_0'}\right)}\left(\ln\left(\frac{r}{k_{2c}-\frac{k_{2c}-k_{2l}}{R_k}r}\frac{k_{2l}}{R_k}\right)\right)+P_k^2 \quad (3.21)$$

here P_0' is the value of the reservoir pressure on the gas-oil border.

Having written the expression (3.2) in the oil filtration equation (3.13) and integrated it with regard to boundary conditions (3.14) and (3.15), we obtain:

$$P_4=\ln\left(\frac{r}{\frac{k_{2c}-k_{2l}}{R_k}r-k_{2c}}\right)\left(\frac{(P_0'-P_c)}{\ln\left(\frac{r_0'}{\frac{k_{2c}-k_{2l}}{R_k}r_0'-k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c-k_{2c}}{r_c}\right)}\right)+$$

$$+P_c - \ln\left(\frac{r_c}{\frac{k_{2c}-k_{2l}}{R_k}r_c - k_{2c}}\right)\left(\frac{(P_0' - P_c)}{\ln\left(\frac{r_0'}{\frac{k_{2c}-k_{2l}}{R_k}r_0' - k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c - k_{2c}}{r_c}\right)}\right) \quad (3.22)$$

We write the expressions (3.21) and (3.22) in the following way :

$$P_3^2 = A_2\left(\ln\left(\frac{r}{k_{2c} - \frac{k_{2c}-k_{2l}}{R_k}r}\frac{k_{2l}}{R_k}\right)\right) + P_k^2 \quad (3.23)$$

$$P_4 = B_2\left(\ln\left(\frac{r}{\frac{k_{2c}-k_{2l}}{R_k}r - k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c - k_{2c}}{r_c}\right)\right) + P_c \quad (3.24)$$

respectively.

Here

$$A_2 = \frac{(P_k^2 - P_0'^2)}{\ln\left(\frac{R_k}{k_{2l}}\frac{k_{2c} - \frac{k_{2c}-k_{2l}}{R_k}r_0'}{r_0'}\right)}$$

$$B_2 = \frac{(P_0' - P_c)}{\ln\left(\frac{r_0'}{\frac{k_{2c}-k_{2l}}{R_k}r_0' - k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c - k_{2c}}{r_c}\right)}$$

The mass flow of injected gas on the contact surface [9]:

$$Q_q = \frac{\pi k_q r h \rho_{atm}}{\mu_q P_{atm}}\frac{\partial P_3^2}{\partial r}\bigg|_{r=r_0'} \quad (3.25)$$

Here h is an effective thickness, k_q is gas permeability in the reservoir,ρ_{atms} is atmospheric density; P_{atms} is atmospheric pressure.

The mass flow of the displaced oil:

$$Q_n = \frac{2\pi k_n r h \rho_n}{\mu_n}\frac{\partial P_4}{\partial r}\bigg|_{r=r_0'} \quad (3.26)$$

k_n is oil permeability, ρ_n is oil density

The continuity condition:

$$Q_q|_{r=r_0'} = Q_n|_{r=r_0'} \quad (3.27)$$

If we write expressions (3.23) and (3.24) in formulas (3.25) and (3.26), taking into account the result in the equation (3.27), we obtain

$$P_{01/2} = \frac{1}{2}\frac{B_3 \pm \sqrt{B_3^2 + 4A_3^2P_k^2 - 4A_3B_3P_c}}{A_3} \quad (3.28)$$

here

$$A_3 = -\frac{\rho_{atm}k_q}{P_{atm}\mu_q}\frac{k_{2c}}{\ln\left(\frac{R_k}{k_{2l}}\frac{k_{2c} + \frac{k_{2l}-k_{2c}}{R_k}r_0'}{r_0'}\right)}\frac{1}{r_0'\left(k_{2c} + \frac{k_{2l}-k_{2c}}{R_k}r_0'\right)}$$

$$B_3 = \frac{2\rho_n k_n}{\mu_n} \frac{k_{2c}}{\ln\left(\frac{r'_0}{k_{2c}+\frac{k_{2l}-k_{2c}}{R_k}r'_0} \frac{k_{2c}+\frac{k_{2l}-k_{2c}}{R_k}r_c}{r_c}\right)} \frac{1}{r'_0\left(k_{2c}+\frac{k_{2l}-k_{2c}}{R_k}r'_0\right)}$$

To determine mass oil flow when displacing oil by gas in the heterogeneous reservoir, allowing for expressions (3.24) and (3.28) in formula (3.26), we obtain the following analytic expression for mass flow of oil:

$$Q_n = \frac{2\pi r_c h k_n \rho_n}{\mu_n}\left[\frac{k_{2c}}{r_c\left(k_{2c}+\frac{k_{2l}-k_{2c}}{R_k}r_c\right)}\right]\times$$

$$\times\left(\frac{\frac{1}{2}\frac{B_3\pm\sqrt{B_3^2+4A_3^2P_k^2-4A_3B_3P_c}}{A_3}}{\ln\left(\frac{r'_0}{\frac{k_{2c}-k_{2l}}{R_k}r'_0-k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c-k_{2c}}{r_c}\right)} - \frac{P_c}{\ln\left(\frac{r'_0}{\frac{k_{2c}-k_{2l}}{R_k}r'_0-k_{2c}}\frac{\frac{k_{2c}-k_{2l}}{R_k}r_c-k_{2c}}{r_c}\right)}\right) \quad (3.29)$$

Oil filtration time during gas displacement process will be:

$$T = \frac{m\mu_n}{k_n}\int_{r_c}^{R_k}\frac{1}{\left.\frac{\partial P_4}{\partial r}\right|_{r=r'_0}}dr'_0 \quad (3.30)$$

4 Conclusion

So, we construct a model of gas-oil displacement process in layered-heterogeneous reservoirs and obtain analytic expressions (3.18) and (3.29) the displacement time for the mass flow of oil and (3.19) for (3.30) for each heterogeneous layer.

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