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Vibrations of longitudinally stiffened elastic medium-contacting cylindrical sheels

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Abstract. *When one of the surfaces of a shell with physical and chemical fields interact, i.e. when they are exposed to intensive temperature effect, radial exposure, action of chemically active media (induced inhomogeneity factors), mechanical characteristics of the material become variable. In this connection, there is a need to determine dynamical characteristics and stress-strain assessment of such constructions. Thus, development of effective methods for calculating inhomogeneous, liquid contacting shells is an actual problem of modern structural mechanics. In this paper we study vibrations of an elastic medium contacting cylindrical shell stiffened with rods along the generatrix and whose material and generatrix has inhomogeneity features along the thickness in circular direction.*
It was accepted that the amplitude of the displacements of the shells points is a linear function of coordinates. The medium effect was taken into account by the Winkler method. Using the Hamilton-Ostrogradsky variational principal, a frequency equation for finding vibration frequency of the construction was constructed, its roots were calculated and characteristical curves were built.

Keywords. shells · elastic medium · generatrix · longitudinal ribs · frequency equation

Mathematics Subject Classification (2010): 74K25

1 Introduction

In the paper [9] based on V.Z. Vlasov's momentless theory, a problem of stability of a cylindrical constructive-orthotropic shell variable along the generatrix under the action of axially-symmetric radial pressure varying along the shell axis is considered. At one thickness and pressure change ratio the exact solution is obtained for finding one of the values in the law of pressure change at which the shell loses its stability.

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In the paper [10] based on the refined theory, a new analytic solution of an axially-symmetric dynamical problem for circular conical shells inhomogeneous in thickness and with finite shear stiffness is constructed by the method of finite integral transformations. Arbitrary dynamic loading for a shell elastically built-in at the ends, is considered. The calculation scheme takes into account the dissipative force of viscous resistance. Stress-strain state and dynamical characteristics of the shell are analyzed depending on inhomogeneity degree of degradable structures.

The paper [11] considers the general case of loading of a shell with an axially-symmetric dynamical load with elastically built-in ends. The modulus of elasticity and density of the material are arbitrary functions of the thickness of the coordinate.

The paper [2] deals with a problem on nonlinear vibrations of three-layer inhomogeneous circular cylindrical shells. It is assumed that the layers were made of various inhomogeneous materials and elastic characteristics are continuous functions of the coordinate of the shell thickness. Accepting validity of Kirchoff-Liaw hypothesis for the whole element, a system of equations of shell motion was obtained with regard to geometrical nonlinearly.

An analytic solution was obtained.

In [12] the necessity of transition in calculations of inhomogeneous as pipelines from the rod theory to semi-momentless theory of thin shells, was substantiated. A cylindrical, two-layer, finite length cylindrical shell consisting of a steel pipe and protective reinforced concrete layer was taken as a calculation scheme. The way how to search for the initial surface of an inhomogeneous pipe was shown, more exactly, equations were given for the reduced stiffness of an inhomogeneous two-layer shell in tension, compression and bending. Proceeding from the assumption of the half-momentless theory of shells, the equation of motion of the shell was derived in two forms in efforts and displacements. The second form of the equation allowed to conclude that the further solution of the problem is reduced to finding the spectrum of frequency of free vibrations.

In [5] the results of experimental study of frequencies and the forms of natural vibrations of inhomogeneous cylindrical shells with holes are given. The studies were performed by the method of holographic interferometry. The influence of holes and other design features on the main dynamical characteristics of shells was established. The methodology of the experiment was described. Experimental data are compared with numerical results obtained by the finite elements method.

The paper [4] considers free vibrations of cylindrical shells with holes, stiffened with ribs and attached to solids. A refined mathematical model of vibrations that takes into account structural heterogeneity of the shell system is developed. The problem is solved in linear statement by the finite elements method with regard to discrete placement of ribs. The results of numerical studies of eigen frequencies and forms of vibrations are given. Dependence of influence of holes, stiffening ribs and attached solids on amplitude frequency characteristics of shells are obtained. Comparative analysis of the obtained numerical results with the known solutions is carried out.

In the paper [8] based on V.Z. Vlasov's half-momentless theory a problem of dynamical stability of an isotropic cylindrical shell variable along the generatrix of thickness and density under the action of a symmetric variable along the generatrix of external pressure is considered under different boundary conditions. At one thickness, pressure and density change ratio, an exact solution was obtained. Structural elements of long and medium length shells with variable density of the material are used in different fields of mechanical engineering and aerospace engineering for weight optimization. In the case of 5 boundary value problems, the minimum values of the excitation coefficients with respect to the possible occurrence of undamped oscillations were obtained for the first and second instability domains, that have great importance for engineering practice. The estimation of the accuracy of the WTB-method was derived for the considered boundary value problems and the laws of change in thickness and density. The numerical results are given.

In [13] long wave vibrations and waves are studied in an infinite anisotropic beam-strip inhomogeneous in thickness. A dispersion equation of second order accuracy was constructed with respect to the relative thickness of the beam. Additional qualitative effects associated with anisotropy were constructed.

In the paper [3] we study one of dynamical strength characteristics, the frequency of natural variations of a fluid-filled cylindrical shell made of a fiber-glass and strengthened with annular ribs heterogeneous in thickness and along the generatrix under the Navier boundary conditions. Using Hamilton-Ostrogradsky's variational principle, the frequency equations for calculating natural vibrations of the system under consideration, are constructed. In the calculation process, the linear laws for heterogeneity function were accepted. Frequency equations were constructed and numerically implemented. The results of calculations of natural frequency of vibrations were represented in the form of dependence of homogeneity parameter on the number of lateral ribs for different values of wave formation parameters. Characteristic curves of dependence were constructed.

In [6], free vibrations of a moving fluid-contacting, longitudinally strengthened orthotropic cylindrical shell heterogeneous in thickness, were studied. Using the Hamilton-Ostrogradsky variational principle, the systems of equations of motion were constructed. Heterogeneity of the shell material in thickness was taken into account accepting that the Young modulus and shell material's density are the functions of normal coordinate. Frequency equations were obtained and numerically implemented. During the calculation process, linear and parabolic laws were accepted for the heterogeneity function. Characteristical curves of dependence were constructed.

The review of the works given in the paper shows that vibrations of a stiffened elastic medium contacting cylindrical shell whose materials have a nonhomogeneous property along the thickness in the circular direction and along the generatrix have not yet been studied. Note that inhomogeneous feature of such material has found its solution in the problem of vibrations of liquid contacting cylindrical shell stiffened with rings whose features are close to the homogeneity property [3].

2 Problem statement

The considered problem is solved by means of the Hamilton-Ostrogradsky variational principle. To use the Hamilton-Ostrogradsky variational principle, we write the total energy of the construction under consideration. The studied construction consists of a cylindrical form inhomogeneous shell and stiffening longitudinal ribs the number of which varies. Furthermore, the studied construction is liquid -contacting (Fig. 1a).

To take into account the inhomogencity in the thickness of the cylindrical shell, we will procced from three-dimensional functional. In this case, the functional of the total energy of the cylindrical shell is of the form:

$$V = \frac{1}{2} \iint \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \\ +\rho \left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \Bigg) dxdydz \tag{2.1}$$

There are various ways to take into account the inhomogeneity of the shell material. One of them is that the Young modulus and shell material density are accepted as functions of normal, lateral and longitudinal coordinate [6]. It is supposed that the Poisson ratio is constant. In this case, the stress-strain ratio is of the form:

$$\sigma_{11} = \frac{E\left(x, \theta, z\right)}{1 - \nu^2} (\varepsilon_{11} + \nu\varepsilon_{22}); \sigma_{22} =$$

$$= \frac{E(x,\theta,z)}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11});\sigma_{12} = G(x,\theta,z)\,\varepsilon_{12} \tag{2.2}$$

$$\varepsilon_{11} = \frac{\partial u}{\partial x};\quad \varepsilon_{22} = \frac{\partial \vartheta}{\partial y} + \frac{w}{R};\ \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x}. \tag{2.3}$$

Assume that

$$E(x,\theta,z) = E_0 f_1(z)\, f_2(x)\, f_3(\theta)\,;\rho(z,x) = \rho_0 f_1(z) f_2(x) f_3(\theta)\,. \tag{2.4}$$

Taking into account, (2.4) in (2.2) we have:

$$\sigma_{11} = \frac{E_0}{1-\nu^2}(\varepsilon_{11}+\nu\varepsilon_{22}) f_1(z)\, f_2(x)\, f_3(\theta)\,;$$
$$\sigma_{22} = \frac{E_0}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11}) f_1(z)\, f_2(x)\, f_3(\theta)\,; \tag{2.5}$$
$$\sigma_{12} = G\varepsilon_{12} = \frac{E_0}{2(1+\nu)}\varepsilon_{12} f_1(z)\, f_2(x)\, f_3(\theta)$$

where E_0 is an elasticity modulus homogeneous material of the shell, ρ_0 is density of the material of the homogeneous shell.

Allowing for (2.5), the functional of total energy of the cylindrical shell is of the form [70]:

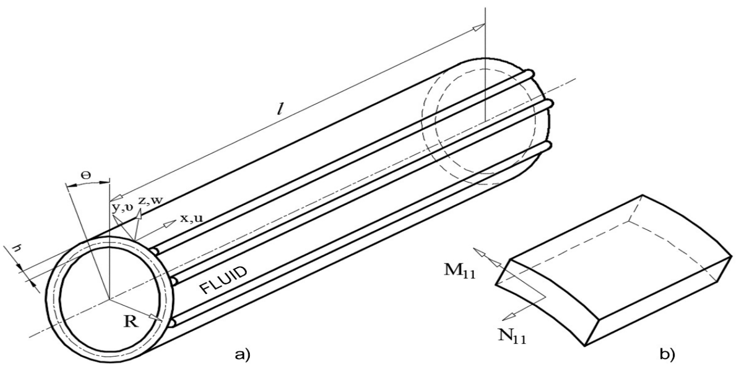

Fig. 1 Longitudinally stiffened inhomogeneous cylindrical shell

$$V = \frac{RE_0}{2(1-\nu^2)}\int_{-h/2}^{h/2} f_1(z)dz \iint \left\{\varepsilon_{11}^2 + 2(1-\nu)\,\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}^2 + \varepsilon_{12}^2\right\} f_2(x) f_3(\theta)\, dx d\theta +$$
$$+\int_{-\frac{h}{2}}^{\frac{h}{2}} f_1(z)dz \iint (\rho_0 \left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right) f_2(x) f_3(\theta) R dx d\theta \tag{2.6}$$

The expression for potential energy of elastic deformation of the i-th longitudinal rib is as follows:

$$\Pi_i = \frac{1}{2}\int_0^l \left[\tilde{E}_i F_i \left(\frac{\partial u_i}{\partial x}\right)^2 + \tilde{E}_i J_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + \right.$$
$$\left. + \tilde{E}_i J_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + \tilde{G}_i J_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right] dx \tag{2.7}$$

Kinetic energy of ribs are written as:

$$K_i = \rho_i F_i \int_0^l \left[\left(\frac{\partial u_i}{\partial t} \right)^2 + \left(\frac{\partial \vartheta_i}{\partial t} \right)^2 + \left(\frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left(\frac{\partial \varphi_{kpi}}{\partial t} \right)^2 \right] dx \tag{2.8}$$

In the expressions (2.7) and (2.8) u_i, ϑ_i, w_i are displacements of the points of rods used in stiffening, F_i are areas of cross section of the i-th rod attached to the shell in the direction of the generatrix, $\tilde{E}_i$ is an elasticity modulus when the i-th rod attached to the cylindrical shell in the direction of the generatrix is stretched, J_{yi}, J_{zi} are inertia moments of the i-th rod with respect to the axis passing through the center of gravity of the cross-section, J_{kpi} are the moments of inertia during torsion of the i- th rod, t- is time, ρ_i is density of the material of the i-th longitudinal rod, $\varphi_i(x), \varphi_{:@i}(x)$are rotation and torsion angles of the cross-section of the rod and through the shell displacements are expressed as follows:

$$\varphi_{kpi}(x) = \varphi_2(x, y_i) = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R} \right) \Big|_{y=y_i} \quad \varphi_i\left(x\right) = \varphi_1\left(x, y_i\right) = -\frac{\partial w}{\partial x} \Big|_{y=y_i}$$

Potential energy of external surface loads acting from the elastic medium and applied to the shell, is determined as the work done by these loads when taking the system from the deformed state to the initial unreformed one and is represented as follows:

$$A_0 = -R \int_0^l \int_0^{2\pi} q_r w dx d\theta \tag{2.9}$$

The total energy of the system equals the sum of the energy of elastic deformations of the shell and all longitudinal ribs and also potential energies of external loads acting as viewed from elastic medium:

$$J = V + \sum_{i=1}^{k_1} (\Pi_i + K_i) + A_0 \tag{2.10}$$

Here k_1 is the amount of longitudinal ribs.

The surface load q_r, acting as viewed from elastic medium on the longitudinally shell is determined by means of the Winkler model:

$$q_r = -\mu w \tag{2.11}$$

where μ is a proportionality factor.

It is assumed that the condition of rigid contact between the shell and rods are satisfied:

$$u_i(x) = u\left(x, y_i\right) + h_i \varphi_1\left(x, y_i\right), \quad \vartheta_i(x) = \vartheta\left(x, y_i\right) + h_i \varphi_2\left(x, y_i\right), \tag{2.12}$$

$$w_i\left(x\right) = w\left(x, y_i\right), \varphi_i(x) = \varphi_1(x, y_i), \varphi_{kpi}(x) = \varphi_2(x, y_i); h_i = 0,5h + H_i^1$$

H_i^1 is the distance of the i- th rod to the surface of the cylindrical shell, h_i is the thickness of the i-th longitudinal rod.

It is assumed that on the lines $x = 0$ and $x = l$ the Navier boundary conditions are fulfilled:

$$\vartheta = 0, w = 0, \ N_{11} = 0, M_{11} = 0 \tag{2.13}$$

Here l is the shell length, T_{11}, M_{11} are forces and moments acting on shell cross-section (Fig. 1b).

The frequency equation of a ribbed inhomogeneous shell with flowing liquid was obtained on the base of Ostrogradsky-Hamilton principle of stationarity of action:

$$\delta W = 0, \tag{2.14}$$

where $W = \int\limits_{t'}^{t''} Jdt-$ is Hamilton's action,t' and t''- are the given arbitrary moments of time.

Supplementing the total energy of the system (2.10) with contact conditions (2.12) and boundary conditions (2.13), we get a problem of natural vibrations of an elastic medium-contacting cylindrical shell stiffened with longitudinal ribs and inhomogeneous in main coordinate directions. In other words, the problem of natural vibrations of an elastic medium-contacting cylindrical shell stiffened with longitudinal ribs and inhomogeneous in principal coordinate directions is reduced to joint integration of the expression for the total energy of the system (2.10), subject to the conditions (2.12) on their contact surface and boundary conditions (2.13).

3 Problem solution

In the expression (2.10) u, ϑ, w are variable values. We approximate these unknown values as follows:

$$\begin{aligned} u &= (u_0 + u_1 x + u_2 R\theta + u_3 z)\cos kx \cos n\theta \sin \omega t \\ \vartheta &= (\vartheta_0 + \vartheta_1 x + \vartheta_2 R\theta + \vartheta_3 z)\sin kx \sin n\theta \sin \omega t \\ w &= (w_0 + w_1 x + w_2 R\theta + w_3 z)\sin kx \cos n\theta \sin \omega t \end{aligned} \tag{3.1}$$

Here $u_i, \vartheta_i, w_i\,(i = 0,1,2,3)-$are unknown constants; $k, n-$are have numbers in longitudinal and peripheral directions, respectively, ω is the desired frequency.

Applying (3.1), we can calculate the work (2.9):

$$A_0 = \frac{\pi\mu hl}{2}\left[w_0^2 + lw_0w_1 + \left(\frac{l^2}{3} - \frac{1}{2k^2}\right)w_1^2 + 2Rw_0w_2 + \right.$$
$$\left. + \pi lRw_1w_2 + \frac{R^2 l}{2}\left(\frac{4\pi^2}{3} + \frac{1}{n}\right)w_2^2 + \frac{h^3}{12}w_3^2\right] \tag{3.2}$$

When simplifying (2.10), the following dependences were accepted [7]:

$$f_1(z) = 1+, \alpha\frac{z}{h}, f_2\,(x) = 1 + \beta\frac{x}{l}.f_3\,(\theta) = 1 + \gamma\frac{\theta}{2\pi R}. \tag{3.3}$$

where α, β, γ-are constant inhomogeneity parameters in the direction along the normal along the generatrix of the shell and peripheral direction, respectively, moreover $\alpha, \beta, \gamma \in [0, 1]$.

Substituting the solution (3.1) in (2.10), taking into account the expression (3.2) for the total energy (2.10), we get a second order polynomial with respect to the constants $u_i, \vartheta_i, w_i\,(i = 0,1,2,3)$:

$$J = t_0 a_1 + \sum_{i=2}^{39} t_i a_i + ?_0?^2\left[\left(u_0^2 + \vartheta_0^2 + w_0^2\right)C_1 + 2\left(u_0u_1 + \vartheta_0\vartheta_1 + \omega_0 w_1\right)C_2 + \right. \tag{3.4}$$

$$+u_1^2 C_3 + \left(\vartheta_1^2 + w_1^2\right)C_4 + 2\left(u_0u_2R + w_0w_2\right)C_5 + 2\vartheta_0\vartheta_2 RC_6 +$$

$$+2\left(u_0u_3 + \vartheta_0\vartheta_3 + w_0w_3\right)C_7 + u_1u_2RC_8 + R\left(\vartheta_1\vartheta_2 + w_1w_2\right)C_9 + u_1u_3C_{10} +$$

$$+\left(\vartheta_1\vartheta_3 + w_1w_3\right)C_{11} + \left(u_2^2 + w_2^2\right)C_{12} + \vartheta_2^2 C_{13} + u_2u_3C_{14} + \left(\vartheta_2\vartheta_3 + w_2w_3\right)C_{15} +$$

$$+\left(u_3^2+\vartheta_3^2+w_3^2\right)C_{16}\big]+\frac{1}{2}\sum_{i=1}^{k_1}\left(E_iF_i+\rho_iF_i\omega^2\right)\frac{k^2l}{2}\left(H_i-\frac{h}{2}\right)\cdot\big[u_0^2+$$

$$+u_0u_1l+u_1^2\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)+2Ru_0u_2\theta_i+u_0u_3\left(H_i+\frac{h}{2}\right)+Ru_1u_2l\theta_i+$$

$$+u_1u_3l\left(H_i+\frac{h}{2}\right)+R^2u_2^2\theta_i^2+Ru_2u_3\theta_i\left(H_i+\frac{h}{2}\right)+\frac{1}{3}u_3^2\left(H_i^2+\frac{H_ih}{2}+\frac{h^2}{4}\right)\bigg]\times$$

$$\times cos^2n\theta_i+\frac{1}{2}\sum_{i=1}^{k_1}\left[E_iJ_{yi}\frac{k^2l}{2}\left(H_i-\frac{h}{2}\right)+\omega^2\rho_iF_i\right]\cdot\big[w_0^2+w_0u_1l+$$

$$+\omega_1^2\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)+2R\omega_0\omega_2\omega_i+\omega_0\omega_3\left(H_i+\frac{h}{2}\right)+R\omega_1\omega_2l\theta_i+$$

$$+\omega_1\omega_3l\left(H_i+\frac{h}{2}\right)+R^2\omega_2^2\omega_i^2+R\omega_2\omega_3\theta_i\left(H_i+\frac{h}{2}\right)+\frac{1}{3}\omega_3^2\left(H_i^2+\frac{H_ih}{2}+\frac{h^2}{4}\right)\bigg]\times$$

$$\times cos^2n\theta_i+\frac{1}{2}\sum_{i=1}^{k_1}\left[E_iJ_{zi}\frac{k^2l}{2}\left(H_i-\frac{h}{2}\right)+\omega^2\rho_iF_i\right]\cdot\big[\vartheta_0^2+\vartheta_0\vartheta_1l+$$

$$+\vartheta_1^2\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)+2R\vartheta_0\vartheta_2\theta_i+\vartheta_0\vartheta_3\left(H_i+\frac{h}{2}\right)+R\vartheta_1\vartheta_2l\theta_i+$$

$$+\vartheta_1\vartheta_3l\left(H_i+\frac{h}{2}\right)+R^2\vartheta_2^2\theta_i^2+R\vartheta_2\vartheta_3\theta_i\left(H_i+\frac{h}{2}\right)+\frac{1}{3}\vartheta_3^2\left(H_i^2+\frac{H_ih}{2}+\frac{h^2}{4}\right)\bigg]\times$$

$$\times sin^2n\theta_i+\frac{1}{2}\sum_{i=1}^{k_1}\left[\frac{l}{2R^2}\left(H_i-\frac{h}{2}\right)G_iJ_{kpi}+\omega^2\rho_iF_i\right]\cdot\Big\{\Big[\left(\vartheta_0-n\omega_0\right)^2+$$

$$+l\left(\vartheta_0-n\omega_0\right)\left(\vartheta_1-n\omega_1\right)+\left(\vartheta_1-n\omega_1\right)^2+2R\left(\vartheta_0-n\omega_0\right)\left(\vartheta_2-n\omega_2\right)\theta_i+$$

$$+\left(\vartheta_0-n\omega_0\right)\left(\vartheta_3-n\omega_3\right)\left(H_i+\frac{h}{2}\right)+Rl\left(\vartheta_1-n\omega_1\right)\left(\vartheta_2-n\omega_2\right)\theta_i+$$

$$+\left(\vartheta_1-n\omega_1\right)\left(\vartheta_3-n\omega_3\right)\frac{l}{2}\left(H_i+\frac{h}{2}\right)+R^2\left(\vartheta_2-n\omega_2\right)\theta_i^2+$$

$$+\frac{R}{2}\left(\vartheta_2-n\omega_2\right)\left(\vartheta_3-n\omega_3\right)\left(H_i+\frac{h}{2}\right)?_i+\frac{1}{3}(\vartheta_3-n\omega_3)^2\left(H_i^2+\frac{H_ih}{2}+\frac{h^2}{4}\right)\bigg]sin^2n\theta_i-$$

$$-\left[\frac{n}{R}\omega_2\left(\vartheta_0-n\omega_0\right)+\frac{l}{2}\left(\vartheta_1-n\omega_1\right)+R\left(\vartheta_2-n\omega_2\right)\theta_i+\frac{1}{2}\left(\vartheta_3-n\omega_3\right)\left(H_i+\frac{h}{2}\right)\right]\times$$

$$\times sin2n\theta_i+n^2\omega_2^2cos^2n\theta_i\big\}+\frac{\pi\mu hl}{2}\left[w_0^2+lw_0w_1+\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)w_1^2+\right.$$

$$\left.+2Rw_0w_2+\pi lRw_1w_2+\frac{R^2l}{2}\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)w_2^2+\frac{h^3}{12}w_3^2\right]$$

$$P_0=ku_1u_0;\quad P_1=ku_1^2;\quad P_2=ku_1u_2;\quad P_3=ku_1u_3;\quad P_4=k^2u_0^2;$$

$$P_5 = 2k^2u_0u_1; \quad P_6 = P_1; \quad P_7 = 2k^2u_0u_2; \quad P_8 = 2k^2u_0u_3; P_9 = 2k^2u_1u_2;$$

$$P_{10} = 2k^2u_1u_3; \quad P_{11} = k^2u_2^2; \quad P_{12} = k^2u_2u_3; \quad P_{13} = u_3^2k^2$$

$$t_0 = u_1^2 + u_2^2; \quad t_{1+i} = -P_i + \frac{1-?}{R}l_i \ (i = 0, 1, 2, 3);$$

$$t_{i+5} = P_{i+4} + \frac{1}{R^2}T_i + l_{4+i}\,(i = 0, 1, 2); \quad t_8 = P_7 + \frac{1}{R^2}T_4 + l_8;$$

$$t_9 = P_8 + \frac{1}{R^2}T_5 + l_7; \quad t_{10} = P_9 + \frac{1}{R^2}T_6 + l_{11}; \quad t_{11} = P_{10} + \frac{1}{R^2}T_7 + l_{12};$$

$$t_{12} = P_{11} + \frac{1}{R^2}T_3 + l_9; \quad t_{13} = P_{12} + \frac{1}{R^2}T_9 + l_{13}; \quad t_{14} = P_{13} + \frac{1}{R^2}T_7 + l_{10};$$

$$t_{15} = S_0R - \frac{1}{2}kRu_0\vartheta_2; \quad t_{16} = S_1R - \frac{1}{2}kRu_1u_2; \quad t_{17} = S_2R - \frac{1}{2}kRu_2\vartheta_2;$$

$$t_{18} = S_3R - \frac{1}{2}kRu_3\vartheta_2; \quad t_{19} = \frac{1}{R^2}\vartheta_2^2; \quad t_{20} = u_0u_2; \quad t_{21} = u_1u_2;$$

$$t_{22} = u_2^2; \quad t_{23} = u_2u_3; \quad t_{24} = \frac{1}{2R}\vartheta_1u_2 + \frac{1-?}{2}u_1\vartheta_2; \quad t_{25} = \vartheta_2^2R;$$

$$t_{26} = n^2u_0^2 + k^2\vartheta_0^2 - 2nku_0\vartheta_0;$$

$$t_{27} = 2n^2u_0u_1 + 2k^2\vartheta_0\vartheta_1 - 2nku_0\vartheta_2 - 2nku_2\vartheta_0;$$

$$t_{28} = 2n^2u_0u_2 + 2k^2\vartheta_0\vartheta_2 - 2nku_0\vartheta_2 - 2nku_2\vartheta_0;$$

$$t_{29} = 2n^2u_0u_3 + 2k^2\vartheta_0\vartheta_3 - 2nku_0\vartheta_3 - 2nk\vartheta_0u_3;$$

$$t_{29+i} = n^2u_i^2 + k^2\vartheta_i^2 - 2nku_i\vartheta_i \quad (i = 1, 2, 3)$$

$$t_{33} = 2n^2u_1u_2 + 2k^2\vartheta_1\vartheta_2 - 2nku_1\vartheta_2 - 2nku_2\vartheta_1$$

$$t_{34} = 2n^2u_1u_3 + 2k^2\vartheta_1\vartheta_3 - 2nku_1\vartheta_3 - 2nku_3\vartheta_1$$

$$t_{35} = n^2u_2u_3 + k^2\vartheta_2\vartheta_3 - 2nku_2\vartheta_3 - 2nku_3\vartheta_2$$

$$t_{36+i} = -nu_i\vartheta_1 + k\vartheta_1\vartheta_i \quad (i = 0, 1, 2, 3)$$

$$S_i = \vartheta_2\,(n\vartheta_i + \omega_i) \quad (i = 0, 1, 2, 3) \quad l_4 = u_1\,(n\vartheta_i + \omega_i) \quad (i = 0, 1, 2, 3)$$

$$l_{4+i} = -nku_0\vartheta_i - nk\vartheta_0\vartheta_i - ku_0\omega_i - k\omega_0u_i \quad (i = 1, 2, 3)$$

$$l_{7+i} = -knu_i\vartheta_i - ku_i\omega_i \quad (i = 1, 2, 3)$$

$$l_{11} = -knu_1\vartheta_2 - nku_2\vartheta_1 - ku_1\omega_2 - ku_2?_1$$

$$l_{12} = -knu_1\vartheta_3 - nku_3\vartheta_1 - ku_1\omega_3 - ku_3?_1$$

$$l_{13} = -knu_2\vartheta_3 - nku_3\vartheta_2 - ku_2\omega_3 - ku_3\omega_2$$

$$T_0 = n^2\vartheta_0^2 + 2n\omega_0\vartheta_0 + \omega_0^2$$

$$T_i = 2n^2\vartheta_0\vartheta_i + 2n\vartheta_0\omega_i + 2n\omega_0\vartheta_i + 2\omega_0\omega_i \quad (i = 1, 2, 3)$$

$$T_{3+i} = n^2\vartheta_i^2 + 2n\omega_i\vartheta_i + \omega_i^2 \quad (i = 1, 2, 3)$$

$$T_7 = 2n^2\vartheta_1\vartheta_2 + 2n\omega_2\vartheta_1 + 2n\omega_1\vartheta_2 + 2\omega_1\omega_2$$

$$T_8 = 2n^2\vartheta_1\vartheta_3 + 2n\omega_3\vartheta_1 + 2n\omega_1\vartheta_3 + 2\omega_1\omega_3$$

$$T_9 = 2n^2\vartheta_2\vartheta_3 + 2n\omega_3\vartheta_2 + 2n\omega_2\vartheta_3 + \omega_2\omega_3$$

$$a_1 = \frac{h\pi l}{2}\left(1 + \frac{\gamma}{2} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right) \quad a_2 = \frac{\pi h}{2n}\left(l + \frac{\gamma l}{2} - \beta - \frac{\beta\gamma}{2}\right)$$

$$a_3 = \frac{R\beta}{2k}\left[\pi^2 + \frac{\gamma l}{2\pi^2}\left(\frac{8\pi^3}{3} - \frac{\pi}{2n^2}\right)\right]$$

$$a_4 = \frac{\pi\alpha\beta h^2}{24k}\left(1 + \frac{\gamma}{2}\right) a_5 = \frac{\pi l h}{2}\left(1 + \frac{\gamma}{2} + \frac{\beta}{2} + \frac{\beta\gamma}{8l}\right)$$

$$a_6 = \frac{\pi h}{2}\left[\frac{l^2}{2} + \frac{\gamma l^3}{4} + \pi h\beta\left(\frac{l^2}{3} + \frac{1}{2k^2}\right) + \frac{\beta\gamma}{2}\left(\frac{l^2}{3} + \frac{1}{2k^2}\right)\right]$$

$$a_7 = \frac{\pi l}{2}\left[\frac{l^2}{3} + \frac{1}{2k^2} + \frac{\gamma h}{2}\left(\frac{l^2}{3} + \frac{1}{2k^2}\right) + \frac{\beta\gamma h}{8}\left(l^2 - \frac{3}{k^2}\right)\right]$$

$$a_8 = \frac{hlR}{2}\left[\pi^2 + \frac{\gamma}{2}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right) + \frac{\beta\pi^2}{2} + \frac{\beta\gamma}{4}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right)\right]$$

$$a_9 = \frac{\pi h^2\alpha}{3}\left(\frac{1}{l} + \frac{\gamma l}{2} + \frac{\beta l}{2} + \frac{\beta\gamma l}{4}\right)$$

$$a_{10} = hR\left[\frac{\pi^2 l^2}{4} + \frac{\gamma h l^2}{8}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right) + \beta h\pi^2\left(\frac{l^2}{6} + \frac{1}{4k^2}\right) + \right.$$
$$\left. + \frac{\beta\gamma h}{2}\left(\frac{l^2}{6} + \frac{1}{4k^2}\right)\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right)\right]$$

$$a_{11} = \alpha h^2\left(\frac{\pi l^2}{6} + \frac{\pi\gamma l^2}{12} - \frac{\beta}{k} - \frac{\pi\beta\gamma}{6k}\right)$$

$$a_{12} = \frac{\pi h l R^2}{2}\left[\frac{8\pi}{3} - \frac{1}{2n^2} + \frac{\beta}{2}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right) + \right.$$
$$\left. + \gamma\left(\pi^2 + \frac{2}{n^2}\right) + \frac{\beta\gamma}{2}\left(\pi^2 + \frac{1}{n^2}\right)\right]$$

$$a_{13} = \frac{Rh^2 l\alpha}{24}\left[\pi^2 + \frac{\gamma}{2}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right) + \frac{\pi^2\beta}{2} + \frac{\beta\gamma}{4}\left(\frac{8\pi^2}{3} - \frac{1}{2n^2}\right)\right]$$

$$a_{14} = \frac{\pi l h^3}{24}\left(1 + \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right) a_{15} = -\frac{\gamma h l}{2}\left(1 + \frac{\beta}{2}\right)$$

$$a_{16} = \frac{\gamma h}{4}\left[l^2 + \beta\left(\frac{l^2}{9} + \frac{1}{k^2}\right)\right] a_{17} = -\pi h l R\left(1 + \frac{\gamma}{2n} + \frac{\beta}{2} + \frac{\beta\gamma}{4n}\right)$$

$$a_{18} = -\frac{\alpha\gamma h^2 l}{24}\left(1 + \frac{\beta}{2}\right)$$

$$a_{19} = \frac{\pi}{2}\left(1 + \frac{\gamma h}{2} + \frac{\beta h}{2} + \frac{\beta\gamma h}{4}\right) a_{20} = -\frac{\gamma h l}{2}\left(1 + \frac{\beta}{2}\right) = a_{15}$$

$$a_{21} = -\gamma h\left[\frac{l^2}{4} - \frac{?}{2}\left(\frac{l^2}{3} - \frac{1}{2k^2}\right)\right] a_{22} = -\pi h R\left(\frac{1}{l} + \frac{\gamma l}{2n} + \frac{\beta l}{2} + \frac{\beta\gamma l}{4n}\right)$$

$$a_{23} = -\frac{\alpha\gamma h^2 l}{24}\left(1 + \frac{\beta}{2}\right) = a_{18}\ a_{24} = -\frac{\beta\gamma h}{2k} a_{25} = \frac{h\pi l}{2}\left(1 + \frac{\pi\gamma}{24} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right) = b_7$$

$$= \frac{h\pi l}{2}\left(1 + \frac{\pi\gamma}{24} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right) = b_7$$

$$a_{26} = \frac{\pi hl}{2R^2}\left(1 + \frac{\pi\gamma}{24} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right) a_{28} = \frac{\pi^2 hl}{2R}\left[1 + \frac{\pi\gamma}{18} + \frac{\beta}{2} + \frac{\beta\gamma}{3}\right]$$

$$a_{27} = \frac{\pi h}{R^2}\left[\frac{l^2}{4} + \frac{\pi\gamma l^2}{96} + \beta\left(\frac{l^2}{6} + \frac{1}{4k2^2}\right) + \frac{\pi\beta\gamma}{2}\left(\frac{l^2}{6} + \frac{1}{4k^2}\right)\right]$$

$$a_{29} = \frac{\pi h^2 l\alpha}{24R^2}\left(1 + \frac{\pi\gamma}{18} + \frac{\beta}{2} + \frac{\beta\gamma}{3}\right) a_{30} =$$

$$= \frac{1}{R^2}\left(\pi h + \frac{\gamma h}{2}\right)\left[\frac{l^3}{6} + \frac{l}{4k^2} + \beta\left(\frac{l^2}{8} + \frac{3l}{8k^2}\right)\right]$$

$$a_{31} = \pi\left(\frac{hl}{2} + \frac{\beta h}{2}\right)\left[\frac{4\pi^2}{3} + \left(\gamma + \frac{\beta\gamma}{l}\right)\left(\pi^2 - \frac{2}{n^2}\right)\right]$$

$$a_{32} = \frac{\pi h^3 l}{24R^2}\left[1 + \frac{\gamma}{2} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right]$$

$$a_{33} = \frac{h\pi^2}{R}\left[\frac{l^2}{4} + \beta\left(\frac{l^2}{6} + \frac{1}{4k^2}\right) + \frac{\gamma l^2}{6} + \frac{2\beta\gamma}{3}\left(\frac{l^2}{6} + \frac{1}{4k^2}\right)\right]$$

$$a_{34} = \frac{\pi h^2}{12R^2}\left[\frac{\alpha l^2}{4} + \alpha\beta\left(\frac{l^2}{6} + \frac{l}{4k^2}\right) + \frac{\alpha\gamma l^2}{8} + \frac{\alpha\beta\gamma}{2}\left(\frac{l^2}{6} + \frac{1}{4k^2}\right)\right]$$

$$a_{35} = \frac{\pi^2\alpha l h^2}{24R}\left[1 + \frac{\beta}{2} + \frac{2\gamma}{3} + \frac{\beta\gamma}{3}\right]$$

$$a_{36} = \frac{\pi\beta h}{2kR^2}\left(1 + \frac{\gamma}{2}\right) a_{37} = \frac{\pi^2\alpha l h^2}{24R^2}\left[l - \beta + \frac{\gamma l}{2} - \frac{\beta\gamma}{2}\right]$$

$$a_{38} = \frac{\pi^2\beta h}{Rk}\left(\frac{1}{2} + \frac{\gamma}{3}\right) a_{39} = \frac{\pi\alpha\beta h^2}{24kR^2}\left(1 + \frac{\gamma}{2}\right)$$

$$c_1 = \frac{\pi hl}{2}\left(1 + \frac{\gamma}{2} + \frac{\beta}{2} + \frac{\beta\gamma}{4}\right)$$

$$c_2 = \frac{\pi h}{2}\left[\frac{l^2}{2} + \frac{\gamma l^2}{4} + \beta\left(\frac{l^2}{3} + \frac{1}{2k^2}\right) + \frac{\gamma l}{2}\left(\frac{l^2}{3} + \frac{1}{2k^2}\right)\right]$$

$$c_3 = \pi h\left[\left(\frac{l^2}{6} + \frac{1}{4k^2}\right) l\left(1 + \frac{?}{2}\right) + \left(\frac{l^2}{8} + \frac{3}{8k^2}\right) l\beta\left(1 + \frac{\gamma}{2}\right)\right]$$

$$c_4 = \pi h\left[\left(\frac{l^2}{6} - \frac{1}{4k^2}\right) l\left(1 + \frac{\gamma}{2}\right) + \left(\frac{l^2}{8} - \frac{3}{8k^2}\right) l\beta\left(1 + \frac{\gamma}{2}\right)\right]$$

$$c_5 = h\left[\frac{\pi^2 l}{2} + \left(\frac{4\pi^2}{3} + \frac{1}{n}\right)\left(\frac{\gamma l}{4} + \frac{\beta\gamma}{4}\right) + \frac{\pi^2\beta}{2}\right]$$

$$c_6 = h\left[\frac{\pi^2 R}{2} + \left(\frac{4\pi^2}{3} + \frac{1}{n}\right)\left(\frac{\gamma l}{4} + \frac{\beta\gamma}{4}\right) + \frac{\pi^2\beta}{2}\right]$$

$$c_7 = \frac{\pi\alpha h^2}{24}\left(1 + \frac{\gamma l}{2} + \frac{\beta l}{2} + 3\beta\gamma l\right)$$

$$c_8 = \pi R\left[\frac{l^2}{2} + \frac{\gamma l^2}{4} + \left(\frac{l^2}{3} + \frac{1}{2k^2}\right)\left(\beta + \frac{\beta\gamma}{2}\right)\right]$$

$$c_9 = \pi R\left[\frac{l^2}{2}+\frac{\gamma l^2}{4}+\left(\frac{l^2}{3}-\frac{1}{2k^2}\right)\left(\beta+\frac{\beta\gamma}{2}\right)\right] c_{10} = \frac{\alpha h^2}{12R}c_8 c_{11} = \frac{\alpha h^2}{12R}c_9$$

$$c_{12} = \frac{\pi R^2}{2}\left[(l+\beta)\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)+(\gamma l+\beta\gamma)\left(\pi^2+\frac{1}{n^2}\right)\right]$$

$$c_{13} = \frac{\pi R^2}{2}\left[(l+\beta)\left(\frac{4\pi^2}{3}-\frac{1}{n}\right)+(\gamma l+\beta\gamma)\left(\pi^2-\frac{1}{n^2}\right)\right]$$

$$c_{14} = \frac{R\alpha h^2 l}{12}\left[\pi^2\left(1+\frac{\beta}{2}\right)+\left(\frac{\gamma}{2}+\frac{\beta\gamma}{4}\right)\left(\frac{4\pi^2}{3}+\frac{1}{n}\right)\right]$$

$$c_{15} = \frac{\alpha h^2 l R}{12}\left[\pi^2\left(1-\frac{\beta}{2}\right)+\left(\frac{\gamma}{2}+\frac{\beta\gamma}{4}\right)\left(\frac{4\pi^2}{3}-\frac{1}{n}\right)\right] c_{16} = \frac{h^2}{12}c_1$$

4 Conclusions

If we vary the expression J by the constants $u_i, \vartheta_i, w_i\,(i = 0,1,2,3)$ and equate the coefficients of independent variations to zero, we get a system of homogeneous algebraic equation. Since the obtained system is a homogeneous system of linear algebraic equations, a necessary and sufficient condition for the existence of its non-zero solution is the equality of its principal determinant to zero. As a result, we obtain a frequency equation. This equation was calculated by the numerical method. The parameters contained in the solution of the problem were accepted as:

$$\rho_0 = \rho_j = 1850kg/m^3, \tilde{E}_i = 6,67\cdot 10^9 N/m^2 m = 1; n = 8;\; h_i = 0,45;$$

$$R = 160\;cm; n = 8; \mu = 10^6 N/m^2; h_i = 0,45\;mm;\; \nu = 0,35;$$

$$\frac{l}{R} = 3, \frac{h}{R} = \frac{1}{6}.\alpha = 0,4;.\,;\; R = 160\;cm;\; F_i = 5,2\;mm^2;$$

$$I_{kp.i} = 0,23\,mm^4;\; I_{yi} = 5,1\,mm^4;\; I_{zi} = 1,3\,mm^4.$$

The results of calculations were given in Fig. 2 in the form of dependence of the frequency parameter $\omega_1 = \sqrt{\frac{\rho_0 R^2\omega^2}{b_{11}}}$ on the amount of longitudinally stiffening rods k_1 on the shell surface, in fig.3. In the form of dependence of the frequency parameter on inhomogeneity parameter in the direction of the shell generatrix β. As can be seen from Fig. 2, increasing the number of longitudinal ribs, the value of the frequency parameter at first increases and then attaining maximum, it decreases. This is explained by the fact that increasing the number of longitudinal ribs at first rigidity of the construction increases. Further, as the inhomogeneity parameter increases in the direction of the shell generatrix β, as can be seen from Fig. 3, the value of the frequency parameter increases. In both figures the dotted curves correspond to displacement approximated by the expression (3.1).

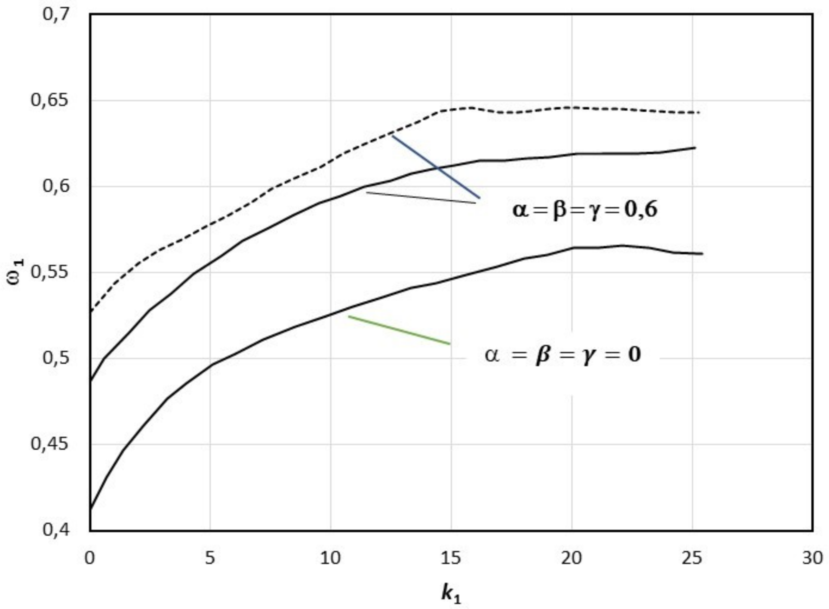

Fig. 2. Dependence of the frequency parameter on k_1.

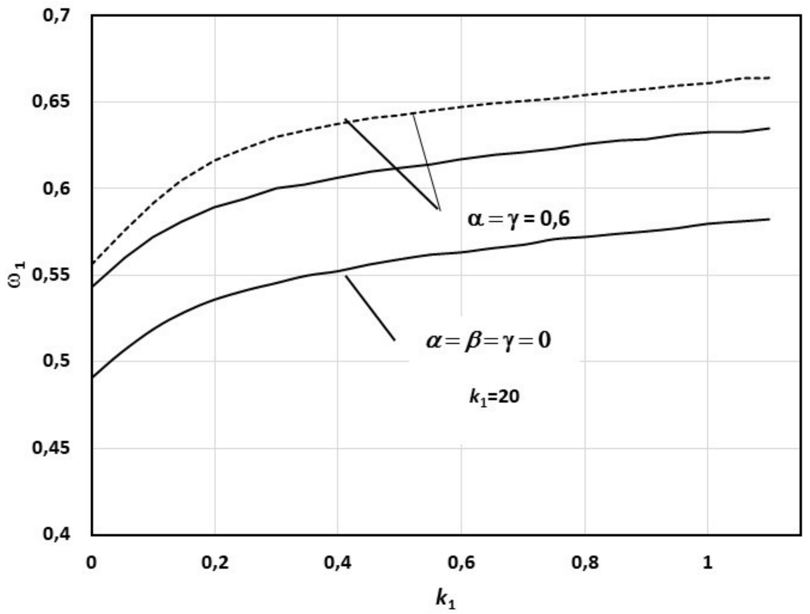

Fig. 3. Dependence of the frequency parameter on b.

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