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Cyclic shock wave in soil and determination of changes in stresses and displacements

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Abstract. *The phenomena of shock wave propagation in soil media are quite common in geoengineering practice. The shock wave, acting on soft soil, causes large stresses in it and displacement of particles of the medium. After passing through a media, the shock wave changes the structure and properties of the soil. Therefore, the determination of stresses and displacements behind the shock wave front can suggest new parameters of the deformed soil. Also in this article, the propagation of several identical shock waves in coherently changing soil is considered.*

Keywords. shock wave · soil · stresses · cyclicity

Mathematics Subject Classification (2010): 74J40

1 Introduction

The study of shock wave propagation in media remains an important aspect of modern science and technology. When a shock wave propagates in solid media like soils or rocks under high pressure, the rock layers are compressed (change their structure), compressed and, turning into a plastic state, move in the radial direction from the center of shock wave propagation [7]. As a result, depending on the magnitude of pressure, soil porosity, the following may occur: a displacement zone, a zone of compacted rock, and at relatively small pressures or when moving away from the center of the shock wave, where the compressive stresses become less than the ultimate strength of the material, but the displacements of the particles of the medium are still large, a zone of rupture of the medium appears. During the propagation of shock disturbances in the soil mass, many characteristic regions arise that differ from each other in the deformation mechanisms implemented in this process. Here

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one can single out the area of super-heavy loads 1, with characteristic pressures of $\sim 10^{6}$ GPa. Evaporation, melting, thermal decomposition, etc. can occur in this zone [10].

As the wave propagates with a decrease in stresses to values of the order of $\sim$100 GPa, phase transformations occur in minerals - region 2. This region is followed by region 3, the movement of the soil, in which it is accompanied by its destruction of the initial structure and with intense all-round compression. Pressures of the order of $\sim$10 GPa - 10^{-5} GPa are typical in this region. Further, with a decrease in amplitude, the relative short-term compressive stress gives way to a longer exposure time to tensile stress, which destroys the rock, leads to cracking - region 3. At stresses less than 10^{-5} GPa, the soil practically does not collapse - region 4. With further propagation, the shock wave degenerates into the acoustic - area 5. When the shock wave moves along the chain from the area to the area: 1-2-3-4-5, the energy of the wave decreases and the mass of the soil covered by the wave increases [10].

In some sufficiently small region of the medium, one-dimensional shock waves propagating along the x axis as a plane front can be considered. The pressure P(t) acts on the boundary. When a certain critical value of pressure *P** for a given soil is reached, the ratio of pressure P(t) and deformation can be expressed in a linear form (P(t)= *k*, *k* - factor, slope angle tangent between the values of the parameters P and). According to the simplified soil model proposed by A.Yu. Illinsky [4], A.P. Sinitsin obtained an equation for determining the law of propagation of the front of a shock wave *x = S(t)* propagating in undisturbed soil without taking into account elastic perturbations [9].

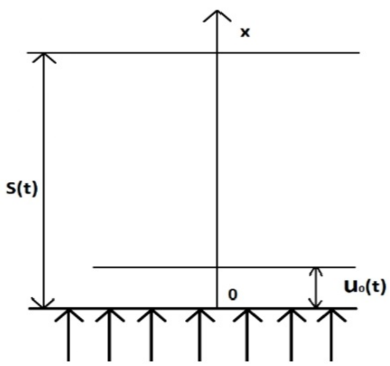


Fig. 1. Propagation of the front of one-dimensional waves in soft ground

In some sufficiently small region of the medium, one-dimensional shock waves propagating along the x axis as a plane front can be considered. The pressure $P(t)$ acts on the boundary. When a certain critical value of pressure P* for a given soil is reached, the ratio of pressure $P(t)$ and deformation ε can be expressed in a linear form $(P(t) = k\varepsilon$, k - factor, slope angle tangent between the values of the parameters P and $\varepsilon)$. According to the simplified soil model proposed by A.Yu. Illinsky [3], A.P. Sinitsin obtained an equation for determining the law of propagation of the front of a shock wave $x = S(t)$ propagating in undisturbed soil without taking into account elastic perturbations (4)

$$P(t) = \rho_0 \varepsilon_* \frac{d(S\dot{S})}{dt}$$

The solution to this equation has the form [9]:

$$S\left(t\right) = \sqrt{\frac{2}{\rho_0 \varepsilon_*} \int_0^t dt \int_0^t P(t)dt}; \dot{S} = \frac{dS}{dt} = \frac{2}{\rho_0 \varepsilon_* S(t)} \int_0^t P(t)dt$$

Considering the case, when $P > P*$, and the soil with the initial density ρ_0 and the velocity of propagation of elastic waves α_0 is pressed to a new state with density ρ, where [8]:

$$P = -\sigma_{xx} = \rho_1 {\alpha_1}^2 \left(\varepsilon - \theta_*\right), \ \varepsilon = \frac{\partial u}{\partial x}, \ \alpha_1 = \sqrt{\frac{P_*}{\rho_1 \left(\varepsilon_* - \theta_*\right)}}, \ \varepsilon_* = \frac{\rho_1 - \rho_0}{\rho_1} \quad (1.1)$$

Here θ_*- the value of the deformation at the intersection of the straight-line $P(t) = k\varepsilon$ with the axis 0ε, $\varepsilon_* > \theta_*$. ρ_1-soil density at $\varepsilon = {}_*$. α_1- the speed of propagation of elastic waves in the pressed soil. u- displacement of the shock wave front. $P*$ and ${}_*$- critical values of pressure and deformation for a given soil.

2 Stresses and displacements behind the front of the shock wave

Definitions of the shock wave front propagation law $x = S(t)$, for $P > P*, \ 1/k \neq 0$is found in the form [1, 8]:

$$u = u_1 + u_2 = f\left(t - \frac{x}{2}\right) + \varphi(t + \frac{x}{2}) \quad (2.1)$$

where is the displacement $u_1 = \varepsilon_* \left[S\left(t\right) - x\right]$ of the soil pressed behind the front, $u_2(x, t)$ are the displacements associated with the propagation of elastic perturbations.
The pressure on the moving boundary is represented as [8]:

$$P\left(t\right) = P_1\left(t\right) + P_2\left(t\right) = -\rho\theta_* {\alpha_1}^2 + \rho\alpha_1 \left[f^{'\left(t - \frac{S}{2}\right) - \varphi^{'\left(t + \frac{S}{2}\right)}}\right], \ x = u_0 = \varepsilon_* S\left(t\right). \quad (2.2)$$

Taking into account the law of conservation of masses and the law of conservation of momentum at the front, the parameters are determined $f^{'(t);\varphi^{'(t)}}$.

$$f^{'(t)=\frac{1}{2}\rho_1\theta_*{\alpha_1}^2 \frac{\alpha_1 + \dot{S}(t+S/\alpha_1)}{\rho_1 {\alpha_1}^2 - \rho_0 \dot{S}^2(t-S/\alpha_1)};\varphi^{'(t)=-\frac{1}{2}\rho_1\theta_*{\alpha_1}^2 \frac{\alpha_1 - \dot{S}(t-S/\alpha_1)}{\rho_1 {\alpha_1}^2 - \rho_0 \dot{S}^2(t-S/\alpha_1)}}}$$

Taking the indicated expression (**??**) as a boundary condition, we obtain an approximate differential equation for determining the law $x = S(t)$ of the form

$$P\left(t\right) = \rho_0\theta_* \frac{d(S\dot{S})}{dt} + \rho_0\theta_* \frac{1 - \varepsilon_*}{{\alpha_1}^2} \left[\left(1 - \varepsilon_*\right)\left(\ddot{S}^2 S^2 + \dddot{S}\dot{S}S^2\right) + 3\ddot{S}\dot{S}^2 S + \dot{S}^4\right] = \left[P\left(t\right)\right]_0 \quad (2.3)$$

The solution of equation (2.3) under initial conditions $\dddot{S}\left(0\right); \ddot{S}\left(0\right); \dot{S}\left(0\right); S\left(0\right) = 0$ gives the law of front propagation $x = S(t)$, knowing which one can use formulas (1.1), (2.1) $f^{'(t);\varphi^{'(t)}}$ to find the distribution of displacements and stresses behind the front of the shock wave.

3 The propagation of the second shock wave shortly after the first

Let us assume that after some time Δt a second shock wave passes through the same soil. In view of the fact that the first shock wave did the work of compacting the porous soil, it can be said that the second wave will move through a new, more compacted medium with density ρ_1 and the pressure $P**$.

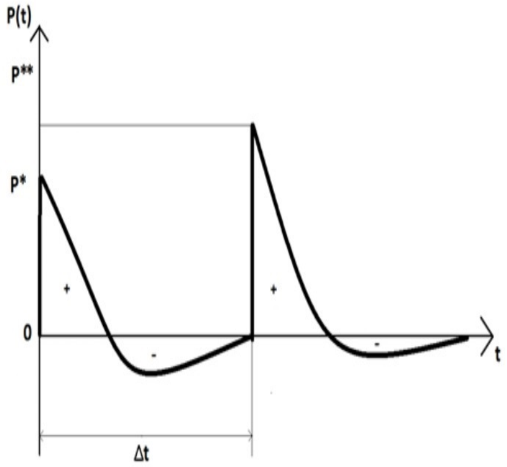


Fig. 2 Shock wave pressure diagram

The unloading area impacting the soil may result in slight loosening or cracking as most soils unable to withstand tensile stresses. In general, this effect depends on the specific type of soil. Also considering that the intensity of the second shock is high enough to neglect its interaction with the elastic waves from the first shock, which it will overtake with sufficient time Δt.

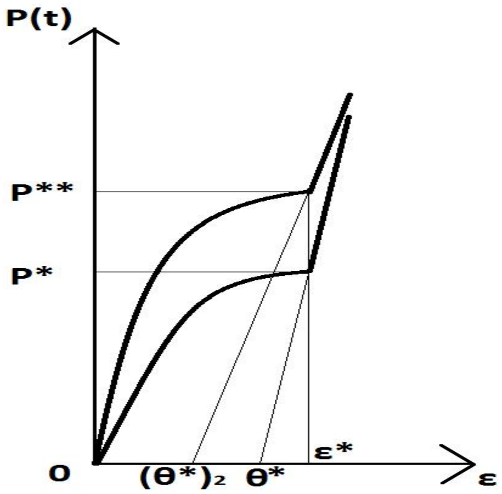


Fig. 3 Diagram of compression during the propagation of two shock waves

Summarizing all the above, relations (1.1) will be rewritten in the form:

$$P_2 = -(\sigma_{xx})_2 = \rho_2 {\alpha_2}^2 \left((\varepsilon)_2 - (\theta_*)_2\right),\ (\varepsilon)_2 = \frac{\partial u_2}{\partial x},$$

$$\alpha_2 = \sqrt{\frac{P^{**}}{\rho_2 \left((\varepsilon_*)_2 - (\theta_*)_2\right)}}, (\varepsilon_*)_2 = \frac{\rho_2 - \rho_1}{\rho_2} \tag{3.1}$$

Here, keeping the notation mentioned above, the following inequality is true: $\rho_2 > \rho_1$; Thus, by analogous reasoning, we can rewrite (2.3) in new variables for the second shock wave:

$$P\left(t\right)=\rho_1(\theta_*)_2\frac{d((S)_2(\dot{S})_2)}{dt}+\rho_1(\theta_*)_2\frac{1-(\varepsilon_*)_2}{{\alpha_2}^2}[(1-(\varepsilon_*)_2)\left((\ddot{S}^2)_2(S^2)_2+(\dddot{S})_2(\dot{S})_2{(S)_2}^2\right)+$$

$$+3(\ddot{S})_2{(\dot{S})_2}^2(S)_2+{(\dot{S})_2}^4]=[P\left(t\right)]_1 \tag{3.2}$$

In a similar way, we can continue the reasoning for subsequent $[P\left(t\right)]_N$ families of curves for each following shock wave. However, if the time between each new wave Δt is short enough that the relaxation of the soil can be freely neglected, then it can be said that the shock waves will very quickly compact the soil to the maximum density values for a given material $\rho_{max}=const$. Therefore, further the problem can be considered as the propagation of a shock wave in a continuous medium with the same parameters. Conversely, at large time period Δt third-party forces that can change the soil to the initial values ρ_0. And even further in geological time scales, obviously, the processes are completely unpredictable.

Thus, it turns out that shock waves can propagate in the soil in the interval between the maximum $P_{\max}$ and the minimum P_{min} - pressure values determined by the initial and maximum values of the densities, respectively.

Possible peak values of acting pressures (Fig. 3) can be estimated using the least squares method, for example, as follows:

$$\begin{cases} \dfrac{\partial\left(\sum\limits_{i,j=1}^{n}([P(t)]_i-a_jt_i^j-b)^2\right)}{\partial a_j}=0 \\ \dfrac{\partial\left(\sum\limits_{i,j=1}^{n}([P(t)]_i-a_jt_i^j-b)^2\right)}{\partial b}=0 \end{cases}$$

$$\begin{cases} bn+\sum\limits_{j=1}^{n}(a_j\sum\limits_{i=1}^{n}t_i^j)=\sum\limits_{i=1}^{n}[P\left(t\right)]_i \\ b\sum\limits_{i=1}^{n}t_i+\sum\limits_{j=1}^{n}(a_j\sum\limits_{i=1}^{n}t_i^{j+1})=\sum\limits_{i=1}^{n}t_i[P\left(t\right)]_i \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ b\sum\limits_{i,j=1}^{n}t_i^j+\sum\limits_{j=1}^{n}(a_j\sum\limits_{i=1}^{n}t_i^{2j})=\sum\limits_{i,j=1}^{n}t_i^j[P\left(t\right)]_i \end{cases}$$

$$|A|=\begin{vmatrix} \sum\limits_{i,j=1}^{n}t_i^{2j} & \dots & \sum\limits_{i,j=1}^{n}t_i^j \\ \dots & \dots & \dots \\ \sum\limits_{i,j=1}^{n}t_i^j & \dots & \sum\limits_{i=1}^{n}t_i^{j+1} \\ \sum\limits_{i=1}^{n}t_i & \dots & n \end{vmatrix}$$

$$|A|^{-1}*\begin{vmatrix} \sum\limits_{i,j=1}^{n}t_i^j[P\left(t\right)]_i \\ \dots \\ \sum\limits_{i=1}^{n}t_i[P\left(t\right)]_i \\ \sum\limits_{i=1}^{n}[P\left(t\right)]_i \end{vmatrix}=|B|$$

$$det\,|B| \neq 0;\; a_j = \frac{det_{a_j}\,|B|}{det\,|B|};\, b = \frac{det_b\,|B|}{det\,|B|}$$

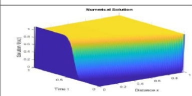
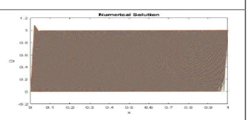
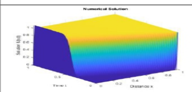
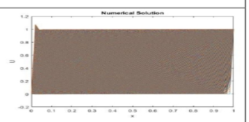
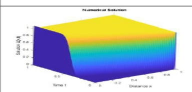
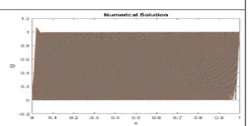
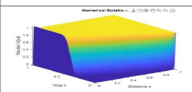
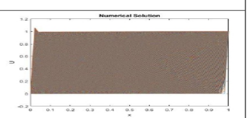
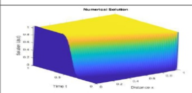
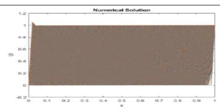
$$\widetilde{P\,(t)} = a_j \sum_{i,j=1}^{n} t_i^j + b \tag{3.3}$$

Where n is the number of shock waves, a_j and b are the approximation coefficients.

Table 1. Numerical estimates of displacements for dolomite and sandstone

	Initial density ρ_0, kg / m 3	**Velocity of propagation of elastic longitudinal waves $\propto_0$, m / s**	**Critical strain value ε_***	**The value of the deformation at the intersection of the straight line P(t)=k with the axis 0 θ_***
Dolomite	2900 [6;7]	4450 [8]	0.09091 [8]	0.063 [3]
Sandstone	1800 [9;7]	3800 [8]	0.04761 [8]	0.033 [3]

Let's carry out numerical estimates for materials: dolomite and sandstone in a continuous form. Computing, we obtain a solution for the displacement parameter $u(x,t)$. Consider five compression shock waves. Substituting the appropriate initial values indicated in Table 1, using the finite difference method, we obtain the following graphs:

	Dolomite				
wave	**Density ρ_0 before the wave passes, g/sm ^{3}**	**Density ρ after the wave passes, g/sm ^{3}**	**Maximum pressure value P, MPa**		
1	2.9	3.19	1722.817 5		
2	3.19	3.509	1895.0992		
3	3.509	3.8599	2084.6091		
4	3.8599	4.24589	2293.0700		
5	4.24589	4.670479	2522.3771		

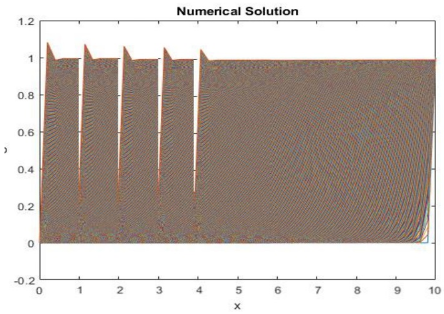
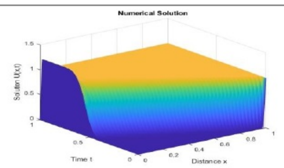
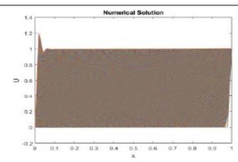
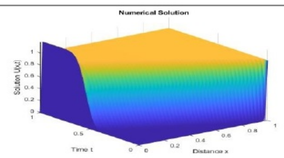
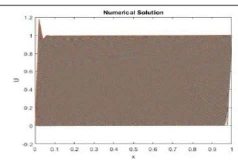
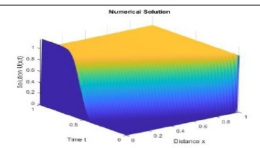
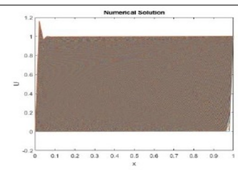
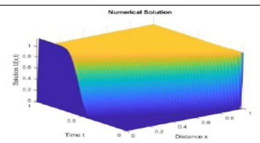
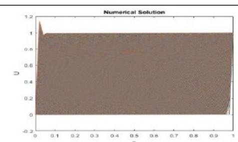
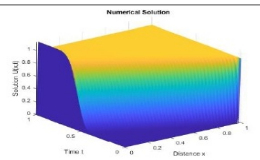
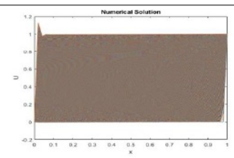


Fig. 4. Diagram of the displacement parameter $U(x,t)$ along the x coordinate for dolomite

	Sandstone				
wave	**Density ρ_0 before the wave passes, g/sm ^{3}**	**Density ρ after the wave passes, g/sm ^{3}**	**Maximum pressure value P, MPa**		
1	1.8	1.89	389.88		
2	1.89	1.9845	409.374		
3	1.9845	2.08372	429.842 7		
4	2.08372	2.18791	451.334 835		
5	2.18791	2.29730	473.901 576		

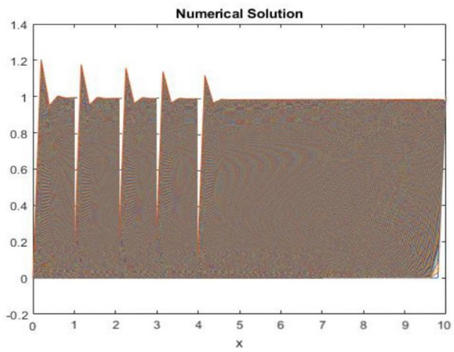
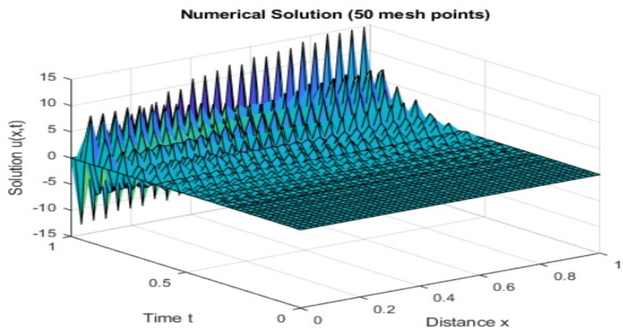


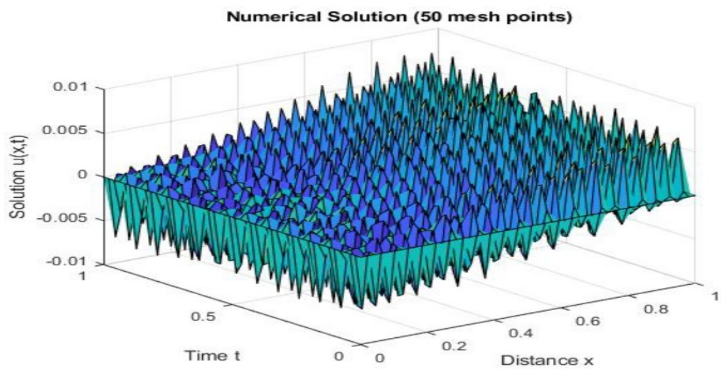
Fig. 5. Diagram of the displacement parameter $U(x,t)$ along the x coordinate for sandstone

Here we see that the decrease in the peak values of the displacement parameter $U(x,t)$ along the X axis is less noticeable for dolomite, but somewhat better for sandstone. This probably happens because in a completely filled medium, particles receive momentum from the shock wave and tend to continue moving after it.

Using the PDE method, for a material sample with parameters 1) $\alpha = 3.406329$; $\rho_0 = 2500$; $\varepsilon = 0.307$; $\theta = 0.0331$ and 2) $\alpha = 3.8$; $\rho_0 = 1800 * 10^6$; $\varepsilon = 0.009$; $\theta = 0.00328$ the following graphs were obtained for the displacement parameter $U(x,t)$. These graphs are remarkable in that the displacement parameter increases somewhat in time and spatial coordinate. In this case, self-action of the wave occurs, like the effect of the resonance.



a)



b)

Fig. 6. Graphical solution of the displacement parameter $U(x,t)$ of materials with parameters 1 and 2, a) and b) respectively

4 Conclusion

The following conclusions can be made: the distribution of displacements and stresses behind the shock wave front are determined through the system of expressions (1.1),(2.1),(2.3) taking into account the initial conditions and relations $f^{'}(t)\,;\varphi^{'}(t)$. Several shock waves propagate in the range of densities ρ_0- ρ_{max} at which the shock wave acts on the ground with peak pressures in the interval P_{min}- P_{max}, where the displacement parameter decreases within U_{max}- U_{min}, respectively.

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