Tran. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.
Mechanics, **42** (7), 32-43 (2022)

Torsional stability of a shell

Ismail R. Sadigov

Received: 02.12.2021 / Revised: 12.03.2022 / Accepted: 07.04.2022

Abstract. *In the paper, the studies on torsional stability of shells are considered and a great attention is paid to circular cylindrical shells. The shells of such a shape usually meet the requirements of the least weight of the structure and ease of manufacture, therefore they are used in various fields of engineering. Cylindrical shells are included in the design of aircraft and submarine engines, reservoirs, pipelines.*

Keywords. buckling · axial compression · cylindrical constructions · aircraft · stability · bending and torsion · linear theory · transversal pressure.

Mathematics Subject Classification (2010): 74K25

1 Introduction

Let us consider torsion of a shell by the pairs M_k, applied at the ends (Fig. 1). The main state of the shell is determined by the tangential stresses s. For a thin shell they can be considered equal.

Ismail R. Sadigov
Azerbaijan University of Architecture and Construction, Baku, Azerbaijan
E-mail: i.sadigov1@mail.ru

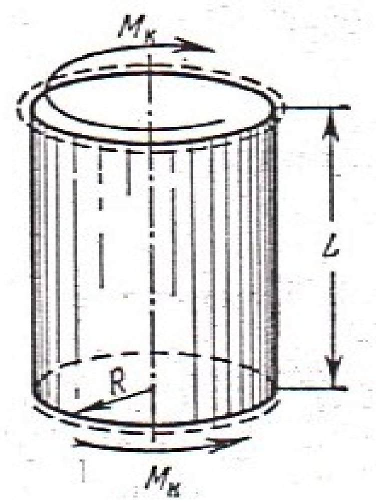


Fig. 1 Torsion of a shell by the pairs M_k, applied at the ends

$$s = \frac{M_k}{2\pi R^2 h}, \tag{1.1}$$

where h is the thickness of the shell.

Buckling under the action of such a load can take place under certain conditions for the shells of aircrafts and engines. At first, we consider the linear problem [3, 6, 7], considering that the medium length shell is hingely supported at the ends. The equation takes the form:

$$\frac{D}{h}\nabla^8 w + \frac{E}{R^2}\frac{\partial^4 w}{\partial x^4} + 2s\nabla^4\left(\frac{\partial^2 w}{\partial x\,\partial y}\right) = 0 \tag{1.2}$$

By analogy with the problem of plate stability under shear we can assume that the buckling of the shell in the case of torsion will be accompanied by the formation of waves regularly arranged around the circumference and inclined at a known angle to the generatrix. Therefore, for w we accept the expression [4, 5]

$$w = f\,cos\frac{\pi x}{L}\,cos\frac{n}{R}\,\left(y + \gamma x\right), \tag{1.3}$$

taking into account that reference point x is located in the middle of the length of the shell; under n we understand the number of complete waves around the circumference; under γ the tangent of the slope angle of the wave crests to the generatrix. The expression (1.3) satisfies the condition $w = 0$ for $x = \pm L/2$. Further we find

$$\begin{cases} \left.\frac{\partial w}{\partial x}\right|_{x=\pm\frac{L}{2}} = -f\frac{\pi}{L}\,cos\frac{n}{R}\left(y \pm \gamma\frac{L}{2}\right), \\ \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=\pm\frac{L}{2}} = f\frac{2\pi n\gamma}{LR}\,sin\frac{n}{R}\left(y \pm \gamma\frac{L}{2}\right). \end{cases} \tag{1.4}$$

As can be seen, the expression chosen by us for the flexure does not satisfy neither hinge support condition, nor the built-in condition. As the same time we get

$$\int_0^{2\pi R}\left(\frac{\partial w}{\partial x}\right)_{x=\pm\frac{L}{2}} dy = \int_0^{2\pi R}\left(\frac{\partial^2 w}{\partial x^2}\right)_{x=\pm\frac{L}{2}} dy = 0,$$

Thus, both of these conditions are satisfied simultaneously in the integral sense. We can rewrite the expression (1.4) in the form $w = w_1 + w_2$, where

$$w_1 = \frac{1}{2}f\left(cos\frac{ny}{R}\cos mx - sin\frac{ny}{R}\sin mx\right), \tag{1.5}$$

$$w_2 = \frac{1}{2} f \left(cos \frac{ny}{R} \cos lx - sin \frac{ny}{R} \sin lx \right). \tag{1.6}$$

moreover

$$m = \frac{n\gamma}{R} + \frac{\pi}{L}, \qquad l = \frac{n\gamma}{R} - \frac{\pi}{L} \tag{1.7}$$

Substituting $w_1 + w_2$ in equation (1.2), we get:

$$\begin{aligned} &\left[\left(m^2 + \tfrac{n^2}{R^2} \right)^4 \tfrac{D}{h} + \tfrac{E}{R^2} m^4 - 2s \tfrac{mn}{R} \left(m^2 + \tfrac{n^2}{R^2} \right)^2 \right] w_1 + \\ &+ \left[\left(l^2 + \tfrac{n^2}{R^2} \right)^4 \tfrac{D}{h} + \tfrac{El^4}{R^2} - 2s \tfrac{ln}{R} \left(l^2 + \tfrac{n^2}{R^2} \right)^2 \right] w_2 = 0 \end{aligned} \tag{1.8}$$

Equating to zero the coefficients w_1 and w_2, we obtain two equations:

$$\frac{2s}{E} = \frac{\left(m^2 R^2 + n^2 \right)^2}{12 \left(1 - \mu^2 \right) mRn} \left(\frac{h}{R} \right)^2 + \frac{m^3 \, R^3}{(m^2 R^2 + n^2)^2 n}, \tag{1.9}$$

$$\frac{2s}{E} = \frac{\left(l^2 R^2 + n^2 \right)^2}{12 \left(1 - \mu^2 \right) lRn} \left(\frac{h}{R} \right)^2 + \frac{l^3 \, R^3}{(l^2 R^2 + n^2)^2 n}. \tag{1.10}$$

The equations (1.9) and (1.11) should give one and the same value of s. Given the number of waves and the ratio R/L, by these equations we can construct the curves $s = s\,(n\gamma)$, as was shown in Fig. (1.2):

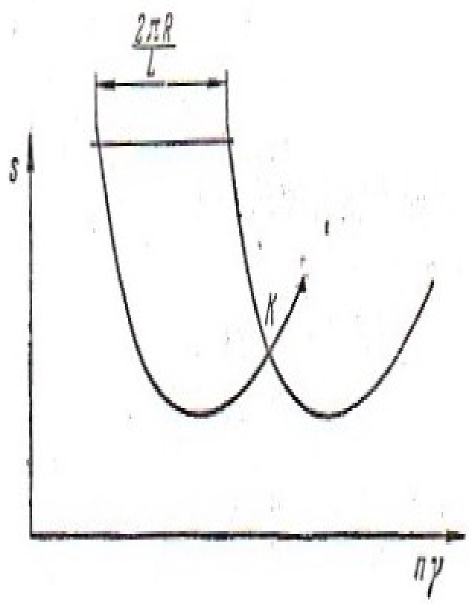


Fig. 2 Tangential stresses on $n\ \gamma$ dependence

These curves are identical in shape (as the quantities m and l in (1.9) and (1.11) are interchangeable), but they are displaced one relative to the other along the axis by the value

$$(m - l)\, R = \frac{2\pi R}{L} \tag{1.11}$$

The ordinate of the point K of intersection of the curves gives the critical value of s under the accepted conditions.

Given various values of n, we look for minimum value of s, determining the upper critical stress s_B. Note that it suffices to construct one of the curves according to (1.9) or (1.11) and write out in it a chord of length $c = 2\pi R/L$ parallel to the abscissa axis. If the curve is constructed according to (1.9), then the point K will be on the right end of the chord. Introduce the denotations:

$$\left.\begin{array}{c} \nu = n\gamma + \frac{\pi R}{L}, \\ \rho^2 = \frac{1}{12(1-\mu^2)} \bullet \left(\frac{h}{R}\right)^2; \end{array}\right\} \tag{1.12}$$

then equation (1.9) takes the form

$$2\frac{s}{E} = \frac{\rho^2\left(\nu^2 + n^2\right)^4 + \nu^4}{\nu n \left(\nu^2 + n^2\right)^2} \tag{1.13}$$

Ignoring ν^2 compared to n^2, we get:

$$2\frac{s}{E} = \frac{n^8\rho^2 + \nu^4}{\nu n^5} \tag{1.14}$$

Taking into account that the values of ν and $\nu + \frac{2\pi R}{L}$ correspond to the same quantities of s, we find

$$\frac{n^8\rho^2 + \nu^4}{\nu n^5} = \frac{n^8\rho^2 + \left(\nu + \frac{2\pi R}{L}\right)^4}{\left(\nu + \frac{2\pi R}{L}\right) n^5} \tag{1.15}$$

Hence, for the given ratio L/R it is necessary to determine n. Solving the equation (1.15), we find [3]

$$n = 4,2\sqrt[8]{1-\mu^2}\,\sqrt{\frac{R}{L}}\,\sqrt[4]{\frac{R}{h}} \tag{1.16}$$

Using (1.16), we determine the upper critical stress:

$$s_B = 0,74\frac{E}{\left(1-\mu^2\right)^{5/8}}\,\frac{h}{R}\,\sqrt{\frac{Rh}{L^2}} \tag{1.17}$$

or, for $\mu = 0,3$,

$$s_B \approx 0,78E\frac{h}{R}\,\sqrt[4]{\frac{Rh}{L^2}}. \tag{1.18}$$

We introduce pure parameters

$$\hat{s} = \frac{s}{E}\,\frac{R}{h}, \quad \vartheta = \frac{\pi R}{nL}, \quad \delta = \frac{Rh}{L^2}; \tag{1.19}$$

and get

$$\hat{s}_B = 0,78\sqrt[4]{\delta}. \tag{1.20}$$

This tine the value of $\hat{s}$ will be

$$\vartheta = \frac{\pi}{4,2\,\sqrt[8]{1-\mu^2}}\,\sqrt[4]{\delta} \approx 0,75\sqrt[4]{\delta},\ \gamma \approx 1,73\sqrt[4]{\delta}. \tag{1.21}$$

According to (1.16) and (1.21) the number of waves formed along the arch drops as the relative thickness h/R increases.

For shells of great length, the wave number becomes equal to $n = 2$, since the cross-section takes the form of an oval. If we proceed from more general equations such as (**??**), we arrive at the following formula for the critical stress [1]:

$$s_B = \frac{1}{3\sqrt{2}} \frac{E}{(1-\mu^2)^{0,75}} \left(\frac{h}{R}\right)^{1,5}; \tag{1.22}$$

and considering $\mu = 0,3$, we get:

$$\hat{s}_B = 0,254 \sqrt{\frac{h}{R}}. \tag{1.23}$$

For very long shells s_B does not depend on the ratio L/R: Here the boundary conditions on the ends become irrelevant.

In Fig. 3 the calculated data were combined for shells of different length [2] at various ratio R/h. Here the ratio L/R is plotted along the abscissa axis, along the ordinate the parameter $s_B^* = sR^2/Eh^2$; logarithmic scale was applied. According to formulas (1.18) and (1.23), in the case of medium length shell in such a graph we should obtain oblique straight lines and for a very long shell a horizontal line. Strictly speaking the oblique straight lines in the lefthand side of the diagram are the envelopes of the curves corresponding to different number of waves n; these numbers are given on the graph. The graphs for considerable $n = 2$ pass to torsional lines L/R turn into horizontal lines. From the graph we can be convinced that the greatest ratio L/R, where the number of waves $n \geq 4$, corresponds approximately to the value $\sqrt{R/h}$; this agrees with the inequality (18 b) which establishes the limits of applicability of the theory of shells of medium length. The quantity γ, equal to the tangent of the slope of the wave crest to the generatrix.

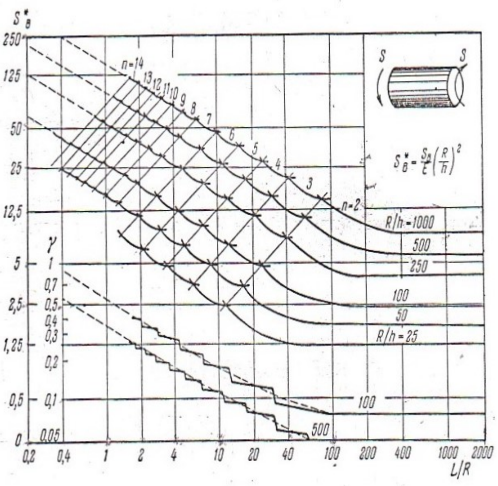


Fig. 3 Calculated data combined for shells of different length at various ratio R/h

The study of the influence of the built-in of edges on the critical stress [2] showed as expected, that it is significant only for relatively short shells. As experiments show, the real shells swell during torsion approximately in the same way as in the case of external pressure, but the dents are located at a certain angle to the generatrix [8] (Fig. 4). Appearance of dents in shells of medium length is usually accompanied by a clap. Therefore, here we must also turn to the solution of the nonlinear problem of stability in large [9].

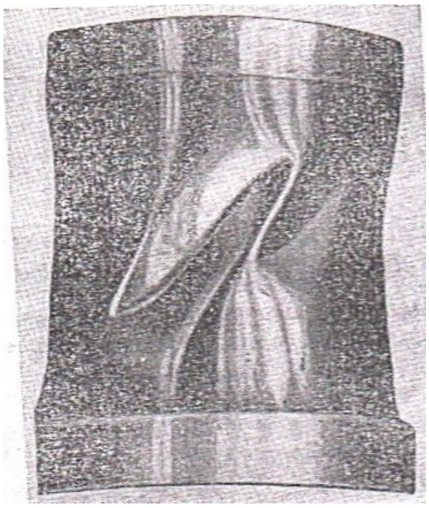


Fig. 4 Swelling of real shell under torsion

We approximate flexure by means of the expression [12, 14]

$$w = f_1 sin\frac{\pi x}{L}\, sin\frac{n}{R}(y-\gamma x) + f_2 sin^2\frac{\pi x}{L} \tag{1.24}$$

The first term is similar to the expression (1.3), related to the linear problem. We second term was selected in the same form as in the cases of axial compression and external pressure. After substitution of (1.24) to the second one of equations (1.4) and integration, we find:

$$\frac{\phi}{E} = \frac{\theta^2}{32}\left[\frac{1}{(1+\gamma^2)^2}\, cos\frac{2n\,(y-\gamma x}{R} + \frac{1}{\theta^4} cos\frac{2\pi x}{L}\right] f_1^2 -$$
$$-\frac{\theta^2}{2}\left[\frac{1}{\left(1+a_1^2\right)^2} cos \times cos\frac{n\,(y-a_1 x)}{R} - \frac{1}{\left(1+b_1^2\right)^2}\times\right.$$
$$\times cos\frac{n\,(y-b_1 x)}{R} - \frac{1}{\left(1+a_3^2\right)^2}\, cos\frac{n\,(y-a_3 x)}{R} +$$
$$\left. + \frac{1}{\left(1+b_3^2\right)^2}\, cos\frac{n\ (y-b_x x)}{R}\right]\ f_1 f_2 +$$
$$\frac{R}{2n^2}\left[\frac{a_1^2}{\left(1+a_1^2\right)^2}\, cos\frac{n\,(y-a_1 x)}{R} - \frac{b_1^2}{\left(1+b_1^2\right)^2}\times\right.$$
$$\left.\times cos\ \frac{n\,(y-b_1 x)}{R}\right] f_1 - \frac{R}{8n^2\theta^2}\ f_2\ cos\frac{2\pi x}{L} + sxy, \tag{1.25}$$

where

$$a_1 = \gamma + \theta, \quad b_1 = \gamma - \theta, \quad a_3 = \gamma + 3\theta, \quad b_3 = \gamma - 3\theta. \tag{1.26}$$

If we assume $\gamma = 0$, we get the previous expression (1.25) for ϕ replaced $\left(-py^2/2\right)$ by sxy.

We determine the total energy of the system $\ni$ from [13]. The quantities $\ni_{\tilde{n}}$ and $\ni_{\grave{e}}$ are calculated by the formulas [10]. The work of external forces equals

$$W = M_k\theta = 2\pi R^2 hs\theta, \tag{1.27}$$

where $\theta-$ is a mutual angle of rotation of the ends. By formula (**??**)

$$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\bullet\frac{\partial w}{\partial y}=\frac{1}{G}\tau\equiv-\frac{1}{G}\frac{\partial^2\phi}{\partial x\,\partial y} \tag{1.28}$$

The angle θ equals $\overline{\gamma}\;L/R$, where $\overline{\gamma}$ is a mean value of the shear strain component which is independent of w:

$$\begin{aligned}\overline{\gamma}=-\tfrac{1}{2\pi RL}\int\int_F\left(\tfrac{\partial u}{\partial y}+\tfrac{\partial v}{\partial x}\right)\partial x\,\partial y=\\=-\tfrac{1}{2\pi RL}\int\int_F\left(\tfrac{1}{G}\tfrac{\partial^2\phi}{\partial x\,\partial y}+\tfrac{\partial w}{\partial x}\bullet\tfrac{\partial w}{\partial y}\right)\partial x\,\partial y.\end{aligned} \tag{1.29}$$

Using the expressions (**??**) and (**??**), we find

$$\theta=\left(\frac{s}{G}+\frac{\pi^2}{4}\frac{h}{L^2}\frac{\gamma}{\theta^2}\,f_1^2\right)\,\frac{L}{R};$$

hence

$$W=2\pi RLh\left(\frac{s^2}{G}+s\frac{\pi^2}{4}\frac{h^2}{L^2}\frac{\gamma}{\theta^2}\,f_1^2\right) \tag{1.30}$$

Then we compose equations by the Riets method:

$$\frac{\partial\ni}{\partial f_1}=0,\;\frac{\partial\ni}{\partial f_2}=0 \tag{1.31}$$

In the first approximation we accept that in the supercritical deformation process the form of wave formation will be the one that corresponds to the solution of the linear problem of stability in small, according to [10]. Then one can determining the ratio between the parameter s and angle of rotation θ and find the lower critical stress $\hat{s}_{\text{í}}$. The relation $\nu=\hat{s}_{\text{í}}/\hat{s}_{\text{â}}$ turns out to depend on the parameter $\delta=Rh/L^2$. For $\delta=1/20;\;1/200;\;1/2000$ we get $\nu=0,94;0,80;0,87$, respectively. Thus, the least value is about 80%. As can be seen, the effect of the nonlinearity of the problem turns out to be somewhat weaker here than in the case of external pressure.

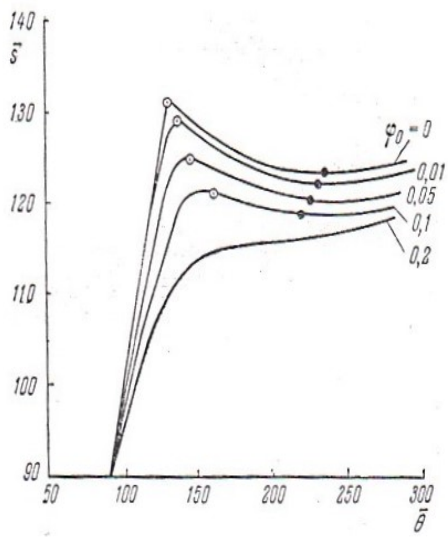


Fig. 5 Relationship between the value of $\overline{s}=(s/G)\,(L/h)^2$ and the parameter of the twisting angle $\overline{\theta}=\theta(L/h)^2$

Geometrical approach to the problem allows in the case of torsion to construct a surface isometric to the initial circular cylinder and reproducing the shape of buckling of the shell in large.

The study of the influence of initial irregularities for a shell of medium length can be carried out in the same way as in [16]. Fig. 5 shows the relationship between the value of $\overline{s} = (s/G)\,(L/h)^2$ and the parameter of the twisting angle $\overline{\theta} = \theta(L/h)^2$ for an ideally shaped shell and for shells with an initial flexure for $\delta = 1/200$. The ratio of the axis of initial flexure to the thickness is denoted by φ_0. The graph shows the points corresponding to the upper and lower critical stresses $s_{\hat{a}}$ and s. The general nature of these curves is the same as in the case external pressure.

Judging by the experimental data, the value of the critical stress usually lies in the plug composed by the upper and lower critical stresses. Therefore, in practical calculations in order to determine $s_{\hat{a}}$ the graph of Fig. 3 or (for shells of medium length) formula (1.18) should be used. The calculated value $s_{cal.}$ should be taken for $R/h \leq 250$ equal to $s_{\hat{a}},\ \ \nu = 0,8$. For large values of R/h the influence of initial flexure will be stronger here too; therefore ν should be reduced approximately according to the table ν for external pressure [8], moreover, the lower bound for $R/h \approx 1500$ equals $\nu \approx 0,5$.

Bending stability

The issue of the stability of cylindrical shells at bending arises, for example when calculating long pipelines; buckling here can occur in those areas where the bending moment of shear forces reaches a maximum. This problem occurs when calculating many aircraft structures. Theory of stability of shells in bending was developed in three directions. The first of these directions is related to the study of local buckling of medium length shells and is expressed in appearance of dents in the compressed zone. This time, the basic stress state is considered to be momentless, and the cross section is assumed to be circular up to the loss of stability. Here essentially the same general approach is retained as in the case of axial compression, but the normal stress is considered to be unevenly distributed over the section.

In the works related to the second direction, the flattering of cross sections is taken into account. This time, ultimate load is determined for a circular cylindrical shell that changes its resistance due to the change in the shape of the section.

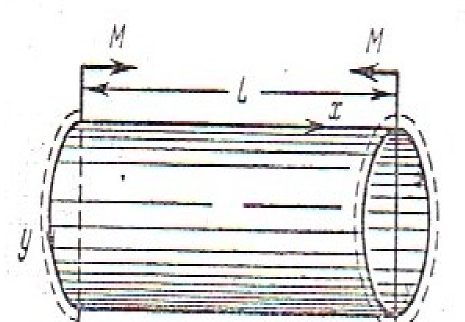


Fig. 6 Shell is subjected to the action of pairs with the moments M, applied in the diametric plane

Finally, the third approach is to determine the moment of occurrence of local dents, taking into account the subcritical process of flattering of the section. Let us cover in more detail the first of these approaches. Let us consider the case of a pure bending of a medium length shell. Assume that the shell is subjected to the action of pairs with the moments M, applied in the diametric plane (Fig. 6). We will count the y coordinate from the point of intersection of the plane of the pair with the medium surface located in the stretched part of the section. The normal stress in the cross section is distributed before buckling of the shell according to the law

$$P_x = P_1 cos\frac{\text{Ó}}{R} = \frac{M}{\pi R^2 h}\, cos\frac{\text{Ó}}{R}. \tag{1.32}$$

Appropriate deformations in the median surface equal

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} = \frac{P_x}{E}, \qquad \varepsilon_{y,0} = \frac{\partial v}{\partial y} - \frac{w}{R} = -\frac{\mu P_x}{E}, \ \ \gamma_0 = 0$$

Assuming $v = 0$ and $w = 0$ on the ends for $x = 0$ and $x = L$, we get the following expressions for the initial displacements:

$$u_0 = \frac{P_1}{E}\left(\frac{L}{2} - x\right) cos\frac{\text{Ó}}{R}, \tag{1.33}$$

$$v_0 = \frac{P_1}{2ER} x\,(L-x)\, sin\frac{\text{Ó}}{R}, \tag{1.34}$$

$$w_0 = \frac{P_1}{2ER}\left[x\,(L-x) - 2\mu R^2\right] cos\frac{\text{Ó}}{R}. \tag{1.35}$$

As can be seen, the sections of the shell do not stay circular: maximum flexures equal

$$w_{0,\ max} = \frac{P_1 R}{2E}\left(\frac{L^2}{4R^2} - 2\mu\right)$$

However, we will not take into account these displacements, understanding in the sequel under w an additional flexure; it is easy to show that the error of the solution will be negligible.

In studying the stability of the shell in small, we will proceed from the equation (1.17); it takes the form:

$$\frac{D}{h}\nabla^8 w + \frac{E}{R^2}\,\frac{\partial^4 w}{\partial x^4} + P_1 \nabla^4\left(\frac{\partial^2 w}{\partial x^2}\, cos\frac{\text{Ó}}{R}\right) = 0 \tag{1.36}$$

Accepting that the end sections of the shell are hingely supported, we choose the solution of the equation (1.36) in the form of a series:

$$w = sin\frac{m\pi x}{L}\,\sum_{n=1}^{\infty} f_n\, sin\frac{ny}{R} \tag{1.37}$$

Substitute this expression in (1.36) and equate the coefficients to zero for homogeneous terms; then we arrive at trinomial equations with respect to the parameters f_n of the form:

$$\left[\frac{D}{h}\left(\frac{m^2\pi^2}{L^2} + \frac{n^2}{R^2}\right)^2 + \frac{E}{R^2}\left(\frac{m\pi}{L}\right)^4\right] f_n - \frac{P_0}{2}\,\frac{n^2}{R^2}\left(\frac{m^2\pi^2}{L^2} + \frac{n^2}{R^2}\right)(f_{n-1} + f_{n+1}) = 0; \tag{1.38}$$

here we take into account that

$$sin\frac{ny}{R}\, cos\frac{\text{Ó}}{R} = \frac{1}{2}\left[sin\frac{(n+1)\,y}{R} + sin\frac{(n-1)\,y}{R}\right]$$

If we restrict ourselves to certain number of parameters f_n and set equal to zero the determinant of the system (1.38), then we can approximately determine the upper critical value P_0. Judging by calculations of A.V. Karamishin, Yu.G. Odinokov and Flugge [15], the value $P_{1,b}$ differs by a few percent from the upper critical stress for a centrally compressed shell. It can be shown that as the number of parameters increases, this discrepancy decreases. This result can be easily explained if one takes into account that the feature of stability loss in both cases is the same. Thus, the upper critical value $\hat{P}_{1,b}$ is determined from

$$\overline{P}_{1,b} = \frac{P_{1,b}R}{Eh} = \frac{1}{\sqrt{3\left(1-\mu^2\right)}} \approx 0,605 \tag{1.39}$$

The experiments show that the buckling of medium length shells under pure bending occurs by clap with the formation of relatively small dents in the compressed zone. Therefore [17] the complete study of behavior of such shells for local buckling should include the solution of the nonlinear problem.

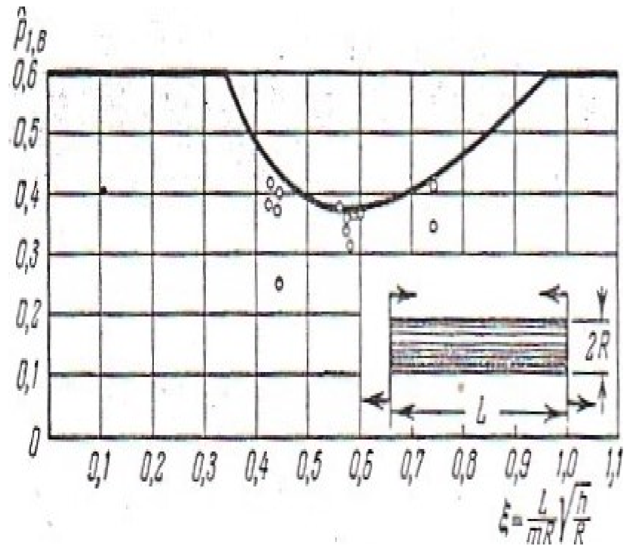
Investigations in this direction have shown that, the lower critical stress for a compressed zone in bending is approximately the same as in axial compression; we mean that of course that calculations are carried out with approximately the same number of variable parameters.

So, for $R/h = 250$ it should be taken $\hat{P}_{1,cal.} = 0,22$ instead of the value 0,18, indicated for the case of central compression.

In general, outside the central compression one should calculate the coefficient α, equal to $\alpha = 1 - P_2/P_1$, where P_1 is a maximum compression stress, P_2 is the stress at the opposite end of the diameter (taking into account the sign). Then for calculation we can use the data of the table for $\hat{P}_{cal.}$ in the case of central compression (depending on the ratio R/h and find the value of the greatest compressive stress by the formula [11]

$$\hat{P}_{1,cal.} = \hat{P}_{cal.}\left(1 + \frac{\alpha}{8}\right) \tag{1.40}$$

In the case of pure bending $\alpha = 2$; hence for $R/h \le 250$ we arrive at the value $\hat{P}_{1,cal.} = 1,25 \bullet 0,18 = 0,225$, that corresponds to the above value 0,22.



Fog. 7 Results of approximate solution

All above given data belong to a shell of medium length. For thin shells with significant ratio of length to the radius, one can expect another phenomenon, buckling along

long half-waves similar to the case of uniform compression. Using the same apparatus of "semi-momentless" theory of shells in linear statement, one can solve a problem of stability loss with the formation of such long-half-waves. The results of approximate solution are given in Fig. 7. The parameter $\hat{P}_{1,\,\hat{\text{A}}}$, was plotted along the coordinate axis, the value $\xi = (L/mR)\ \sqrt{h/R}$ along the abscissa axis; here m is the number of longitudinal semi-waves. The least value $P_{1,\,\hat{\text{A}}} = 0,38$ holds for $\xi = 0,58$. For example, for $R/h = 330$ this minimum is obtained for the shell with the ratio $L/R = 10$ as along short half-waves when $P_{1,\,\hat{\text{A}}} \approx 0,605$ occurs starting with $\xi = 0,35$. In Fig. 7 in calculations ξ should be calculated by substituting $m = 1,2$ etc. and then to choose the least value of $\hat{P}_{1,\,\text{A}}$.

References

1. Alfutov A.N. Stability of cylindrical and conical stiffened shells loaded by external pressure. Phd. Thesis. Moscow Higher Technical School, 1956; Stability analysis of shells by the energy method. Inzh. sborn. 22 (1955).
2. Volmir A.S. Theory of stability and large deformations of cylindrical shells at compression and shear "Calculation of spatial constructions' I, Mashgiz (1950). On the influence of initial irregularities on the stability of cylindrical shells under external pressure, DAN SSSR 113, no 2 (1957), 291 – 293.
3. Darevsky V.M. Stability of a cylindrical shell under simultaneous action of torgue and normal pressure. Izv. AN SSR, OTN, no 11 (1957), 137 – 147; Stability of a cantilevered cylindrical shell under bending by transverse forces with torsion and internal pressure, "Pochnost cilindricheskih obolochek", Oborongiz, 1959, 72–94, "calculation of spatial constructions" 5 (1959), 431 – 449.
4. Leontev N.N. To the stability of a closed cylindrical shell. Nauchniye doklady Vysshey shkoli. Stroitelstvo, no 1 (1958), 26 – 34.
5. Lukas P.A. On stability of orthotropic cylindrical shells. Nauchniye doklady Vysshey shkoli. Stroitelstvo, no 2 (1958) 5–13.
6. Marin V.A. Stability of a cylindrical shell under torsion and internal pressure. "Calculation of spatial constructions" 5 (1959), 475–484. Stability of a cylindrical panel under share, ib p, 485–501.
7. Mushtari Kh.M., Sachenkov A.V. On stability of cylindrical and conical shells of circular cross-section under joint action of axial compression and external normal pressure. Prikl. Mati. mech. 18, ¹ 6 (1954), 667 – 674.
8. Nagayev V.A. Determining the lower critical load of a cylindrical shell under external transversal pressure. Izv. vysshego uchebnogo zavedeniya.Ser. mashinostr. 6 (1959), 46 – 52.
9. Paliy O.M., Stability of a circular cylindrical shell built in at curvilinear edges. Izv. AN SSR, OTN, 1 (1958), 126 – 128.
10. Pogorelov A.V. Cylindrical shells under supercritical deformations. Parts 1–4, Kharkov Univ., 1962, Geometrical theory of shell stability. Nauka M., 1966.
11. Turkin K.D. Stability of a stiffened cylindrical shell under compression and pure bending. "Calculation of spatial constructions" 4 (1958), 477–498. General stability of a stiffened cylindrical shell under transversal bending. 5 (1959), 450 – 474.
12. Aleksandrovskiy S.V. On stability of a cylindrical shell under great flexure. "Calculation of spatial constructions" 3, Stroiizdat (1955), 453 – 492.
13. Alumiae N.A. Critical load of a long cylindrical shell under torsion. Priklad. mat. mech. 18, ¹ 1 (1954), 27–34.
14. Alekseev S.A. Flexible plates and shells in a supercritical domain. Doctor's dissertation. Ì., 1956.

15. Krivosheev N.I. On stability of a cylindrical shell-under shear. Izv. Kazan branch of AS USSR 12 (1958), 133–142, On stability of a cylindrical shell under joint action of torsion and uniform transverse pressure. ib. p.143–148.
16. Nash, John Forbes (1951). ”Non-cooperative Games”. Annals of Mathematics. Princeton, New Jersey: Princeton University. **54** (2), 286
17. Shevlyakov Yu.A., Manevich L.I.. DAN Ukr. SSR, ¹ 5 (1960), p. 605-608.