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Longitudinal shear of a piecewise homogeneous medium in the case when the binder and inclusions are weakened by doubly periodic rectilinear cracks

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Abstract. *A doubly periodic lattice with circular apertures of a plane filled with washers without tension from an isotropic elastic material whose surface is uniformly covered with a uniform film is considered. Each fiber and medium (binder) is weakened by straight-line cracks. Each washer has a centrally located crack, the length of which is less than the diameter of the washer. The presented stresses and their displacements are expressed in terms of the analytical function.*

Keywords. isotropically elastic material · doubly periodic lattice · rectilinear cracks · stress intensity factor · average stress · critical load · circular hole · longitudinal shift.

Mathematics Subject Classification (2010): 74A40 · 74S15

1 Introduction

The solution is based on the well-known proposition that the displacement in the case of anti-plane shear is a harmonic function. The well-known representation of the solution in each area is applied through the corresponding complex analytic function. Three analytic functions are represented by Laurent series. Satisfying the boundary condition on the contours of the holes and the cracks, the problem reduces to two infinite algebraic systems with respect to the desired coefficients and two singular integral equations with a Cauchy – type kernel. Then the singular integral equation by the Multhopp –Qalandia method is reduced to a finite algebraic system of equations. The procedure for calculating the stress intensity factors is given. For the numerical implementation of the described method, we took the

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cases of the location of the holes at the vertices of the triangular and square grids. The resulting analytical formulas were studied using the MATLAB program to determine the external load depending on the crack length. The results of calculations of the critical load depending on the crack length and elastic geometric parameters of the perforated medium are presented.

2 Formulation of the problem

Let a doubly periodic lattice with circular holes, has a radius λ ($\lambda < 1$) and centers at points:

$$P_{mn} = m\omega_1 + n\omega_2;\ (m, n = 0, \pm 1, \pm 2, \ldots);$$

$$\omega_1 = 2;\ \omega_2 = \omega_1 h e^{i\infty};\ h > 0;\ Im\omega_2 > 0;$$

The circular holes of the lattice are filled with washers (fibers) from an isotropic elastic material whose surface is uniformly covered with a uniform cylindrical film.

The banks of the cracks are free from external forces (Fig. 1). In the lattice, the average stresses $\tau_y = \tau_y^{\infty}, \tau_x = 0$ (shift at infinity) take place. By virtue of the symmetry of the boundary conditions and the geometry of the domain S occupied by the coupling medium, the stresses are doubly periodic functions with the main periods ω_1 and ω_2.

$$\left(1 + \frac{\mu_b}{\mu_t}\right) \phi_b(\tau_1) + \left(1 - \frac{\mu_b}{\mu_t}\right) \overline{\phi_b(\tau_1)} = 2\phi_t(\tau_1);$$
$$\left(1 + \frac{\mu_t}{\mu_s}\right) \phi_t(\tau) + \left(1 - \frac{\mu_t}{\mu_s}\right) \overline{\phi_t(\tau)} = 2\phi_s(\tau); \tag{2.1}$$

$$\phi_s'(t) - \overline{\phi'(t)} = 0;\ \ \phi_b'(t) - \overline{\phi_b'(t)} = 0. \tag{2.2}$$

The coordinates of points on the outer surface of the coating are hereinafter referred to as $\tau = \lambda^{i\theta} + m\omega_1 + n\omega_2$, $m, n = 0, \pm 1, \pm 2, \ldots$; and on the inside, $\tau_1 = (\lambda - h^*) e^{i\theta} + m\omega_1 + n\omega_2$, $m, n = 0, \pm 1, \pm 2, \ldots$ μ_t, μ_b and μ_t elastic permanent coating material, fibers and binder, respectively, t is the affix of points of cracks on the abscissa axes, h^* is the coating thickness, the value related to the coating, the fiber and the binder, are subsequently marked with the subscripts t, b and s, respectively.

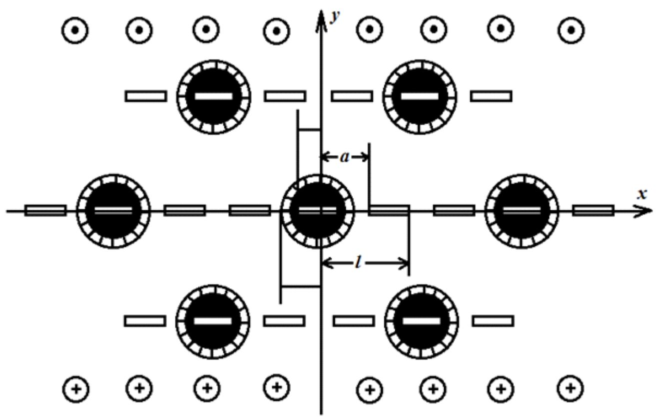

Fig. 1 Lattice scheme of a weakened doubly periodic system of rectilinear cracks

The solution of the boundary value problem is written in the form

$$\phi_s(z) = \phi_1(z) + \phi_2(z); \quad \phi_b(z) = \phi_{1b}(z) + \phi_{2b}(z); \tag{2.3}$$

$$\phi_{1b}(z) = \sum_{k=0}^{\infty} a_{2k} \tfrac{z^{2k+1}}{2k+1}; \quad \phi_t(z) = \sum_{k=-\infty}^{\infty} b_{2k} z^{2k+1};$$

$$\phi_1'(z) = \tau_y^{\infty} + \sum_{k=0}^{\infty} \alpha_{2k+2} \tfrac{\lambda^{2k+2}\gamma^{(2k)}(z)}{(2k+1)!}; \tag{2.4}$$

$$\phi_2'(z) = \tfrac{1}{\pi i} \int_L g(t)\,\xi(t-z)\,dt + A;$$

$$\phi_{2b}(z) = \tfrac{1}{\pi i} \int_{-l}^{l} \tfrac{g(t)dt}{t-z}; \tag{2.5}$$

where the integrals in (2.5) are taken along the line $L = \{[-l, -a] + [a, l]\}$, $\gamma(z)$ and $\xi(z)$ Weierstrass functions [2], A is a constant, $g(t)$ is the function

$$g(x) = \frac{\mu_s}{2} \frac{d}{dx} \left[W^+(x,\ 0)\, W^-(x, 0)\right] \quad \text{on the } L,$$

To the relations (2.3)–(2.5), an additional condition should be added, which follows from the physical meaning of the problem

$$\int_{-l}^{-a} g(t)\,dt = 0; \quad \int_a^l g(t)\,dt = 0; \quad \int_{-l}^{l} g(t)\,dt = 0. \tag{2.6}$$

The condition of constancy of the principal vector of all forces acting on an arc connecting two congruent points in S, taking into account (2.6) and the properties of the functions $\gamma(z)$ and $\xi(z)$ at congruent points, leads to the relation

$$Im\left[A\omega_1 + i\delta_j b - \alpha_2 \lambda^2 \delta_j\right] = 0; \quad (j = 1, 2);$$

$$b = -\frac{1}{\pi} \int_L t g(t)\,dt.$$

The solution of the boundary problem. The unknown function $g(t)$ and the constants $a_{2k}, b_{2k}, \alpha_{2k}$ must be determined from the boundary conditions (2.1)-(2.2).

To compose equations for the coefficients α_{2k}, the function $\phi_1'(z)$ will represent the boundary condition (2.1) in the form

$$\left(1 + \frac{\mu_b}{\mu_t}\right) \phi_{1b}(\tau_1) + \left(1 - \frac{\mu_b}{\mu_t}\right) \overline{\phi_{1b}(\tau_1)} = 2\phi_t(\tau_1) + i\phi_2^*(\theta); \tag{2.7}$$

where

$$i\phi_2^*(\theta) = -\left(1 + \frac{\mu_b}{\mu_t}\right) \phi_{2b}(\tau_1) - \left(1 - \frac{\mu_b}{\mu_t}\right) \phi_{2b}(\tau_1); \tag{2.8}$$

$$\left(1 + \frac{\mu_t}{\mu_s}\right) \phi_t(\tau) + \left(1 - \frac{\mu_t}{\mu_s}\right) \overline{\phi_t(\tau)} = 2\left[\phi_1(\tau) + i\phi_2(\theta)\right], \tag{2.9}$$

where

$$i\phi_2(\theta) = \phi_2(\tau). \tag{2.10}$$

With respect to the function $i\phi_2^*(\theta)$ and $i\phi_2(\theta)$, we assume that it decomposes $|\tau| = \lambda$ into a Fourier series. Due to symmetry, this series has the form:

$$i\phi_2^*(\theta) = \sum_{k=-\infty}^{\infty} B_{2k} e^{zki\theta}; \; ReB_{2k} = 0; \; i\phi_2(\theta) = \sum_{k=-\infty}^{\infty} C_{2k} e^{2ki\theta}; \; ReC_{2k} = 0; \tag{2.11}$$

$$B_{2k} = \frac{1}{2\pi}\int_0^{2\pi} i\phi_2^*(\theta)\, e^{-2ki\theta} d\theta;\ C_{2k} = \frac{1}{2\pi}\int_0^{2\pi} i\phi_2(\theta)\, e^{-2ki\theta} d\theta;\ (k = 0, \pm 1, \pm 2, \ldots).$$

Substituting the hostilities here (2.8) and (2.10) with regard to (2.5) and changing the order of integration, after calculating the integrals using the residue theory, we find

$$B_{2k} = -\frac{1}{\pi i}\int_{-l}^{l} g(t)\, \phi_{2k}^*(t)\, dt; \tag{2.12}$$

$$\phi_0^*(t) = -\frac{1}{t};\ \phi_{2k}^*(t) = \left(1 - \frac{\mu_b}{\mu_t}\right)\frac{\lambda^{2k}}{(2k)!t^{2k}} + \left(1 + \frac{\mu_b}{\mu_t}\right)\frac{\lambda^{2k}}{(2k)!t^{2k}};$$

$$\phi_{-2k}(t) = -\left(1 - \frac{\mu_b}{\mu_t}\right)\frac{\lambda^{2k}}{2t^{2k+1}};\ C_{2k} = \frac{1}{\pi i}\int_L \phi_{2k}(t)\, g(t)\, dt; \tag{2.13}$$

$$\phi_{2k}(t) = \frac{\lambda^{2k}}{(2k)!}\xi^{(2k)}(t);\ (k = 0, \pm 1, \pm 2, \ldots.).$$

Substituting in the boundary conditions (2.1) instead of $\phi_b(z)$, $\phi_t(z)$, $\phi_1'(z)$ expansions into Laurent's series, and instead of $\phi_2(z)$, $\phi_{2b}(z)$ the Fourier series is $|\tau| = \lambda$ and comparing the coefficients with the same powers of $exp(i\theta)$, we obtain an infinite system of linear algebraic controls:

$$b_{2k} = \left(1 + \frac{\mu_b}{\mu_t}\right)\frac{a_{2k}}{2(2k+1)!} - \frac{B_{2k}}{2(\lambda - h)^{2k+1}};$$

$$b_{-2k-2} = \left(1 - \frac{\mu_b}{\mu_t}\right)\overline{a_{2k}}\frac{(\lambda - h)^{4k+2}}{(2k+1)} - \frac{B_{-2k-2}}{2(\lambda - h)^{-2k-2}};$$

$$\frac{a_0}{4}\left[g_1 + f^2 h_1\right] = \tau_y^{\infty} + A + C_0 + \sum_{k=1}^{\infty}\alpha_{2k+2}\lambda^{2k+2}A_{0,k} + \frac{B_0}{2\lambda_*}; \tag{2.14}$$

$$\frac{\overline{a_0}}{4}\left[h_2 f^2 + g^2\right] = -\alpha_2;$$

$$\frac{\overline{a_{2k}}}{4}\lambda^{2k}\left[g_2 + f^{4k+2}h_2\right] = -\alpha_{2k+2};$$

$$\frac{a_{2k}}{4}\left[g_1 + f^{4k+2}h_1\right] = \lambda\alpha_2 A_{k,0} + \sum_{P=1}^{\infty}\alpha_{2p+1}\lambda^{2p+2} + \frac{C_{2k}}{\lambda_{2k}} + \frac{B_{2k}}{2\lambda_*^{2k+1}}.$$

Here

$$g_1 = \left(1 + \frac{\mu_b}{\mu_t}\right)\left(1 + \frac{\mu_t}{\mu_s}\right); \qquad g_2 = \left(1 + \frac{\mu_b}{\mu_t}\right)\left(1 - \frac{\mu_t}{\mu_s}\right);$$

$$h_1 = \left(1 - \frac{\mu_t}{\mu_s}\right)\left(1 - \frac{\mu_b}{\mu_t}\right); \qquad h_2 = \left(1 + \frac{\mu_t}{\mu_s}\right)\left(1 - \frac{\mu_b}{\mu_t}\right);$$

$$A_{P,k} = \frac{(2P + 2k + 1)!g_{p+k+1}^*}{(2P)!\,(2k+1)!2^{2p+2k+z}};\quad A_{0,0} = 0;\ \ \lambda_* = \lambda - h^*;$$

$$g_{p+k+1}^* = \sum_{mn}{}' \frac{1}{T^{2p+2k+2}};\quad T = \frac{1}{2}P_{mn};\quad f = \frac{\lambda - h^*}{\lambda}.$$

Requiring that functions (2.3) satisfy the boundary condition on the bank of the cut L, we obtain a singular integral equation for $g(x)$

$$\frac{1}{\pi}\int_L g(t)\,\xi(t-z)\,dt - Im\left[A + f_1'(x)\right] = 0;\ \text{on the }\ L, \tag{2.15}$$

$$\frac{1}{\pi}\int_{-l}^{l}\frac{g(t)\,dt}{t-x} - Im\left[f_{1b}'(x)\right] = 0. \tag{2.16}$$

The system (2.14) together with the singular equation (2.15) and (2.16) are the basic equations of the problem allowing to determine $g(x)$ and the coefficients $a_{2k}, b_{2k},\ \alpha_{2k}$. Recall that system (2.16) contains the coefficients C_{2k}, B_{2k} and depending on the desired function $g(x)$. The system (2.14) and the equation (2.15) and (2.16) turned out to be related should be solved together.

Knowing the functions $\phi_s(z)$, $\phi_b(z)$, $\phi_t(z)$, one can find the stress–strain state of the plate. By changing the ratio of the stiffness of the fiber to the stiffness of the bonding medium, you can get all the options, starting from the free from the forces of the circular opening and ending with the absolutely rigid fibers.

Taking advantage of the expansion of the function $\xi(z)$, taking into account $g(x) = -g(-x)$ and applying the change of variables, control (2.15) and (2.16) holds to the standard form

$$\frac{1}{\pi}\int_{-1}^{1}\frac{P(\tau)\,d\tau}{\tau-\eta} + \frac{1}{\pi}\int_{-1}^{1}P(\tau)\,B(\eta,r)\,d\tau - Im\left[A + \phi_1'(\eta)\right] = 0; \tag{2.17}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{P(\tau)}{\tau-\eta}d\tau - Im\phi_{1b}'(\eta) = 0. \tag{2.18}$$

Here

$$P(\tau) = g(t);\ \ B(\eta,\tau) = \frac{1-\lambda_1^2}{2}\sum_{j=0}^{\infty} g_{j+1}\left(\frac{l}{2}\right)^{2j+2} U^j A_j;$$

$$A_j = \left\{(2j+1) + \frac{(2j+1)(2j)(2j-1)}{1\cdot 2\cdot 3}\left(\frac{U}{U_0}\right) + \ldots + \left(\frac{U}{U_0}\right)^j\right\};$$

$$U = \frac{1-\lambda_1^2}{2}(\tau+1) + \lambda_1^2;\ \ U_0 = \frac{1-\lambda_1^2}{2}(\eta+1) + \lambda_1^2;\ \ \lambda_1 = \frac{a}{l};$$

$$x = \eta_0 l;\ \ t = \eta l;\ \ \eta_0^2 = U;\ \ \eta^2 = U;\ \ (j = 1, 2, \ldots).$$

Imagine the solution (2.17) and (2.18) in the form:

$$P(\eta) = \frac{P_0(\eta)}{\sqrt{1-\eta^2}} \tag{2.19}$$

the function $P_0(\eta)$ is replaced by the Lagrange interpolation polynomial constructed from Chebyshev nodes. Using quadrature formulas

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{P(\tau)\,d\tau}{\tau-\eta} = \frac{1}{n\ sin\theta}\sum_{v=1}^{n} P_v^0 \sum_{m=0}^{n-1} cos\ m\theta_v * \sin m\theta; \tag{2.20}$$

$$\frac{1}{2\pi}\int_{-1}^{1}P(\tau)\,B(\eta,\tau) = \frac{1}{2n}\sum_{\nu=1}^{n} P_v^0 B(\eta,\tau_v);\ \ \tau_\nu = \eta_\nu; \tag{2.21}$$

$$C_{2k} = -\frac{1-\lambda^2}{2}\frac{1}{2n}\sum_{\nu=1}^{n} P_\nu^0 f_{2k}^*(\tau_\nu)\,; \;\; B_{2k} = -\frac{1-\lambda_1^2}{2}\frac{1}{2n}\sum_{\nu}^{n} P_\nu^0 f_{2k}^*(\tau_\nu)\,.$$

Here

$$f_{2k}^{**}(\tau) = f_{2k}^{**}\left(\xi^2\right);\; \xi f_{2k}^{*}\left(\xi^2\right) = i f_{2k}(t)\,;\; f_{2k}^{**}(\tau) = f_{2k}^{**}\left(\xi^2\right);\; \xi_{2k}^{**}\left(\xi^2\right) = i f_{2k}^{*}(t)\,.$$

The furmules (2.20), (2.21) allow replacing the basic equations (2.17) and (2.18) with an infinite system of linear algebraic equations for the approximate values $g(t)$ of the unknown function at the nodal points, as well as the coefficients $\alpha_{2k} = \alpha_{2k}' + \alpha_{2k}''$. In this case, by successively excluding the constants a_{2k} in relations (2.14) and determining the real parts of the imaginary, we obtain two systems of equations for α_{2k}' and α_{2k}''

$$\sum_{\nu=1}^{n} a_{m\nu} P_\nu^0 - \frac{1}{2}\left[A + \phi_1'(\zeta_m)\right] = 0; \;\; \sum_{\nu=1}^{n} b_{mn} P_\nu^0 - \frac{1}{2} Im\ \phi_{1b}'(\zeta_m) = 0. \qquad (2.22)$$

Here

$$a_{m\nu} = \frac{1}{2n}\left[\frac{1}{sin\theta_m} ctg\frac{\theta_m + (-1)^{|m-\nu|}\theta_\nu}{2} + B(\eta_m,\ \tau_\nu)\right];\;\; \tau_m = \eta_m;$$

$$b_{m\nu} = \frac{1}{2n}\left[\frac{1}{\sin\theta_m} ctg\frac{\theta_m + (-1)^{|m-v|}\theta_\nu}{2}\right].$$

To system (2.22) it is necessary to add an additional condition, which in the discrete form has the form

$$\sum_{\nu=1}^{n} \frac{P_\nu^0}{\sqrt{\frac{1}{2}\left(1-\lambda_1^2\right)(\tau_\nu + 1) + \lambda_1^2}} = 0. \qquad (2.23)$$

System (2.21)-(2.23) is connected (closed) by infinite systems (2.14), in which instead of C_{2k} and B_{2k} relation (2.21) is substituted. The three systems noted completely determine the solution to the problem. After finding the values of P_ν^0, the stress intensity factor K_{III} is determined on the basis of relations (2.13), (2.16), (2.17), (2.19):

$$K_{III}^{a} = \sqrt{\frac{\pi l\left(1-\lambda_1^2\right)}{\lambda_1}}\frac{1}{2n}\sum_{\nu=1}^{n}(-1)^{\nu+n} P_\nu^0 tg\frac{\theta_\nu}{2};$$

$$K_{III}^{l}\sqrt{\pi l\left(1-\lambda_1^2\right)}\frac{1}{2n}\sum_{\nu=1}^{n}(-1)\, P_\nu^0 ctg\frac{\theta_\nu}{2};$$

$$K_{III}^{-l_1} = \sqrt{\pi l}\,\frac{1}{n}\sum_{k=1}^{n}(-1)^{k+n} P_k^0 tg\frac{\theta_k}{2}; \qquad (2.24)$$

$$K_{III}^{l_1} = \sqrt{\pi l}\frac{1}{n}\sum_{k=1}^{n}(-1)^{k} P_k^0 ctg\frac{\theta_k}{2}.$$

Analysis of the decision. For numerical calculations, we took the case of the location of the hole at the vertices of the triangular $\omega_1 = 2$, $\omega_2 = 2e^{\frac{1}{3}i\pi}$ and square $\omega_1 = 2$, $\omega_2 = 2i$ lattices. The calculations were performed on the IBM computer using the MATLAB program. It was assumed that $n = 10$ and $n = 20$, which corresponds to dividing the interval into 10 and 20 Chebyshev nodes, respectively. The resulting systems were solved by the Gauss method with the choice of the main element.

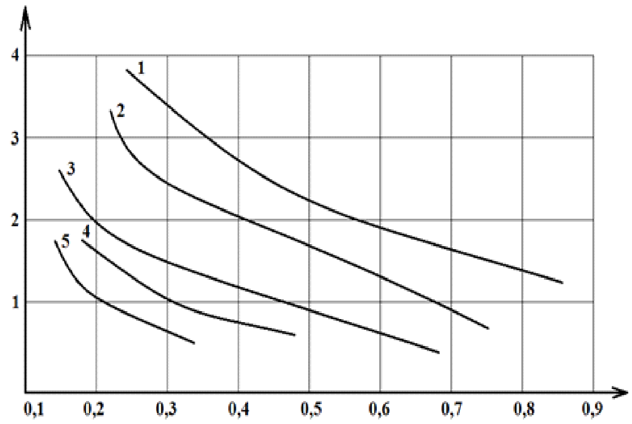

Fig. 2 Dependences of the critical load $\tau^* = \tau_y^\infty \sqrt{\omega_1}/K_{III\ c}$ on the crack length $l_* = (l - \lambda)/l$ for some values of the hole radius $\lambda = 0,2; 0,3; 0,4; 0,5; 0,6$ (curves 1–5)

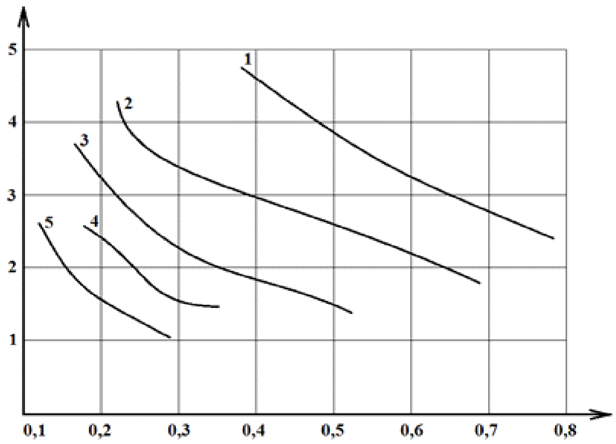

Fig. 3 Dependences of the critical load $\tau^* = \tau_y^\infty \sqrt{\omega_1}/K_{III\ c}$ on the crack length $l_* = (l - \lambda)/l$ for some values of the hole radius $\lambda = 0,2;\ 0,3;\ 0,4;\ 0,5;\ 0,6$ (curves 1–5)

It is believed that the law of deformation between partial bonds in the pre-failure zone is linear for $(W^+ - W^-) \leq \delta^*$. The nonlinear part of the bond deformation curve was approximated by a bilinear dependence [9], the ascending part of which corresponded to the deformation with the maximum bond force. When $(W^+ - W^-) > \delta^*$, the law of deformation was described by a nonlinear dependence determined by points (δ^*, τ^*) and (δ_c, τ_c) and with $\tau_c \geq \tau^*$ there was an increasing linear dependence (linear hardening corresponding to elastoplastic deformation of bonds).

Prefracture zones are in a binder; therefore, the dimensions of the holes are taken. The quantity λ is related to the radius λ_1 of the isotropic inclusion $\lambda = \lambda_1 + h^*$.

Based on the results obtained in fig. 2, 3 and 4 graphs of the critical (maximum) load are plotted $\tau^* = \tau_y^\infty \sqrt{\omega_1}/K_{III\ c}$ for both crack tips from the crack length $l_* = l - a$ for some values of the hole radius $\lambda = 0.2; 0.3; 0.4; 0.5; 0.6$ (curves 1–5).

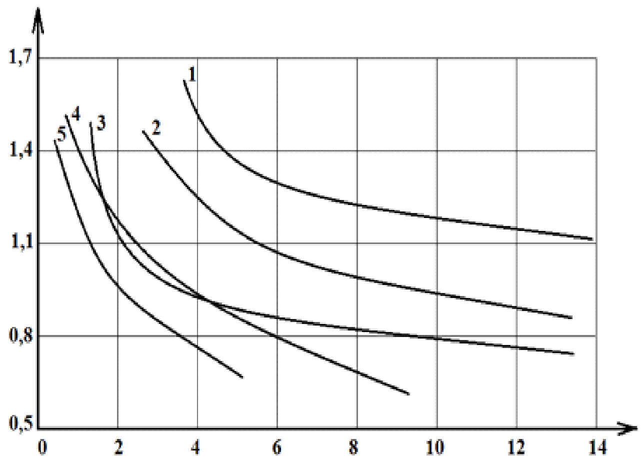

Fig. 4 Dependences of the critical load $\tau^* = \tau_y^\infty \sqrt{\omega_1}/K_{III\ c}$ on the crack length $l_* = (l-\lambda)/l$ for some values of the hole radius $\lambda = 0,2;\ 0,3;\ 0,4;\ 0,5;\ 0,6$ (curves 1–5)

Calculations were carried out for fiberglass EDC–B with parameters $\frac{\mu_b}{\mu_s} = 25$, $\frac{\mu_b}{\mu_t} = 50$.

3 Conclusions

The problem of longitudinal fracture cracks in composites with a doubly periodic structure was solved for the first time.

Analysis of the critical equilibrium state in a composite with a doubly periodic structure, at which cracks appear, reduces to a parametric study of the combined algebraic system (2.14), (2.22) - (2.24) and the crack appearance criterion with different laws of deformation of bonds, elastic constant materials and geometric characteristics of the composite. Directly from the solution of the obtained algebraic systems, the tangential stresses in the connections and the displacement of the banks of the pre-failure zones are determined. The relations obtained allow us to investigate the cracking in the composite under longitudinal shear.

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