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Waves established during viscous fluid flow in elastic tube

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Abstract. *The propagation of waves generated during the movement of the viscous fluid in a linear viscous elastic tube is confirmed. Taking into account the hydraulic resistance of the compressed viscous fluid, the mathematical model of the wave process generated during the movement of the fluid based on the averaged law of linear viscosity for differential equations of continuity and movement of the tube along the cross section.*

Keywords. viscous fluid · elastic tube · wave · hydraulic resistance · wave velocity.

Mathematics Subject Classification (2010): 76D55

1 Introduction

In solving many practical problems, there is a need to study the process of wave propagation, taking into account the interaction between the deformed tube wall and the fluid. In this case, the forces of interaction between a system consisting of a liquid, a gas and a deformable body must be applied taking into account the properties of the gas, liquid and solid.

The study of the propagation of nonlinear waves occurring during the flow of fluid in a deformed tube is of theoretical and practical importance. In particular, the construction and analysis of a mathematical model in which the nonlinear flow of a fluid in a viscous elastic tube is taken into account together is very interesting. Thus, the obtained scientific results are used in the design of main tubelines and medicine.

Assume that, displacement of the walls of the tube - ξ and its thickness - h , is relatively small compared to the radius of equilibrium- R_0, characteristic wavelengths -L are considerably larger than R_0. Deformation of the tube is characterized by a change in its radius. The liquid and the tube are isotropic, and the characteristic velocity of the wave is very high compared to the flow velocity of the liquid. The pressure drop in the tube during friction is determined by the Darcy-Weisbach formula.

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2 Problem statement

Taking into account the hydraulic resistance of the compressed viscous fluid, we can write the differential equations of average continuity and motion along the cross section of the tube as follows [12]:

$$\frac{\partial(\rho s)}{\partial t}+\frac{\partial(\rho s u)}{\partial X}+\frac{2s\rho}{R}\left(\frac{\partial\xi}{\partial t}+u\frac{\partial\xi}{\partial X}\right)=0, \tag{2.1}$$

$$\frac{\partial(\rho s u)}{\partial t}+\frac{\partial\left[(1+\beta)\rho s u^2\right]}{\partial X}=-s\frac{\partial P}{\partial X}+\mu s\frac{\partial^2 u}{\partial X^2}-\frac{\lambda\rho s u^2}{4R}+\rho s g\sin\alpha. \tag{2.2}$$

Let us accept the law of linear viscosity for a thin-walled tube material as follows [23]:

$$\left(1+\sum_{l=1}^{m}b_l\frac{D^l}{Dt^l}\right)\mathit{\Delta}P=\frac{h}{R^2}\left(a_0+\sum_{l=1}^{n}a_l\frac{D^l}{Dt^l}\right)\xi. \tag{2.3}$$

Given that the change in the radius of the pipe-R_0 depends only on the displacement of the wall $\xi\left(s=\pi R^2(\xi)\right)$, then the system of equations (2.1) - (2.3) is closed by the equation of state of the viscous elastic fluid.

$$\rho=\rho(P). \tag{2.4}$$

Equations (2.1) - (2.4) have the following designations:

$\xi(X,\,t)$ - is the radial displacement radius R_0 and thickness h of a pipe ; $u(X,\,t)$ - is the average velocity of the fluid flow; $\mathit{\Delta}P=P-P_0$; $P(X,\,t)$ - is hydrodynamic pressure; $P_0(X),\ u_0=const$ - respectively, the steady-state pressure and the average velocity of the fluid up to agitation; ρ - is the average density of the liquid; s- cross-sectional area of the tube; λ- is the coefficient of hydraulic resistance in the Darcy-Weisbach formula; μ- is the dynamic viscosity of the fluid; β- is a Coriolis correction in the irregular distribution of velocities. (Note that during the parabolic distribution $\beta=1/3$,, and indefinitely, there will be a variable quantity [13]); g- free fall acceleration; α- the angle of ascent of the X axis from the horizon; $a_0,\ a_l,\ b_l$- are fixed coefficients determined from certain viscous elastic models [2, 4, 5].

The flow of fluid to the excitation is described by the stationary equations of momentum retention (2.2).

$$\frac{dP_0}{dX}-\frac{\lambda}{4R_0}\rho_0 u_0^2+\rho_0\sin\alpha=0. \tag{2.5}$$

Let's adopt a new coordinate system $x=\eta X$, $\tau=c_s^{-1}X-t$ related to small-scale and acoustic wave front $\eta<<1$. Let's write equations (2.1) - (2.5) in this system. Then:

$$-\frac{\partial(\rho s)}{\partial\tau}+\eta\frac{\partial(\rho s u)}{\partial x}+c_s^{-1}\frac{\partial(\rho s u)}{\partial\tau}+\frac{2s\rho}{R}\left(-\frac{\partial\xi}{\partial\tau}+\eta u\frac{\partial\xi}{\partial x}+c_s^{-1}u\frac{\partial\xi}{\partial\tau}\right)=0, \tag{2.6}$$

$$-\frac{\partial(\rho s u)}{\partial\tau}+\eta\frac{\partial\left[(1+\beta)\rho s u^2\right]}{\partial x}+c_s^{-1}\frac{\partial\left[(1+\beta)\rho s u^2\right]}{\partial\tau}=-s\left(\eta\frac{\partial P}{\partial x}+c_s^{-1}\frac{\partial P}{\partial\tau}\right)+$$

$$+\mu s\left(\eta^2\frac{\partial^2 u}{\partial x^2}+2\eta c_s^{-1}\frac{\partial^2 u}{\partial x\partial\tau}c_s^{-2}\frac{\partial^2 u}{\partial\tau^2}\right)-\frac{\lambda\rho s u^2}{4R}+\rho s g\sin\alpha, \tag{2.7}$$

$$\left[1+\sum_{l=1}^{m} b_l \prod_{q=1}^{l}\left(-\frac{\partial}{\partial\tau}+\eta u\frac{\partial}{\partial x}+c_s^{-1}u\frac{\partial}{\partial\tau}\right)^q\right]\Delta P=$$

$$=\frac{h}{R^2}\left[a_0+\sum_{l=1}^{n} a_l \prod_{q=1}^{l}\left(-\frac{\partial}{\partial\tau}+\eta u\frac{\partial}{\partial x}+c_s^{-1}u\frac{\partial}{\partial\tau}\right)^q\right]\xi. \tag{2.8}$$

The characteristic velocity of the wave is very large compared to the flow velocity of the liquid and given the convergence of $[x]\sim c_s\,[\tau]$, we obtain that

$$\left|\frac{\partial}{\partial\tau}\right|>>\eta\left|u\frac{\partial}{\partial x}\right|,\left|\frac{\partial}{\partial\tau}\right|>>\left|\frac{u}{c_s}\frac{\partial}{\partial\tau}\right|.$$

Therefore, equation (2.8) is simplified in the new variables

$$\left(1+\eta\sum_{l=1}^{m}(-1)^l\,\frac{b_l}{\eta}\frac{\partial^l}{\partial\tau^l}\right)\Delta P=\frac{h}{R^2}\left(a_0+\eta\sum_{l=1}^{n}(-1)^l\,\frac{a_l}{\eta}\frac{\partial^l}{\partial\tau^l}\right)\xi. \tag{2.9}$$

The system of equations (2.6) - (2.9) includes a small parameter η . (2.6) - (2.9) looking for the solution of the system of equations in the form of a sequence of unknowns according to the small parameter η [2,3]:

$$\xi=\eta\xi_1+\eta^2\xi_2+\cdots=\sum_{k=1}^{\infty}\eta^k\xi_k,c_f=\sqrt{\frac{\partial P}{\partial\rho}},$$

$$R=R_0+\eta\xi_1+\eta^2\xi_2+\cdots=R_0+\sum_{k=1}^{\infty}\eta^k\xi_k,$$

$$u=u_0+\eta u_1+\eta^2u_2+\cdots=u_0+\sum_{k=1}^{\infty}\eta^k u_k,$$

$$s=\pi\left(R_0^2+2\eta R_0\xi_1+\eta^2\left(\xi_1^2+2R_0\xi_2\right)+\cdot\cdots\right),$$

$$\rho=\rho_0+\frac{ha_0}{R_0^2c_f^2}\left(\eta\xi_1+\eta^2\xi_2+\cdots\right)=\rho_0+\frac{ha_0}{R_0^2c_f^2}\sum_{k=1}^{\infty}\eta^k\xi_k, \tag{2.10}$$

$$\Delta P=\eta\Delta P_1+\eta^2\Delta P_2+\cdots=\sum_{k1}^{\infty}\eta^k\Delta P_k,$$

$$\beta=\beta_0+\eta\beta_1+\eta^2\beta_2+\cdots=+\beta_0+\sum_{k=1}^{\infty}\eta^k\beta_k,.$$

Then

$$s\rho=\pi\rho_0R_0^2+\eta\pi\left(2R_0\rho_0+c_f^{-2}ha_0\right)\xi_1+$$

$$+\eta^2\pi\left[\left(2R_0\rho_0+c_f^{-2}ha_0\right)\xi_2+\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)\xi_1^2\right]+\cdots,$$

$$s\rho u = \pi\rho_0 R_0^2 u_0 + \eta\pi\left(R_0^2\rho_0 u_1 + u_0\xi_1\left(2R_0\rho_0 + c_f^{-2}ha\right)\right) + \eta^2\pi\left[\left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0\xi_1^2 + \right.$$

$$\left. + \rho_0 R_0^2 u_2 + \left(2R_0\rho_0 + \frac{ha_0}{c_f^2}\right)u_0\xi_2 + u_1\xi_1\left(2R_0\rho_0 + c_f^{-2}ha\right)\right] + \cdots, \qquad (2.11)$$

$$s\rho u^2 = \pi R_0\rho_0 u_0^2 + \eta\pi\left[2R_0^2\rho_0 u_0 u_1 + \left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_0^2\xi_1^2\right] +$$

$$+\eta^2\pi\left[(\rho_0 + + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0^2\xi_1^2 + +2R_0^2\rho_0 u_0 u_2 + + \left(2R_0\rho_0 + \frac{ha_0}{c_f^2}\right)u_0^2\xi_2 +$$

$$\left. + R_0^2\rho_0 u_1^2 + 2u_0 u_1\xi_1\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\right] + \cdots,$$

Write the expressions (2.11) in equations (2.6) - (2.9):

$$-\frac{\partial}{\partial\tau}\left(\pi\rho_0 R_0^2 + \eta\pi\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\xi_1 + \eta^2\pi\left(\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\xi_2 + \right.\right.$$

$$\left(\rho_0\frac{c_f^{-2}ha_0}{R}\right)^2 \xi_1)) + R\eta\frac{\partial}{\partial x}$$

$$\left(\pi\rho_0 R_0^2 u_0 + \eta\pi\left(R_0^2\rho u_1\left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_0\ \xi_1\right) + \eta^2\pi\left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_0\xi_2 + \right.$$

$$+\left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0\xi_1^2 + \left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_1\ \xi_1) +$$

$$+Rc_5^{-1}\frac{\partial}{\partial\tau}\left(\pi\rho_0 R_0^2 u_0 + \eta\pi\left(R_0^2\rho u_1\ (2R_0\ \rho_0 + c_f^{-2}ha_0\right)u_0\ \xi_1\right) +$$

$$+\left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0\xi_1^2 + \left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_1\ \xi_1) +$$

$$+2\left(\pi\rho_0 R_0^2 + \eta\pi\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\xi_1 + \eta^2\pi\,(2R_0\ \rho_0 + c_f^{-2}\ ha_0)\,u_0\xi_2 + \right.$$

$$+\eta^2\pi\left(\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\xi_2 + \left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)^2\xi_1\cdot\left(-\frac{\partial\xi}{\partial\tau} + 2u\frac{\partial\xi}{\partial x} + c_5^{-1}u\frac{\partial\xi}{\partial\tau}\right) = 0;\right.$$

$$-\frac{\partial}{\partial\tau}\left(\pi\rho_0 R_0^2 u_0 + \eta\pi\left(R_0^2\rho_0 u_1 + c_f^{-2}ha_0\right)u_0\xi_1\right) + \eta^2\pi\left(2R_0\rho_0 + \left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_0\xi_2\right.$$

$$(2R_0\rho_0 + c_f^{-2}ha_0)u_1\xi_1) + \eta\frac{\partial}{\partial\chi}\left((1+\beta)\left(\pi\rho_0 R_0 u_0^2 + \right.\right.$$

$$+\eta\pi(2R_0^2\rho_0 u_0 u_1 + (2R_0\rho_0 + c_f^{-2}ha_0)u_0^2\xi_1) + \left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0\xi_1^2 +$$

$$+\eta^2\pi 2R_0^2\rho_0 u_0 u_2 + \left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_0^2\xi_2 + \left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0^2 +$$

$$+\rho_0 R_0^2 u_1 + \rho_0 R_0^2 u_1 + 2u_0\left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_1\xi_1))+$$

$$+c_5^{-1}\frac{\partial}{\partial\tau}((1+\beta(\pi\rho_0 u_0^2 R_0 + \eta\pi(2R_0^2\rho_0 u_0 u_1 + (R_0\rho_0 + c_f^{-2}ha_0 u_0^2\xi_1^2) + \eta^2\pi$$

$$+(2R_0^2\rho_0 u_0 u_2 + (2R_0\rho_0 + c_f^{-2}ha_0)u_0^2\xi_2) + \left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0^2\xi_1^2 + \rho_0 R_0^2 u_1 +$$

$$+2u_0\left(2R_0\rho_0 + c_f^{-2}ha_0\right)u_1\xi_1)) = -\pi\left(R_0^2 + 2\eta R_0\xi_1 + \eta^2\xi_1^2 + 2R_0\xi_2\right)\times$$

$$\times\left(\eta\frac{\partial\rho}{\partial x} + c_5^{-1}\frac{\partial\rho}{\partial\tau}\right) + \mu\pi(r_0^2 + 2\eta R_0\xi_1 + \eta^2(\xi_1^2 + 2R_0\xi_2)$$

$$\times\left(\eta^2\frac{\partial^2 u}{\partial x^2} + 2\eta c_5^{-1}\frac{\partial^2 u}{\partial x\partial\tau}c_5^{-2}\frac{\partial^2 u}{\partial\tau^2}\right) -$$

$$-\frac{\lambda\pi\rho_0 u_0^2 R_0 + \eta\pi(2R_0\rho_0 u_0 u_1 + (2R_0\rho_0 + c_f^{-2}ha_0)}{4\left(R_0 + \eta^1\xi_1 + \eta^2\xi_2\right)}\cdot u_0^2\xi_1^2)+$$

$$+\eta^2\pi\left(2R_0^2\rho_0 u_0 u_2 + (2R_0\rho_0 + c_f^{-2}ha_0\right)u_0^2\xi_2 + \left(\rho_0 + \frac{2c_f^{-2}ha_0}{R_0}\right)u_0^2\xi_1^2 +$$

$$+\rho_0 R_0^2 u_1 + 2u_0\left(2R_0\,\rho_0 + c_f^{-2}\,ha_0\right)u_1\,\xi_1)$$

$$+\rho\pi\left(R_0^2 + 2\eta R_0\xi_1 + \eta^2\left(\xi_1^2 + 2R_0\xi_2\right)g\sin x\right.$$

$$-\pi R_0\eta\left[\left(2R_0\rho_0 + \frac{ha_0}{c_f^2}\right)\frac{\partial\xi_1}{\partial\tau} - c_5^{-1}u_0\left(\frac{ha_0}{c_f^2} + 2R_0\rho_0\right)\frac{\partial\xi_1}{\partial\tau} - c_5^{-1}\rho_0 R_0^2\frac{\partial u_1}{\partial\tau}\right] +$$

$$+2\rho_0 R_0\frac{\partial\xi_1}{\partial\tau} - \frac{2}{c_5}uR_0\rho_0 u_0\frac{\partial\xi_1}{\partial\tau} = 0 \qquad (2.12)$$

If we equate the coefficients of the same degree limits of the parameter η in the system of equations (2.12), we get in the first approximation:

$$\left(4R_0\rho_0 - \frac{4R_0\rho_0 u_0}{c_s}\right)\frac{\partial\xi_1}{\partial\tau} + \left({c_f}^{-2}ha_0 - \frac{{c_f}^{-2}ha_0}{c_s}\right)\frac{\partial\xi_1}{\partial\tau} - \frac{{R_0}^2\rho_0}{c_s}\frac{\partial u_1}{\partial\tau} = 0$$

$$u_0\left(2R_0\rho_0 + c_f^{-2}ha_0\right)\left(1 - \frac{(1+\beta)\,u_0}{c}\right)\frac{\partial\xi_1}{\partial\tau} - \frac{ha_0}{c}\frac{\partial\xi_1}{\partial\tau} +$$

$$+\left(R_0^2\rho_0 - \frac{2R_0^2\rho_0\,(1+\beta)\,u_0}{C_5}\right)\frac{\partial u_1}{\partial\tau} = 0 \qquad (2.13)$$

After making the appropriate grouping, we get:

$$\left(4R_0\rho_0 + c_f^{-2}ha_0\right)\left(1 - \frac{u_0}{c_s}\right)\frac{\partial\xi_1}{\partial\tau} - \frac{R_0^2\rho_0}{c_s}\frac{\partial u_1}{\partial\tau} = 0, \qquad (2.14)$$

$$\left[u_0\left(2R_0\rho_0+c_f^{-2}ha_0\right)\left(1-\frac{(1+\beta_0)\,u_0}{c_s}\right)-\frac{ha_0}{c_s}\right]\frac{\partial\xi_1}{\partial\tau}+$$
$$+R_0^2\rho_0\left[1-\frac{2\,(1+\beta_0)\,u_0}{c_s}\right]\frac{\partial u_1}{\partial\tau}=0.$$

It is known from the higher algebra course that for the existence of non-trivial solutions of the system (2.14) it is necessary to reduce its main determinant to zero [2], ie:

$$\begin{vmatrix} \left(4R_0\rho_0+c_f^{-2}ha_0\right)\left(1-\frac{u_0}{c_s}\right) & -\frac{R_0^2\rho_0}{c_s} \\ u_0\left(2R_0\rho_0+c_f^{-2}ha_0\right)\left(1-\frac{(1+\beta_0)u_0}{c_s}\right)-\frac{ha_0}{c_s} & R_0^2\rho_0\left[1-\frac{2(1+\beta_0)u_0}{c_s}\right] \end{vmatrix}=0$$

Then we get,

$$\left(4\rho_0+\frac{ha_0}{c_fR_0}\right)c_s^2-2u_0\left((5+4\beta_0)\,\rho_0+(1+\beta_0)\,\frac{ha_0}{c_fR_0}\right)c_s+$$
$$+u_0^2\,(1+\beta_0)\left(6\rho_0+\frac{ha_0}{c_fR_0}\right)-\frac{a_0h}{R_0}=0. \tag{2.15}$$

The obtained equation (2.15) is the variance equation. In the special case, if $(u_0/c_s)<<1$, then the variance relation (2.15) falls as follows:

$$c_s=\sqrt{\frac{ha_0}{2R_0\rho_0+c_f^{-2}ha_0}}. \tag{2.16}$$

Equations (2.15) - (2.16) show that the excitations generated during the flow of viscous fluid in a viscous elastic tube propagate in the form of linear-acoustic waves.

Consider the propagation of nonlinear waves in an elastic tube with an oscillator. To do this, using (2.9) - (2.14), let's express the variables $\varDelta P_1$ and u_1 in the first approximation with ξ_1 :

$$\varDelta P_1=\frac{ha_0}{R_0^2}\xi_1, \tag{2.17}$$

$$u_1=\frac{c_s\left(4R_0\rho_0+c_f^{-2}ha_0\right)}{R_0^2\rho_0}\left(1-\frac{u_0}{c_s}\right)\xi_1.$$

Considering the relations (2.17) in equation (2.12), we obtain from the second approximation by choosing the limits of the coefficient η^2:

$$\left(2R_0\rho_0+\frac{ha_0}{c_f^2}\right)\frac{\partial\xi_2}{\partial\tau}-c_f^{-1}u_0\left(2R_0\rho_0+\frac{ha_0}{c_f^2}\right)\frac{\partial\xi_2}{\partial\tau}-c_f^{-1}R_0^2\rho_0\frac{\partial u_2}{\partial\tau}+$$
$$+2R_0\rho_0\frac{\partial\xi_2}{\partial\tau}-\frac{2}{c_5}u_0R_0\rho_0\frac{\partial\xi_2}{\partial\tau}=\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)\frac{\partial\xi_1^2}{\partial\tau}+u_0\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)\frac{\partial\xi_1^2}{\partial\tau}+$$
$$+u_1\left(2R_0\rho_0+c_f^{-2}ha_0\right)\frac{\partial\xi_1}{\partial\tau}+c_5\left(4R_0\rho_0+\frac{ha_0}{c_f^2}\right)\frac{\partial\xi_1}{\partial x} \tag{2.18}$$

$$R_0^2\rho_0\frac{\partial u_1}{\partial\tau}+u_0\left(2R_0\rho_0+c_f^{-2}ha_0\right)\frac{\partial\xi_2}{\partial\tau}-\frac{2\left(1+\beta_0\right)R_0^2\rho_0 u_0}{c_5}\frac{\partial u_2}{\partial\tau}-$$

$$-\frac{\left(1+\beta_0\right)u_0^2\left(2R_0\rho_0+c_f^{-2}ha_0\right)}{c_5}\frac{\partial\xi_2}{\partial\tau}-\frac{ha_0}{c_5}\frac{\partial\xi_2}{\partial\tau}=-\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)u_0\frac{\partial\xi_1^2}{\partial\tau}-$$

$$-\left(2R_0\rho_0+2c_f^{-2}ha_0\right)\frac{\partial\left(u_1\xi_1\right)}{\partial\tau}-2\left(1+\beta_0\right)R_0^2\rho_0u_0\frac{\partial u_1}{\partial x}-\left(2R_0\rho_0+c_f^{-2}ha_0\right)\times$$

$$\times\left(1+\beta_0\right)u_0^2\frac{\partial\xi_1}{\partial x}+\frac{\left(1+\beta_0\right)}{c_5}\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)u_0^2\frac{\partial\xi_1^2}{\partial\tau}+\frac{\left(1+\beta_0\right)R_0^2\rho_0}{c_5}\frac{\partial u_1^2}{\partial\tau}+$$

$$+\frac{2\left(1+\beta_0\right)u_0}{c_5}\left(2R_0\rho_0+c_f^{-2}ha_0\right)\frac{\partial\left(u_1\xi_1\right)}{\partial\tau}-ha_0\frac{\partial\xi_1}{\partial x}+\frac{2ha_0}{c_5R_0}\xi_1\frac{\partial\xi_1}{\partial\tau}+$$

$$+\frac{h}{\eta c_5}\sum_{l=1}^{n}(-1)^{l+1}s_{l+1}\frac{\partial^{1+1}\xi_1}{\partial\tau^{l+1}}--2\frac{\mu R_0^2}{c_5}\frac{\partial^2u_1}{\partial x\partial\tau}+\frac{\mu R_0^2}{c_5^2}\frac{1}{\eta}\frac{\partial^2u_1}{\partial\tau^2}-\frac{\lambda u_0R_0\rho_0}{2}\frac{u_1}{\eta}-\frac{\lambda R_0\rho_0}{4}u_1^2-$$

$$-g\sin\alpha\left(2R_0\rho_0+c_f^{-2}ha\right)\frac{\xi_1}{\eta}-g\sin\alpha\left(\rho_0+\frac{2c_f^{-2}ha_0}{R_0}\right)\xi_1^2.$$

After grouping the system of equations (2.18) falls into the following form:

$$\left[\frac{ha_0}{c_s}-\left(2R_0\rho_0+\frac{ha_0}{c_f^2}\right)\left(u_0-\frac{u_0^2\left(1+\beta_0\right)}{c_s}\right)\right]\frac{\partial\xi_2}{\partial\tau}-R_0^2\rho_0\left[1-\frac{2u_0\left(1+\beta_0\right)}{c_s}\right]\frac{\partial u_2}{\partial\tau}=$$

$$\left(4R_0\rho_0+\frac{ha_0}{c_f^2}\right)\left(1-\frac{u_0}{c_s}\right)\frac{\partial\xi_2}{\partial\tau}-\frac{R_0^2\rho_0}{c_s}\frac{\partial u_2}{\partial\tau}=\left(10\rho_0+\frac{4ha_0}{R_0c_f^2}+\frac{h^2a_0^2}{R_0^2c_f^4\rho_0}\right)\times$$

$$\times\left(1-\frac{u_0}{c_s}\right)\frac{\partial\xi_1^2}{\partial\tau}+N_1\frac{\partial\xi_1}{\partial x},$$

$$\left[\frac{ha_0}{c_s}-\left(2R_0\rho_0+\frac{ha_0}{c_f^2}\right)\left(u_0-\frac{u_0^2\left(1+\beta_0\right)}{c_s}\right)\right]\frac{\partial\xi_2}{\partial\tau}-R_0^2\rho_0\left[1-\frac{2u_0\left(1+\beta_0\right)}{c_s}\right]\frac{\partial u_2}{\partial\tau}=$$

$$N_2\xi_1^2-N_4\frac{\partial\xi_1}{\partial x}+N_5\frac{\partial\xi_1^2}{\partial\tau}+\frac{\mu}{\rho_0}\left(4R_0\rho_0+\frac{ha_0}{c_f^2}\right)\left(1-\frac{u_0}{c_s}\right)\left(\frac{\partial}{\partial x}+\frac{1}{\eta c_s}\frac{\partial}{\partial\tau}\right)\frac{\partial\xi_1}{\partial\tau}+$$

$$+\frac{h}{\eta c_s}\sum_{l=1}^{n}\left(-1\right)^{l+1}S_{l+1}\frac{\partial^{l+1}\xi_1}{\partial\tau^{l+1}}\tag{2.19}$$

Here,

$$N_1=c_s\left(4R_0\rho_0+\frac{ha_0}{c_f^2}\right),$$

$$N_2=-\frac{\lambda c_s^2}{4R_0^3\rho_0}\left(4R_0\rho_0+\frac{ha_0}{c_f^2}\right)^2\left(1-\frac{u_0}{c_s}\right)^2+g\sin\alpha\left(\rho_0+\frac{2ha_0}{R_0c_f^2}\right),$$

$$N_3=-\frac{1}{\eta}\left(g\sin\alpha\left(2R_0\rho_0+\frac{ha_0}{c_f^2}\right)-\frac{\lambda u_0\rho_0 N_1}{2R_0}\left(1-\frac{u_0}{c_s}\right)\right),$$

$$N_4=ha_0+u_0c_s\left(1+\beta_0\right)\left[8R_0\rho_0+\frac{2ha_0}{c_f^2}-\frac{u_0}{c_s}\left(6R_0\rho_0+\frac{2ha_0}{c_f^2}\right)\right],$$

$$N_5=\frac{ha_0}{c_sR_0}-\rho_0N_1\left(1-\frac{u_0}{c_s}\right)\left(\left(2+4\beta_0\right)\rho_0+\frac{ha_0}{R_0c_f^2}\frac{\beta_0c_s+u_0\left(1-\beta_0\right)}{c_s}\right)+$$
$$+\frac{u_0c_s-u_0^2\left(1-\beta_0\right)}{c_s}\left(\rho_0+\frac{2ha_0}{R_0c_f^2}\right),$$

$$S_{l+1}=a_1\mathit{\Gamma}_{n-1}-b_1a_0\mathit{\Gamma}_{m-1},n>m,\mathit{\Gamma}_{n-1}=0,n-l<0,\mathit{\Gamma}_{n-1}=1,n-l\geq 0.$$

Thus, the main determinant of the system of linear equations (2.19) with respect to $\partial u_2/\partial\tau$ and $\partial\xi_2/\partial\tau$ gives the variance relation (2.15), ie this determinant is equally zero, then according to Kroneker-Kappelli theorem we obtain the evolutionary equation from (2.19):

$$\frac{\partial\xi_1}{\partial x}+A_1\xi_1\frac{\partial\xi_1}{\partial\tau}-\frac{A_2}{\eta c_s}\sum_{l=1}^{n}\left(-1\right)^{l+1}S_{l+1}\frac{\partial^{l+1}\xi_1}{\partial\tau^{l+1}}-$$
$$-A_4\frac{\partial}{\partial\tau}\left(\frac{\partial}{\partial x}+\frac{1}{2\eta c_s}\frac{\partial}{\partial\tau}\right)\xi_1+A_6\frac{\xi_1}{\eta}+A_7\xi_1^2=0 \qquad (2.20)$$

Here,

$$b=c_s\left[4R_0+\frac{ha_0}{c_f^2\rho_0}+\frac{ha_0}{c_s^2\rho_0}-\left(1+\beta_0\right)\left(6R_0+\frac{ha_0}{c_f^2\rho_0}\right)\right]\frac{u_0^2}{c_f^2}\approx\frac{ha_0}{\rho_0c_s};A_2=\frac{h}{2c_sb}\approx\frac{1}{2a_0};$$

$$A_1=\frac{1}{b}\left[\left(10+4\frac{ha_0}{R_0c_f^2}+\frac{h^2a_0^2}{R_0c_f^4\rho_0^2}\right)\left(1-\frac{2u_0\left(1+\beta_0\right)}{c_s}\right)\left(1-\frac{u_0}{c_s}\right)-\frac{u_0}{c_s}\times\right.$$
$$\times\left(1-\frac{u_0\left(1+\beta_0\right)}{c_s}\right)\left(1+\frac{2ha_0}{R_0c_f^2\rho_0}\right)+$$
$$\left.+\left(4+\frac{2ha_0}{R_0c_f^2\rho_0}\right)\left(2\left(1+2\beta_0\right)+\frac{ha_0}{R_0c_f^2\rho_0}\left(\beta_0+\frac{u_0\left(1+\beta_0\right)}{c_s}\right)\right)\left(1-\frac{u_0}{c_s}\right)-\frac{ha_0}{R_0c_s^2\rho_0}\right]\approx$$
$$\approx\frac{1}{b}\left[\left(10-\frac{3ha_0}{R_0\rho_0c_f^2}+\frac{\left(1+\beta_0\right)h^2a_0^2}{\rho_0^2R_0^2c_s^4}\right);\right.$$

$$A_4=\frac{\mu}{c_sb}\left(1-\frac{u_0}{c_s}\right)\left(4+\frac{ha_0}{R_0\rho_0c_f^2}\right)\approx\frac{\mu}{\rho_0c_s^2};$$

$$A_6=-\frac{1}{b}\left[\frac{gR_0\sin\alpha}{c_s}\left(2+\frac{ha_0}{c_f^2R_0\rho_0}\right)-\frac{\lambda u_0}{2}\left(4+\frac{ha_0}{c_f^2\rho_0}\right)\left(1-\frac{u_0}{c_s}\right)\right]\approx$$

$$\approx \frac{gR_0 \sin\alpha}{c_s}\left(2+\frac{ha_0}{c_f^2 R_0\rho_0}\right);$$

$$A_7 = -\frac{\lambda c_s}{4R_0 b}\left(4+\frac{ha_0}{c_f^2\rho_0}\right)^2\left(1-\frac{u_0}{c_s}\right)^2+\frac{g\sin\alpha}{c_s}\left(1+\frac{2ha_0}{R_0\rho_0 c_f^2}\right)\approx$$

$$\approx -\frac{\lambda c_s}{4R_0 b}\left(4+\frac{ha_0}{c_f^2\rho_0}\right)^2+\frac{g\sin\alpha}{c_s}\left(1+\frac{2ha_0}{R_0\rho_0 c_f^2}\right).$$

According to the Busenesk approximation, it can be assumed that $\partial/\partial x \approx c_s^{-1}\partial/\partial\tau$ at the dispersion and dissipative limits. Taking this assumption into account, if we substitute $w = \eta A_1\xi_1 = A_1\xi$ and $X = x/\eta$, we can bring equation (2.20) to the following form:

$$\frac{\partial w}{\partial X}+w\frac{\partial w}{\partial\tau}-\frac{1}{a_0 c_s}\sum_{l=1}^{`n}(-1)^{l+1}S_{l+1}\frac{\partial^{l+1}w}{\partial\tau^{l+1}}-\frac{3\mu}{2\rho_0 c_s^2}\frac{\partial^2 w}{\partial\tau^2}-A_6 w+\frac{A_7}{A_1}w^2=0. \quad (2.21)$$

Thus, to describe the excitation during the flow of a weakly compressed fluid in an oscillating viscous elastic tube, a generalized nonlinear wave equation with nonlinearity and quadratic hydraulic resistance (2.21) is obtained.

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