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Investigation of waves propagation in a plane net

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Abstract. *On the basis of the equations of motion of the net, in the general case, the equations of motion in flat net are constructed. The purpose of this work is to study waves in plane net. The diagonal movement of the net is considered. In this case, the study examines the strong discontinuities in the plane, which occur when the edges of the mesh are stretched, and continuous waves, which occur when the edges are first stretched and then unloaded.*
The problem is solved by the method of characteristics and illustrated by calculations. The results are presented in the form of graphs and tables.

Keywords. net · wave · speed · deformation · diagonal movement

Mathematics Subject Classification (2010): 74J15, 74J05

1 Introduction

Actions of intensive short-term loads of mesh systems are usually encountered in many industries such as land and underwater nets of an obstacle, shock waves and wind gusts, seismic and various explosive loads of wide-span mesh overlapping, etc. Besides, mesh systems are successfully being deployed in distinct areas of the modern equipment, aircraft, fishery, construction, etc.

Problem of distribution of waves in deformable filamentous systems considering a significant deviation of a form of threads from initial rectilinear in the mathematical relation is a very challenging task as the equations of the movement represent the system of the nonlinear differential equations in private derivatives. The equations of the movement of a grid are extracted based on H.A. Rakhmatulin's equations about the movement of thread [7].

A research of an unloading wave in cylindrical net of not linearly elastic fibers is carried out in work [8]. An attempt of the solution of a task on continuous waves is initiated considering many options of distribution of waves in cylindrical nets.

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Articles [2] are concerned with the investigation of sets in rectangular Cartesian system of coordinates. Here waves in a cylindrical system of co-ordinates are explored. In works [1, 4, 5, 6, 8] a non-one-dimensional dynamic problem and set equilibrium were studied. To study the wave propagation, the equations of motion of the net in a rectangular Cartesian coordinate system and in natural coordinates are given. Equations of characteristic with local parameters were found in the grid, which made it possible to determine the type of wave.

2 The general equations of movement of a net

Planar net is being considered. The equation of one-dimensional movement is given in [7], where automodel movement of a semi- infinite set is investigated:

$$\begin{aligned} &\tfrac{\partial}{\partial z}\left(\sigma_1 \cos\gamma_1\right)\sin\alpha + \tfrac{\partial}{\partial z}\left(\sigma_2 \sin\gamma_2\right)\cos\alpha = 2\rho\tfrac{\partial^2 x}{\partial t^2} \\ &\tfrac{\partial}{\partial z}\left(\sigma_1 \sin\gamma_1\right)\sin\alpha + \tfrac{\partial}{\partial z}\left(\sigma_2 \cos\gamma_2\right)\cos\alpha = 2\rho\tfrac{\partial^2 y}{\partial t^2} \end{aligned} \tag{2.1}$$

$$\begin{aligned} &1+\tfrac{\partial x}{\partial z}\sin\alpha = (1+e_1)\cos\gamma_1;\ \tfrac{\partial y}{\partial z}\sin\alpha = (1+e_1)\sin\gamma_1; \\ &\tfrac{\partial x}{\partial z}\cos\alpha = (1+e_2)\sin\gamma_2;\ 1+\tfrac{\partial y}{\partial z}\cos\alpha = (1+e_2)\cos\gamma_2; \end{aligned} \tag{2.2}$$

where $\sigma_1,\ \ \sigma_2$ - the conditional tension determined as the sum of tension of separate threads of one family crossing the site of threads of other family, carried to the initial length of the considered element. $e_1,\ e_2$ - relative lengthenings, ρ- the mass of net having per unit area in an initial state $x,\ y$- coordinates of particles of net, t - time, $\gamma_1,\ \ \gamma_2$- angles of rotation of threads of the corresponding families (Fig. 1)

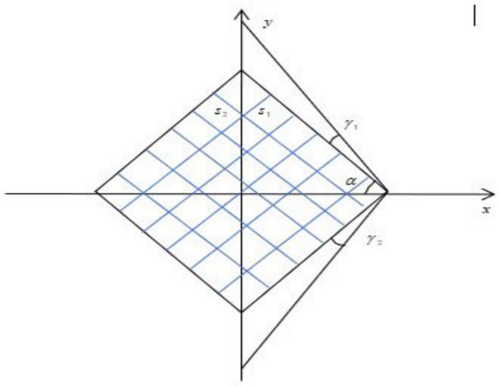

Fig. 1

$$z = s_1 \sin\alpha + s_2 \cos\alpha$$

where $s_1,\ s_2$- the Lagrangian coordinates of particles of threads counted from the chosen threads α- tilt angle of one of families to a counting straight line.

Next, diagonal movement of the net is being considered, i.e.$\alpha = \pi/4$.

Then the equations [1] will take a form:

$$\frac{\partial}{\partial z}\left[\sigma\left(\cos\gamma + \sin\gamma\right)\right] = 2\sqrt{2}\rho\frac{\partial^2 x}{\partial t^2} \tag{2.3}$$

From [2] follows

$$1+\frac{\partial x}{\partial z}\frac{\sqrt{2}}{2}=(1+e)\cos\gamma;\quad \frac{\partial x}{\partial z}\frac{\sqrt{2}}{2}=(1+e)\sin\gamma; \tag{2.4}$$

If to stretch the edge of net, then in the plane of net the strong gap will spread since, the speed of wave will, apparently, increase with stretching. If the set is previously stretched, and then unloads, on net will find continuous waves.

3 Stretching of set

Let the net with edge stretch (Fig. 2). D - speed of distribution of a strong gap, υ - speed of particles of net. According to the mass conservation law

$$D\rho_0=(D-\upsilon)\,\rho \tag{3.1}$$

and number of the movement

$$\vartheta\rho\,(D-\upsilon)=\rho_0 D\upsilon=-2\sigma\cos\gamma \tag{3.2}$$

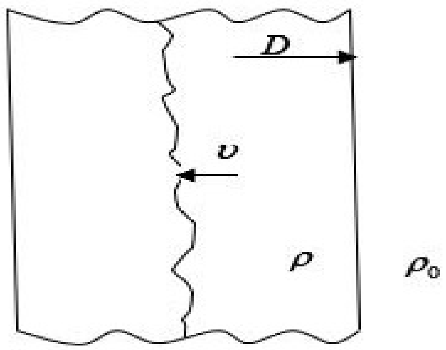

Fig. 2

If mass of an element of net dM (Fig. 3).

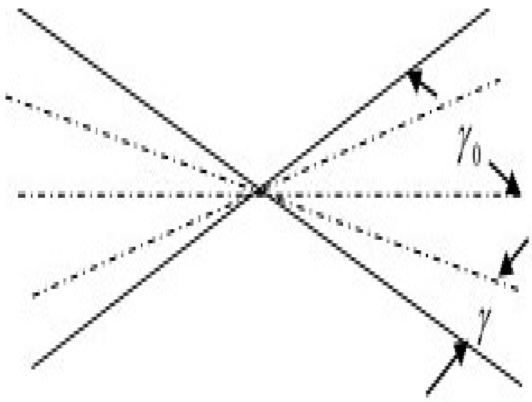

Fig. 3

that $\rho_0 = \frac{dM}{\cos\gamma_0 dz}$ and $\rho = \frac{dM}{(1+e)\cos(\gamma_0-\gamma)dz}$ or

$$\rho_0 = \frac{(1+e)\cos(\gamma_0-\gamma)}{\cos\gamma_0}\rho \tag{3.3}$$

From (3.1) and (3.3)

$$\upsilon = -\left[\frac{(1+e)\cos(\gamma_0-\gamma)}{\cos\gamma_0} - 1\right] D \tag{3.4}$$

From (3.2) and (3.4) follows

$$\frac{2\sigma\cos(\gamma_0-\gamma)}{\rho D} = D\left[\frac{(1+e)\cos(\gamma_0-\gamma)}{\cos\gamma_0} - 1\right]$$

Since $\sigma = Ee$, we will get

$$D^2 = a^2\frac{\cos\gamma_0\cos(\gamma_0-\gamma)}{(1+e)\cos(\gamma_0-\gamma)-\cos\gamma_0}e \tag{3.5}$$

where $a^2 = E \,/\, \rho_0$ considering (2.4),

$$(1+e)(\cos\gamma - \sin\gamma) = 1 \tag{3.6}$$

we will get

$$D = a\sqrt{\frac{\cos\gamma_0\cos(\gamma_0-\gamma)(1-\cos\gamma+\sin\gamma)}{\cos(\gamma_0-\gamma)-\cos\gamma\cos\gamma_0+\sin\gamma-\cos\gamma}} \tag{3.7}$$

In Fig. 4 the dependence of speed of a strong gap D from γ at $\gamma_0 = \frac{\pi}{4}$ is presented.

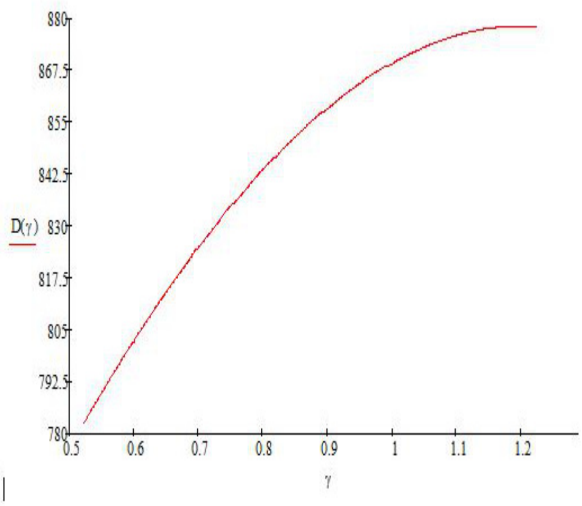

Fig. 4 Dependence of speed of a strong gap D from γ at $\gamma_0 = \frac{\pi}{4}$

4 Unloading wave in previously stretched net

For previously stretched net, correlations of (2.3) and (2.4) will take a form:

$$\sigma_1 \left(\cos\gamma_1 + \sin\gamma_1\right) = c \tag{4.1}$$

$$1 + \frac{\partial x}{\partial z}\frac{\sqrt{2}}{2} = (1+e_1)\cos\gamma_1; \qquad \frac{\partial x}{\partial z}\frac{\sqrt{2}}{2} = (1+e_1)\sin\gamma_1; \tag{4.2}$$

where $\sigma_1,\ \gamma_1,\ e_1$ - values of sizes in a static state.

From (4.1) and (4.2)

$\gamma_1 = \frac{1}{2}\arcsin\left(\frac{c^2}{\sigma_1^2} - 1\right)$ or $\gamma_1 = \frac{1}{2}\arcsin\left[1 - \frac{1}{(1+e_1)^2}\right]$

Semi-infinite net with initial data is being considered which is being unloaded from the edge $z = 0$.

The equation of the movement has a form of (2.3) and (2.4). Let's reduce to the equation with one unknown x.

From (2.4)

$$e = \sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2} - 1 \tag{4.3}$$

$$\cos\gamma = \frac{1 + \frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}}{\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}}; \qquad \sin\gamma = \frac{\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}}{\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}}$$

since $\sigma = Ee$

$$\sigma = E\left(\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2} - 1\right)$$

$$\frac{\partial \sigma}{\partial z} = E\frac{\sqrt{2}\frac{\partial^2 x}{\partial z^2} + 2\frac{\partial x}{\partial z}\frac{\partial^2 x}{\partial z^2}}{2\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}};$$

$$\frac{\partial \cos\gamma}{\partial z} = \frac{\frac{\sqrt{2}}{2}\frac{\partial^2 x}{\partial z^2}\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2} - \left(1 + \frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\right)\frac{\sqrt{2}\frac{\partial^2 x}{\partial z^2} + 2\frac{\partial x}{\partial z}\frac{\partial^2 x}{\partial z^2}}{2\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}}}{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}$$

$$\frac{\partial \sin\gamma}{\partial z} = \frac{\frac{\sqrt{2}}{2}\frac{\partial^2 x}{\partial z^2}\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2} - \frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\frac{\sqrt{2}\frac{\partial^2 x}{\partial z^2} + 2\frac{\partial x}{\partial z}\frac{\partial^2 x}{\partial z^2}}{2\sqrt{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}}}{1 + \sqrt{2}\frac{\partial x}{\partial z} + \left(\frac{\partial x}{\partial z}\right)^2}$$

From (2.3)

$$\frac{\partial \sigma}{\partial z}\left(\cos\gamma + \sin\gamma\right) + \sigma\frac{\partial \cos\gamma}{\partial z} + \sigma\frac{\partial \sin\gamma}{\partial z} = 2\sqrt{2}\rho\frac{\partial^2 x}{\partial t^2}$$

Having substituted expressions of derivatives from the previous formulas, we will get the equation of the movement in a form:

$$\left\{\begin{array}{c} \frac{\left(\sqrt{2}+2\frac{\partial x}{\partial z}\right)\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)}{2\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)}+\left(\sqrt{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2}-1\right)\times \\ \times\frac{\sqrt{2}}{2}\frac{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2+\left(1+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\right)\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)}{\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)^{\frac{3}{2}}}+ \\ +\left(\sqrt{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2}-1\right)\frac{\sqrt{2}}{2}\frac{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)}{\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)^{\frac{3}{2}}} \end{array}\right\}\frac{\partial^2 x}{\partial z^2}=\frac{4\rho}{E}\frac{\partial^2 x}{\partial t^2}$$

or

$$a^2\frac{\partial^2 x}{\partial z^2}=\frac{\partial^2 x}{\partial t^2};\quad\left(a^2=\frac{E}{4\rho}\right)\tag{4.4}$$

$$a^2=\frac{\sqrt{2}\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)^2}{2\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)}+\left(\sqrt{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2}-1\right)\times$$

$$\times\frac{\sqrt{2}}{2}\frac{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2+\left(1+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\right)\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)}{\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)^{\frac{3}{2}}}+$$

$$+\left(\sqrt{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2}-1\right)\frac{\sqrt{2}}{2}\frac{1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)}{\left(1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2\right)^{\frac{3}{2}}}$$

$$b=1+\sqrt{2}\frac{\partial x}{\partial z}+\left(\frac{\partial x}{\partial z}\right)^2$$

$$a^2=\frac{\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)^2 b^{\frac{1}{2}}+\left[1+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}+\frac{5}{2}\left(\frac{\partial x}{\partial z}\right)^2\right](b-1)}{b^{\frac{3}{2}}}$$

$$a=\sqrt{\frac{\left(1+\sqrt{2}\frac{\partial x}{\partial z}\right)^2 b^{\frac{1}{2}}+\left[1+\frac{\sqrt{2}}{2}\frac{\partial x}{\partial z}+\frac{5}{2}\left(\frac{\partial x}{\partial z}\right)^2\right](b-1)}{b^{\frac{3}{2}}}}$$

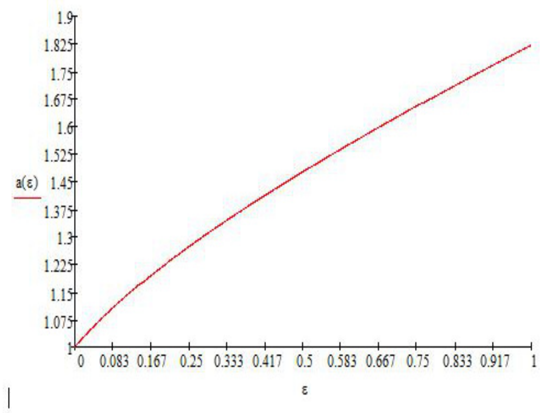

Fig. 5 $\left(\varepsilon=\frac{\partial x}{\partial z}\right)$

The equations of the movement of net come down to the quasi linear equation with private derivatives of the second order with coefficients dependent on $\frac{\partial x}{\partial z}$.

In a task of initial deformation from (4.2) follows

$$\left.\frac{\partial x}{\partial z}\right|_{\substack{t=o\\ z=0}} = \frac{-1+\sqrt{-1+2(1+e_1)^2}}{\sqrt{2}} \tag{4.5}$$

On border $z = 0$, e_1 therefore, $\frac{\partial x}{\partial z}$ kills. On border $z = 0$ the net unloads. Characteristic of the equation (4.3) have a form

$$dz = \pm a dt \tag{4.6}$$

Conditions on characteristics

$$dx_t = \pm a dx_z; \qquad \left(\frac{\partial x}{\partial s} = x_z \, \frac{\partial x}{\partial t} = x_t\right) \tag{4.7}$$

The front of an unloading wave moves with a speed $a\,(e_1)$. In the area SOA (Fig. 6) in the rest condition.

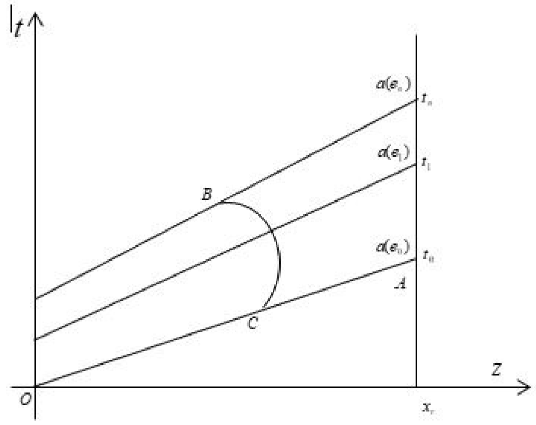

Fig. 6

From a condition on negative characteristics BC follows $x_t = -\int_{x_z'}^{x_z} a dx$.

Differentiating in the direction of positive characteristic we have $dx_t = -a dx_z$. When comparing results we have, $x_t = const$, $x_z = const$, , i.e. on positive characteristics x_t and x_z are constant. Considering constancy x_z on characteristic of a positive inclination, from (4.6) we have:

$$z = a(t - t_0) \tag{4.8}$$

At $z = 0$ we choose t_0 and define we x_z . From (4.8)

$$t_0 = t - \frac{z}{a}$$

and respectively,

$$x_t = x_t^0\,(t_0) \text{ or } x_t = x_t^0\left(t - \frac{z}{a}\right) \tag{4.9}$$

Let the net unload with a speed of x_t on border $z = 0$. From (4.7) for negative characteristic

$$x_t = -\int_{x_z'}^{x_z} a\,(x_z)\,dx_z \tag{4.10}$$

Equation (4.10) is an equation for determining the axial deformation of the set x_z (in contrast to the deformation of the fibers e).

Approximately having presented integral (4.10) in the form of the sum we have

$$x_t \int_{x_z}^{x'_z} a(x_z) dx_z$$

$$\begin{array}{l} x_t^0 = a(x_z^1) \Delta x_z \\ x_t^1 = \left(a(x_z^1) + a(x_z^2)\right) \Delta x_z \\ x_t^2 = \left(a(x_z^1) + a(x_z^2) + a(x_z^3)\right) \Delta x_z \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_t^n = \left(a(x_z^1) + a(x_z^2) + a(x_z^3) + ... + a(x_z^n)\right) \Delta x_z \end{array}$$

or

$$x_t \cong f(x_z)$$

i.e. the opposite dependencies of $x_z \to x_t$ on border. Since positive characteristics are rectilinear, it is possible to define x_z in all area of the movement. Functional dependencies of speed of the movement - the speed of a wave and deformation are illustrated in the Table 1:

Table 1.

ε_0	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9	ε_{10}	ε_{11}
0.778	0.770	0.762	0.754	0.746	0.738	0.730	0.722	0.714	0.706	0.698	0.690
$e(\varepsilon_0)$	$e(\varepsilon_1)$	$e(\varepsilon_2)$	$e(\varepsilon_3)$	$e(\varepsilon_4)$	$e(\varepsilon_5)$	$e(\varepsilon_6)$	$e(\varepsilon_7)$	$e(\varepsilon_8)$	$e(\varepsilon_9)$	$e(\varepsilon_{10})$	$e(\varepsilon_{11})$
0.1	0.094	0.089	0.083	0.078	0.072	0.067	0.061	0.056	0.050	0.045	0.040
x_t^0	x_t^1	x_t^2	x_t^3	x_t^4	x_t^5	x_t^6	x_t^7	x_t^8	x_t^9	x_t^{10}	x_t^{11}
747	1.493x 10^3	2.237 x10^3	2.980 x10^3	3.721 x10^3	4.461 x10^3	5.199 x10^3	5.936 x10^3	6.672 x10^3	7.406 x10^3	8.139 x10^3	8.87 x10^3
$a(\varepsilon_0)$	$a(\varepsilon_1)$	$a(\varepsilon_2)$	$a(\varepsilon_3)$	$a(\varepsilon_4)$	$a(\varepsilon_5)$	$a(\varepsilon_6)$	$a(\varepsilon_7)$	$a(\varepsilon_8)$	$a(\varepsilon_9)$	$a(\varepsilon_{10})$	$a(\varepsilon_{11})$
$7.753 \cdot 10^4$	$7.732 \cdot 10^4$	$7.711 \cdot 10^4$	$7.689 \cdot 10^4$	$7.667 \cdot 10^4$	$7.644 \cdot 10^4$	$7.621 \cdot 10^4$	$7.598 \cdot 10^4$	$7.575 \cdot 10^4$	$7.551 \cdot 10^4$	$7.526 \cdot 10^4$	$7.502 \cdot 10^4$

$$x_t = f\left(x_z^0 - x_z\right)$$

Calculated values of the utilized parameters for $\gamma_0 = \frac{\pi}{4}$

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