Tran. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.
Mechanics, **41** (7), 7–13 (2021).

Effect of the features of the Earth's crust on wave processes during earthquake

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Received: 27.05.2021 / Revised:21.06.2021 / Accepted: 02.08.2021

Abstract. *The article examines the reasons for the features of the occurrence of an earthquake in one area (Lenkoran, AR). As a result, an interesting outcome was obtained that the upper layer of objects located on the surface of the liquid, does not experience the longitudinal impact load.*

Keywords. Lenkoran · earthquake · elasticity · fluid · layer.

Mathematics Subject Classification (2010): 74J05, 76N30

1 Introduction

Usually during earthquakes there are two consecutive impulses. The first corresponds to the type of longitudinal waves and appears in the form of oscillatory movements in a plane parallel to the surface of the earth with increasing amplitude. The second is a single-instantaneous impulse, most likely corresponding to the surface Rayleigh waves, in which the movement vertical, i.e. perpendicular to the surface of the earth.

We observed that in the city of Lenkran in the south of Republic of Azerbaijan during earthquakes the first type of waves is not felt at all even for high-magnitude earthquakes which happened several years ago in this area.

In an attempt to find the reason, we turned to the fact of the peculiarities of the structure of the earth's crust in this area. And it differs in that there are a lot of water wells here, and the level of groundwater is quite close to the surface. This level ranges from 3 to 5 meters above the surface of the earth.

Taking into account these features and the fact that Lankaran is located between the Caspian Sea and the Talysh mountains, and that usually the epicenters of shocks are located on the seabed, the problem was posed of the non-stationary dynamics of an elastic semi-infinite layer, the lower part of which borders on a compressible ideal fluid (Fig. 1). The fluid movement is considered potential, i.e. irrotational. Some or all of the end face of the layer is subjected to impact.

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To solve this problem, we used some results in [1], devoted to the study of the dynamics of rectangular prisms, from the standpoint of the exact three-dimensional theory of elastodynamic. However, in the present problem the existence of media of different types bordering each other significantly complicates the solution process. We propose a new method for determining the originals of functions — transformations, which in the present work have a very complex form; they are represented through determinants of the fifth rank. In a sense, this method is a generalization of a similar method first proposed in [1], and later for axially symmetric cases in [2].

We developed an exact solution valid in the initial short time of the process, which also provides a good picture of the whole process for subsequent times. The results confirm with high accuracy our hypothesis outlined above.

2 Statement and method of solution

Taking into account the location of the city of Lenkoran of the Republic of Azerbaijan, the problem under consideration is modelled as follows.

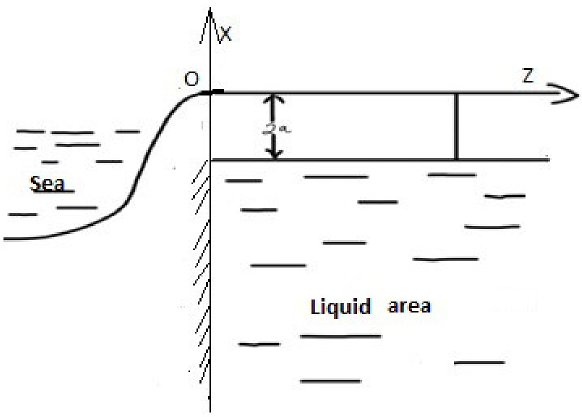

Fig. 1

An elastic, semi-infinite layer, $2a$ thick, is located on the surface of an ideal compressible liquid area of infinite depth (Fig.1). The existence of an impermeable wall is assumed at the boundary of the liquid region $z = 0$. It is assumed that the impact is applied to the end region of the layer, and the movement of the fluid is considered potential. Under these assumptions, the problem posed can be formulated as the following initial-boundary value problem for a given construction, consisting of two different environments.

$$\rho \frac{\partial U}{\partial t^2} = (\lambda + \mu)\, grad\, div \overrightarrow{U} - \mu \Delta \overrightarrow{U}$$

$$\overrightarrow{U} = \overrightarrow{U}\,(u, w) \tag{2.1}$$

$$u = w = 0,\ \frac{\partial u}{\partial t} = \frac{\partial w}{\partial t} = 0,\ \text{when } t = 0 \tag{2.2}$$

$$\sigma_{zz} = \sigma_0 \cdot f\,(t)\,, u = 0 \ \text{ when } \quad z = 0 \tag{2.3}$$

$$\sigma_{xx} = \sigma_{xz} = 0 \ \text{ when } x = 0$$

In what follows, we will assume that $f(t) = H(t)$, where $H(t)$ is the Heaviside function.

At the boundary between the liquid region and the layer, the following conditions take place:

$$\sigma_{xx} = -\rho_0 \ \frac{\partial \varphi}{\partial t} \ \sigma_{xz} = 0 \ \text{ when } x = -2a \tag{2.4}$$

$$\frac{\partial \varphi}{\partial x} = u$$

and

$$\frac{\partial \varphi}{\partial z} = O \qquad \text{when} \qquad z = 0$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{1}{a_0^2} \cdot \frac{\partial^2 \varphi}{\partial t^2} \tag{2.5}$$

Here $\overrightarrow{U} = \overrightarrow{U}\,(u, w)$ is the vector of displacement of the elastic layer, λ and μ are the Lamé coefficients, a_0 is the speed of propagation of sound waves in a liquid medium, the motion of which is described by a potential function – φ, ρ, ρ_0 are the densities of the layer and the liquid, respectively, t is time.

To solve this system, a similar method will be applied, which was developed and used in [1]. Thanks to this method, the system of Lamé equations is reduced to the simplest system of inhomogeneous Helmholtz equations, in the right-hand sides of which there are boundary functions of impact loading. This method involves the application of double integral transforms, along with the method of replacing the sought functions, which leads to the above-mentioned excellent result. However, this fact does not yet relieve us of the difficulty associated with the transition from transformations to originals. And to overcome these difficulties, the most universal method is also proposed there - for finding the originals of double integral transformations.

So, using the ready-made equations of this work for two-dimensional motion and after simple calculations in relation to the equation of motion of the liquid part of this structure, we can obtain the following algebraic system of linear equations to determine five unknown constants featuring in the composition of new potential functions:

$$\left\{ \begin{matrix} C_{01} \\ C_{02} \\ A_{01} \\ A_{02} \\ g_0 \end{matrix} \right\} \cdot \{D\} = \left\{ \begin{matrix} 0 \\ \Omega \cdot q^2 \\ 0 \\ \Omega \cdot q^2 \\ 0 \end{matrix} \right\} \text{ where } \Omega = -\frac{\sigma_0}{(\lambda + 2\mu)\, \nu_1^2 q} \tag{2.6}$$

Here $\{D\}$=$e^{2a\nu_1} \cdot e^{2a\nu_2} \cdot e^{-2a\nu_0} \cdot D_0$ $\{D_0\}$ is the matrix of 5^{th} rank:

$$\{D_0\} = \{a_{ik}\}$$

$$a_{11} = 2q\nu_1$$

$$a_{21} = \left(1 + \frac{2\mu}{\lambda}\right) \nu_1^2 - q^2$$

$$a_{31} = 2q\nu_1 e^{-2a\nu_1}$$

$$a_{41} = \left[\left(1 + \frac{2\mu}{\lambda}\right) \nu_1^2 - q^2\right] e^{-2a\nu_1}$$

$$a_{51} = p\nu_1 e^{-2a\nu_1}$$

$$a_{12} = -2qv_1 e^{-2a\nu_1}$$

$$a_{22} = \left[\left(1+\frac{2\mu}{\lambda}\right)\nu_1^2 - q^2\right] e^{-2a\nu_1}$$
$$a_{32} = -2q\nu_1$$
$$a_{42} = \left(1+\frac{2\mu}{\lambda}\right)\nu_1^2 - q^2$$
$$a_{52} = p\nu_1$$
$$a_{13} = -\left(q^2+\nu_2^2\right)\nu_2$$
$$a_{23} = \frac{2\mu}{\lambda} q\nu_2^2$$
$$a_{33} = \left(q^2+\nu_2^2\right)\nu_2 \times e^{-2a\nu_2}$$
$$a_{43} = \frac{2\mu}{\lambda} q\nu_2^2 e^{-2a\nu_2}$$
$$a_{53} = pq\nu_2 e^{-2a\nu_2}$$
$$a_{14} = \left(q^2+\nu_2^2\right)\nu_2 e^{-2a\nu_2}$$
$$a_{24} = -\frac{2\mu}{\lambda} q\nu_2^2 e^{-2a\nu_2}$$
$$a_{34} = \left(q^2+\nu_2^2\right)\nu_2$$
$$a_{44} = -\frac{2\mu}{\lambda} q\nu_2^2$$
$$a_{54} = pq\nu_2$$
$$a_{15} = 0$$
$$a_{25} = 0$$
$$a_{35} = 0$$
$$a_{45} = \rho_0\, p\, \frac{1}{\lambda}$$
$$a_{55} = \nu_0$$

The following designations are adopted here:

$$\nu_k = \sqrt{\left(\frac{p^2}{c_k^2}+q^2\right)}\; k=1,2 \qquad \text{and } \nu_0 = \sqrt{\left(\frac{p^2}{c_0^2}+q^2\right)},$$

$c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$, $c_2 = \sqrt{\frac{\mu}{\rho}}$ - are the velocities of propagation of longitudinal and transverse waves in the layer material and it is evident that

$$\dot{\tilde{\varphi}} = g_0 e^{x\nu_0}$$

The longitudinal velocity transformation on the free surface of the layer, at $x = 0$ according to [1], is expressed by the following formula:

$$\dot{\tilde{w}} = -\frac{\sigma_0}{(\lambda+2\mu)\cdot\nu_1^2} + C_{01}\cdot q + C_{02}\cdot q - \nu_2^2 A_{01} - \nu_2^2 A_{02} \tag{2.7}$$

Thus, the solution is completely defined in the variable parameters of the transformations. However, it is expressed through the determinants of the 5^{th} rank, and therefore, finding the originals in the conventional ways is almost impossible. In such cases, it is appropriate to apply the method based on [1]. The principle on which this method is based becomes more relevant when we deal with very complex transformation functions

Above all, according to the method above, it is necessary to determine the behaviour at infinity (when $p \to \infty$) of that part of the transformations that need to be expanded in convergent series by functions [1] (ν_k^n) in the vicinity of the same point.

First, we may take the main determinant $\{D_0\} = |a_{ik}|$ and modify it as follows; on the right-hand side of the equality, we put 0 in those places where the expression $e^{-2a\nu_k}$, $k = 1, 2$ appears, since all the items with the participation of these terms, formed during the disclosure of this determinant, in the sum approach zero faster than any power ν_1^{-n}.

In this case, the expression for $\{D_0\}$ is noticeably simplified, and it only has eight terms of the total. Of these 8 terms, we keep the term that has the highest power at infinity $p \to \infty$:

$$|D_0| \approx -\Omega q^2 \cdot a_{21} \cdot a_{13} \cdot a_{34} \left\{ \left(a_{55} \cdot a_{42} - a_{45} \cdot a_{52} \right) \right\}$$

In the same way, we define the principal terms and other determinants $|D_n| \cdot e^{-2a\nu_0}$ ($n = 1, 2, 3, 4$), formed from system (2.6), according to Cramer's rule for determining the constants $C_{01}, C_{02}, A_{01}, A_{02}$..

$$|D_1| \approx -\Omega q^2 \cdot a_{13} \cdot a_{34} \left\{ \left(a_{55} \cdot a_{42} - a_{45} \cdot a_{52} \right) \cdot e^{2a\nu_1} \cdot e^{2a\nu_2} \right.$$

$$|D_2| \approx -\Omega q^2 \cdot a_{21} \cdot a_{13} \cdot a_{34} \cdot a_{55} \cdot e^{2a\nu_2}$$

$$|D_3| \approx -\Omega q^2 \cdot a_{11} \cdot a_{34} \left\{ \left(a_{55} \cdot a_{42} - a_{45} \cdot a_{52} \right) \cdot e^{2a\nu_1} \cdot e^{2a\nu_2} \right.$$

$$|D_4| \approx -\Omega q^2 \cdot a_{21} \cdot a_{13} \cdot a_{32} \cdot a_{55} \cdot e^{2a\nu_1}$$

For short intervals of time, during which the impact load acts, the behaviour of the ratios $\frac{D_k}{D_0}$ ($k = 1, ..4$) at infinity, of course, will be determined mainly from the ratios of the very same terms of the highest power; then:

$$A_{01} = \frac{\Omega q^2}{a_{21}}$$

$$A_{02} = \frac{\Omega q^2 \cdot a_{12} \cdot a_{55} \cdot e^{-2a\nu_1}}{a_{21} \cdot \left(a_{55} \cdot a_{42} - a_{45} \cdot a_{52} \right)}$$

$$A_{01} = \frac{\Omega q^2 \cdot a_{11}}{a_{21} \cdot a_{13}}$$

$$A_{02} = -\frac{\Omega q^2 \cdot a_{32} \cdot a_{55} \cdot e^{-2a\nu_2}}{a_{34} \cdot \left(a_{55} \cdot a_{42} - a_{45} \cdot a_{52} \right)}$$

Plugging the values of these constants in the formulas (2.7), we obtain the expression of the desired solution in the parameters of the transformations. Employing existing analytical methods and tables provided in [3], it is easy to determine the double originals of these transformations.

$$1) - \frac{\sigma_0}{(\lambda + 2\mu) \cdot \nu_1^2} - \frac{\sigma_0 \cdot \tilde{n}_1}{(\lambda + 2\mu)} \cdot H \left(t - \frac{z}{\tilde{n}_1} \right)$$

$$2) C_{01} \cdot q = \frac{\Omega q^3}{a_{21}} = - \frac{\sigma_0 \cdot q^2}{(\lambda + 2\mu) \cdot \nu_1^2} \cdot \frac{1}{\left(1 + \frac{2\mu}{\lambda} \right) \nu_1^2 - q^2} =$$

$$= -\frac{\sigma_0 \cdot \tilde{n}_1^2}{(\lambda + 2\mu)} \left[\frac{H\left(t - \frac{z}{\sqrt{2}\tilde{n}_2}\right)}{\sqrt{2} \cdot \tilde{n}_2} - \frac{H\left(t - \frac{z}{\tilde{n}_1}\right)}{\tilde{n}_1} \right].$$

$$-\nu_2^2 A_{01} = -\nu_2^2 \cdot \frac{\Omega q^2 \cdot a_{11}}{a_{21} \cdot a_{13}} =$$

$$= \frac{\sigma_0 \cdot \nu_2^2}{(\lambda + 2\mu) \cdot \nu_1^2} \cdot \frac{2q^2 \nu_1}{[\left(1 + \frac{2\mu}{\lambda}\right) \nu_1^2 - q^2] \cdot [\left(q^2 + \nu_2^2\right) \nu_2]} \leftrightarrow$$

$$\leftrightarrow \frac{\sigma_0 \cdot (\tilde{n}_1^2 - 2c_2^2)}{(\lambda + 2\mu)} \cdot \frac{\tilde{n}_1}{\tilde{n}_2} \cdot \frac{H\left(t - \frac{z}{\sqrt{2}\tilde{n}_2}\right)}{\sqrt{2} \cdot \tilde{n}_2}.$$

Please note that the last formula uses the obvious approximation.

$$\frac{\nu_2}{\nu_1} \approx \frac{c_1}{c_2} \text{ when } p \to \infty$$

As can be observed from these formulas, their sum is identically equal to zero for the relation $c_1 = 2c_2$, which is valid for the value of Poisson's ratio $\nu = \frac{1}{3}$. and that for most materials this value is the same.

This is an unusually striking result, confirming the high accuracy of the choice of the cause of the phenomenon under study, because the main tone in the formation of longitudinal movements on the surface of the layer is made by precisely these components in expressions (2.7). The other two components in (2.7) represent diffraction waves arriving from the lower side in contact with the liquid. The insignificance of these waves is obvious; nevertheless, Fig. 2 shows the distribution of the longitudinal velocity on the free surface of the layer, what essentially proves this statement.

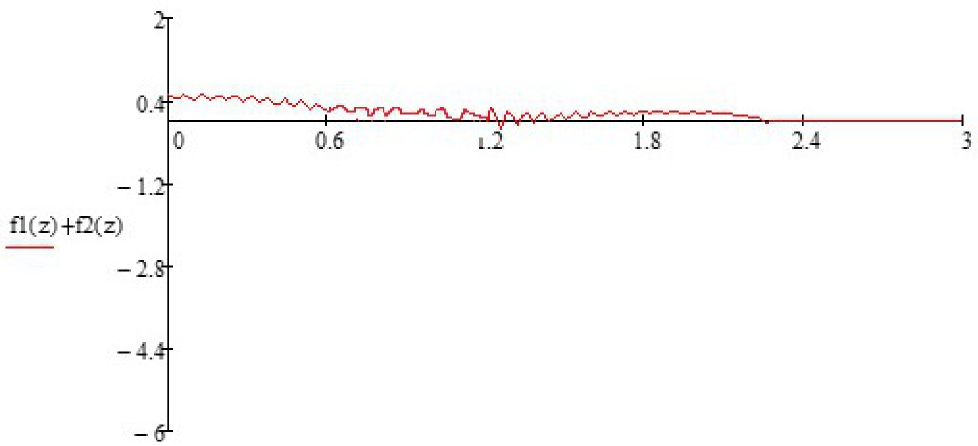

Fig. 2 Distribution of longitudinal velocity on the surface $x = 0$ for the moment

$$t = 3\frac{a}{c_1}$$

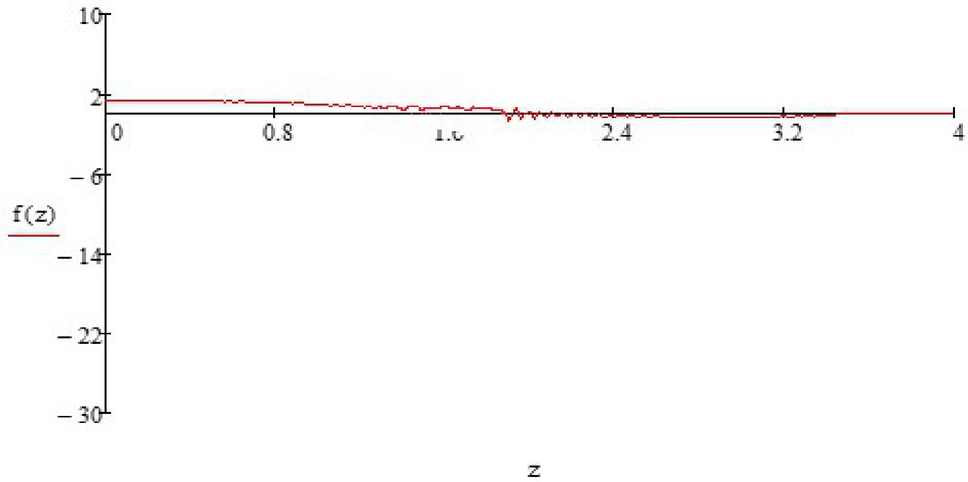

Fig. 3 Distribution of longitudinal velocity on the surface $x = 0$ for the moment $t = 4\frac{a}{c_1}$

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