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Development of approximate methods for determinatıon of stability in displaced and displacing systems wıth different rheophysical properties

Ibrahim J. Mamedov

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Abstract. *The issues of stability of solutions obtained during the study of fluid displacement in a porous medium are of great importance. In order to increase productivity in oil recovery, viscous-plastic, viscous-elastic and and other liquids having complex rheological properties in water injection are used. In these cases, giving a complete hydrodynamic description of the process becomes practically impossible due to the complexity of studying the continuity of displacement. Studies were carried out in this direction, considering the importance of the assessment of the stability of the boundary flow between displacement and displacing liquids. The study of the resistance of hydrocarbon liquid to displace by water and foam was carried out here. The proposed approach formulated the problem of stability in the movement of the separation boundary for various states of liquids.*

Keywords. displacement · porous medium · fingering · viscous-elastic fluids · foam · oil recovery

Mathematics Subject Classification (2010): 76M25

1 Introduction

The fluids flow in non-homogeneous porous medium expressed as an extremely complex system's flow. This creates a surface with a complex structure distribution in porous media, especially when two immiscible liquids flow together during the displacement process.

In this regard, the method of hydrodynamic description of the displacement process is changed depending on the current flow forces and impact ratios.

The Bakley-Leverett scheme of two-phase flow theory, written for a layer covering a large number of pores compared to its dimensions, does not fully cover the process. The Bakley-Leverett scheme correctly reflects the displacement process on a relatively small scale. In fact, the process creates principled difficulties. Difficulties are associated with

Ibrahim J. Mamedov
Institute of Mathematics and Mechanics of NAS Azerbaijan
Baku, Azerbaijan

E-mail: imamedoff1997@list.ru

the heterogeneity of reservoir, permeability, almost chaotic distribution, which leads to the creation of an extremely complex displacement front in the form of "fingering".

For displacing and displaced liquids on a layer scale in the hydrocarbon saturated formations, the filtration theory of the homogeneous system is determined by studying the flow of the boundary layer between them.

Here, the issues of stability of solutions obtained during the study of fluid displacement in a porous medium are of great importance.

The occurrence of physical instability is due to the fact that as a result of arbitrary influences, the penetration of more mobile fluid particles into a less mobile fluid arises under the influence of the difference in pressures that make it move, where the movement of particles is accelerated.

If the displacement fluid has a higher mobility, this leads to a displacing of the displacement fluid.

As a result of such an elementary approach [3], the same sustainability conditions that fit into a more rigorous theory can be obtained.

In the research on stability, the change in the range of viscosity relations and the issues of stability in the displacement of the liquids involved have not been fully resolved. This problem arises as it becomes difficult to carry out research with continuous increase $(\sim \sqrt{t})$ of the width of the transition zone over time and change of conditions at the border during displacement of mainly mixed liquids.

Recently, in order to increase productivity in oil recovery, viscous-plastic, viscous-plastic and other liquids having complex rheological properties in water injection are used [4, 7, 9]. In these cases, giving a complete hydrodynamic description of the process becomes practically impossible due to the complexity of studying the continuity of displacement.

Therefore, studies were carried out in this direction, considering the importance of the assessment of the stability of the boundary flow between displacement and displacing liquids. The study of the resistance of hydrocarbon liquid (transformate oil) to displace by water and foam was carried out here.

The proposed approach formulated the problem of stability in the movement of the separation boundary for various states of liquids [2]. A number of works have been reviewed here, which are analyzed in detail in the simplest porous medium - glass crack model [1, 5, 6, 8].

At the first stage, the porous medium was saturated with viscous hydrocarbon liquid, and after a while it began to inject water into the medium to liquefy it.

Experiments to study the structure of the displacement front of hydrocarbon liquids with water were carried out on a two-dimensional transparent model from a porous medium. The working part of the model consists of two glass boards, which are tightly pressed to each other with rough (protruding) surfaces and are selected with smoothness, forming the porous cavity of the model. The roughing profile is determined by the size of the "grinding" and the duration of the work of the board.

In experiments, glass boards with dimensions of $0,2 \times 0,4$ m were used, thoroughly cleaned, washed and dried with water.

2 Laboratory experiments

The experiments are carried out in the range of fluid flow rate of $3,4 \cdot 10^{-7}$- $5,1 \cdot 10^{-5}$ $\mathrm{m/s}$.

The permeability $m \approx 0.39$ and porosity of the measured model $K \sim 20 \cdot 10^{-12}$ m^2 have been evaluated. In the process of compaction, the entire flow area is divided into two parts. It is assumed that only "hydrocarbon fluid" is contained in one of them with water. Here, let's look at the water permeability of the medium K_w and its oil permeability K_n, the rectilinear displacement of water from a homogeneous medium with m - porosity.

The viscosity ratios of the circulating and displaced liquids were varied from 1,2 to 14,0. The surface tension between the liquids was equal to $6,2 \cdot 10^{-3}$. The formation of capillary forces in pore channels the two-phase flow zone on the displacement front (transition zone) did not exceed 6 - 11 pore lengths in experiments.

The results of studies taken at various displacement speed are shown in Fig. 2.1 – 2.3.

Here is shown the structure of the boundary layer of displacement of transformator oil with water with the displacement flow rate of $Q = 0,3$ mm^3/s, $Q = 0,4$ mm^3/s, $Q = 0,5$ mm^3/s, , $\mu^* = \mu_0/\mu_\omega = 17$.

Fig. 2.1

Fig. 2.2

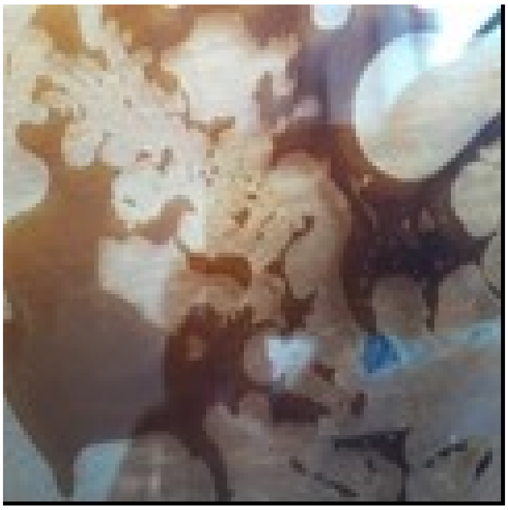

Fig. 2.3

Apparently, when the porous medium of a viscous fluid (“transformer oil”) is displaced with less viscous water, the separating boundary (contact) between the liquids becomes unstable, as a result of which the development of various “fingers” occurs.
This occurs due to the clogging of the fluid by capillary forces [6].

3 Solution method

At the early stage of the formation of ”fingers”, due to the arbitrary character of the pores, they are formed having different wave numbers. This produces effects that have the highest growth rate in the displaced fluid (Fig. 2.1 and 2.2). The highest growth rate is observed at maximum in the effect from the largest wavelength [5].

$$\lambda_m = 2\pi\sqrt{3}\left[\frac{K_\omega \sigma_e}{\upsilon_0 \mu_\omega}\left(M_c - 1\right)\right]^{0,5} \tag{3.1}$$

From the Darcy law and the discontinuity equation for pressure distribution, we can write the following:

$$\frac{\partial^2 P_w}{\partial x^2} = 0, \qquad 0 < x < l\ (t) \tag{3.2}$$

$$\frac{\partial^2 P_h}{\partial x^2} = 0, \qquad l\ (t) < x < L \tag{3.3}$$

$$\left.\begin{array}{l} P_h = P_w = P_0 \\ \gamma_h = \gamma_w\,, \qquad or \\ \frac{K_w}{\mu_w}\frac{\partial P_w}{\partial x} = \frac{k_h}{\mu_h}\frac{\partial P_h}{\partial x} \end{array}\right\} \quad x = l\ (t) \tag{3.4}$$

$$P_w = P_1, \qquad x = 0 \tag{3.5}$$

$$P_h = P_2, \qquad x = L \tag{3.6}$$

Conditional (3.4) are the conditions for the performance of pressure discontinuities and material balance when crossing the boundary flow.

Equations (3.2), (3.3) under conditions of (3.4), (3.5), (3.6) provide testimony for pressure distribution in oil and water areas:

$$P_w = P_1 - \frac{\Delta P}{\phi L + (1-\phi)\, l\ (t)} x \tag{3.7}$$

$$P_h = P_2 - \frac{\phi\, \Delta P}{\phi\, L + (1-\phi)\, l\ (t)}\, (x - L) \tag{3.8}$$

Here: $\Delta P = P_1 - P_2$- the difference in pressure written for the sample.

$\phi = \frac{K_w}{K_h} \frac{\mu_h}{\mu_w}$- mobility factor.

Therefore, the separating surface coordinates for the flow speed (the speed of particles of the liquid located in the "front") are true in the form of the following equalities:

$$\upsilon_0 = \frac{dl}{dt} = \frac{\upsilon_w}{m}$$

Then we get from (3.7):

$$\frac{dl}{dt} = \frac{K_w}{m\mu_w} \cdot \frac{\Delta P}{\phi L + (1-\phi)\, l\ (t)} \tag{3.9}$$

Let's assume that the coordinate changes in particles from arbitrary influence are changed in the front ε (in principle, very little effect $\varepsilon << l$)
Now we are investigating when ε impact conditions (resistance conditions) will turn off (resistance conditions)

$$\frac{d\varepsilon}{dt} = \frac{d\,(l+\varepsilon)}{dt} - \frac{dl}{dt}$$

$$\frac{d\varepsilon}{dt} \approx \frac{K_w \Delta P\,(\phi - 1)\,\varepsilon}{m\mu_w\,[\phi L + (1-\phi)\ l]^2} \tag{3.10}$$

If $\frac{d\varepsilon}{dt} < 0, \ \ \phi < 1$, then $\frac{d\varepsilon}{dt} < 0$ from (3.10) is taken
or

$$\frac{K_w}{\mu_w} < \frac{K_h}{\mu_h} \tag{3.11}$$

That is, if the mobility coefficient of the displaced fluid is less than the mobility coefficient of the compressible fluid, then the formation of "fingers" occurs on the front, and the flow does not stagnate.
In the second stage, let's consider displacing fluid at a constant speed for the case when it is recumbent and the displacement fluid is viscous.
Here, the displacement of the hydrocarbon "oil" with a stable foam is considered. Sulfanol was used as a surfactant for foaming. $Q_1 = 0,3\ \mathrm{mm}^3/\mathrm{san}$, $Q_2 = 0,4\ \mathrm{mm}^3/\mathrm{san}$, $Q_3 = 0,5\ \mathrm{mm}^3/\mathrm{san}$ was adopted in accordance with the first stage of the studies. The received naps are shown in Figure 3.1 – 3.3.

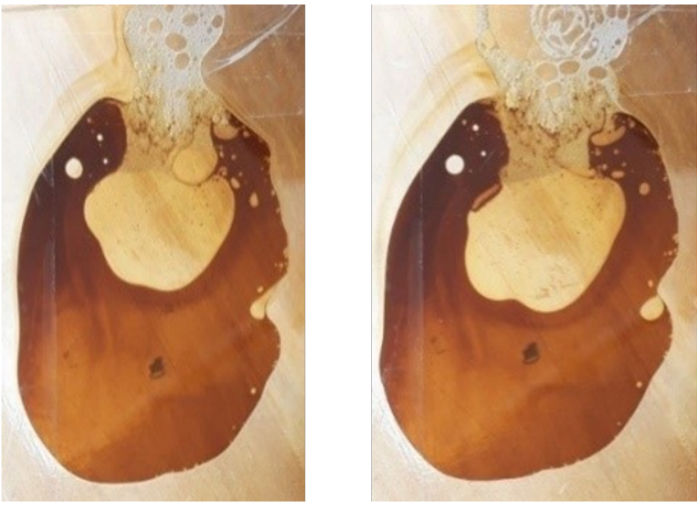

Fig. 3.1

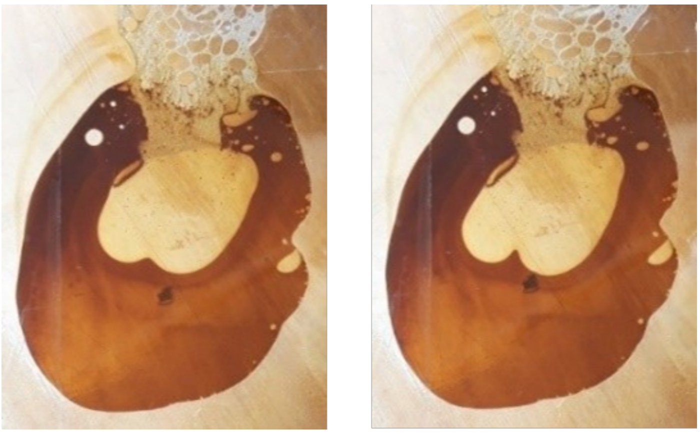

Fig. 3.2

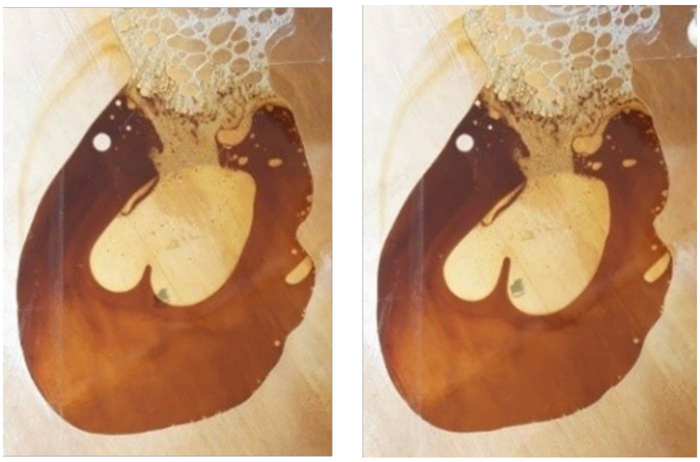

Fig. 3.3

As can be seen from the pictures, in contrast to displacement with water in displacement with foam, which is an elastic system, fractality at the border is eliminated.

Consider displaced fluid (3.12) with recumbent foam and displacing fluid (3.13) with constant velocity in case it is viscous.

$$\upsilon_1 = -\frac{k_1}{\mu_1}\frac{\partial}{\partial x}\left(P + \lambda\frac{\partial P}{\partial t}\right), 0 < x \le l\,(t) \tag{3.12}$$

$$\upsilon_2 = -\frac{k_2}{\mu_2}\frac{\partial P}{\partial x},\ \ l\ (t) \le x \le L \tag{3.13}$$

Such are the conditions of pressure and speed at the border discontinuities:

$$\upsilon_1 = \upsilon_2 = m\frac{dl}{dt} = c = const \tag{3.14}$$

$$P\ \ (x = 0) = P_1 \tag{3.15}$$

$$P\ \ (x = l) = P_0 \tag{3.16}$$

$$P\ \ (x = L) = P_2 \tag{3.17}$$

(3.12) and then (3.13), (3.14), (3.15), (3.16), (3.17) if we integrate with the terms υ_1 and υ_2, we get the following expressions:

$$\upsilon_1 = \frac{k_1}{\mu_1}\left[\frac{P_1 - P_0}{l} - \frac{\lambda}{l}\frac{\partial P_0}{\partial t}\right] \tag{3.18}$$

$$\upsilon_2 = \frac{k_2}{\mu_2}\frac{P_0 - P_2}{L - l} \tag{3.19}$$

To determine $P;(t)$ and $l\ (t)$ from the condition (3.14), we get the following system of equations.

$$\begin{array}{l} \lambda\frac{\partial P_0}{\partial t} = P_1 - P_0 + \frac{k_2\mu_1}{L-l}\cdot l\ (P_2 - P_0) \\ \frac{dl}{dt} = \frac{k_2}{m\mu_2}\frac{P_0-P_2}{L-l} \end{array} \tag{3.20}$$

If we apply small effects to $l\ (t)$ and $P_0\,(t)$, respectively, $\varepsilon\,(t)$ and $\eta\,(t)$, then taking into account the small effects of $\eta\,(t)$ according to the system (3.20), it will be written only in the first compilation in the following form:

$$\begin{array}{l} \frac{d\eta}{dt} = -\frac{1}{\lambda}\left(1 + \frac{\mu_1}{\mu_2}\frac{k_2}{k_1}\frac{l}{L-l}\right)\eta - \frac{\mu_1}{\mu_2}\frac{k_2}{k_1}\frac{(P_0-P_2)L}{(L-l)^2}\varepsilon \\ \frac{d\varepsilon}{dt} = \frac{k_2}{\mu_2 m(L-l)}\eta + \frac{k_2}{\mu_2 m}\frac{P_0-P_2}{(L-l)^2}\varepsilon \end{array} \tag{3.21}$$

Here $l\ (t)$ and P_0- it is found from the system (3.20) or designated as follows.

$$\begin{cases} \frac{d\eta}{dt} = a_{11}\eta + a_{12}\varepsilon \\ \frac{d\varepsilon}{dt} = a_{21}\eta + a_{22}\varepsilon \end{cases} \tag{3.22}$$

$$\begin{array}{ll} a_{11} = -\frac{1}{\lambda}\left(1 + \frac{\mu_1 k_2}{\mu_2 k_1}\frac{l}{L-l}\right); & a_{12} = -\frac{\mu_1 k_2 L(P_0-P_2)}{\lambda\mu_2 k_1 (L-l)^2} \\ a_{21} = \frac{k_2}{m\mu_2(L-l)}; & a_{22} = \frac{k_2}{m\mu_2}\frac{P_0-P_2}{(L-l)^2} \end{array} \tag{3.23}$$

The characteristic equation of the system (3.22) is written as follows

$$\lambda^2 - (a_{22} + a_{11})\,\lambda + a_{11}a_{22} - a_{12}a_{21} = 0 \tag{3.24}$$

If $\eta\,(t)$ and $\varepsilon\,(t)$ are affected by time, then the roots of the characteristic equation will be negative. In this case, the following conditions must be met:

$$\begin{cases} b = a_{11} + a_{22} < 0 \\ c = a_{11}a_{22} - a_{12}a_{21} > 0 \end{cases} \tag{3.25}$$

Taking into account the expression (3.23) we get for *C* :

$$C = \frac{k_2^2 \mu_1 (P_0 - P_1)}{\lambda \mu_2^2 k_1 (L-l)^2} \left(1 - \frac{\mu_2 k_1}{k_2 \mu_1}\right) \tag{3.26}$$

The condition of stability of the limit of viscous liquids arises from the condition $C > 0$:

$$1 - \frac{\mu_2 k_1}{k_2 \mu_1} > 0 \text{ or } \frac{\mu_1 k_2}{k_1 \mu_2} > 1 \tag{3.27}$$

That is, the mobility coefficient of the displaced fluid should be greater than the mobility coefficient of the displacing fluid.
At the same time, the $b < 0$condition must be met. If we consider the expression (3.23), we get the following expression for b:

$$b = -\frac{1}{\lambda} - \frac{\mu_1 k_2}{\lambda \mu_2 k_1} \frac{l}{L-l} + \frac{k_2}{m \mu_2} \frac{P_0 - P_2}{(L-l)^2} \tag{3.28}$$

If we replace the expression $\left[\frac{k_2}{m\mu_2}\frac{P_0-P_2}{L-l}\right]$ in (3.28) with υ_2, then the expression (3.30) will take the following form :

$$b = -\frac{1}{\lambda} - \frac{\mu_1 k_2}{\lambda \mu_2 k_1} \frac{l}{L-l} + \frac{\upsilon_2}{L-l} \tag{3.29}$$

We can write from the condition $b < 0$:

$$\frac{\upsilon_2}{L-l} < \frac{1}{\lambda} \left(1 + \frac{\mu_1 k_2}{\mu_2 k_1} \frac{l}{L-l}\right) \tag{3.30}$$

By replacing the low estimate of the expression $\frac{\mu_1 k_2}{\mu_2 k_1}$ in (3.30) equal to the unit (3.27), we would have taken the conditions of stability in the boundary flow.

$$\begin{cases} \upsilon_2 < \frac{L}{\lambda} \\ \frac{\mu_1 k_2}{\mu_2 k_1} > 1 \end{cases} \text{ or } \begin{cases} \lambda < \frac{L}{\upsilon_2} \\ \frac{\mu_1 k_2}{\mu_2 k_1} > 1 \end{cases} \tag{3.31}$$

$$\upsilon_2 < \frac{L}{\lambda} \tag{3.32}$$

$$\lambda < \frac{L}{\upsilon_2} \tag{3.33}$$

That is, it is necessary to limit the displaced fluid with the selection of displacing fluid in accordance with the established rheological $(\mu; \lambda)$properties (3.32). If (υ_2) is given in the output, then it is necessary to select rheological parameters according to (3.33).
Thus, oil displacement from homogeneous porous medium with viscous, relaxing fluid is more durable.

References

1. Bing Wei, Laura Romero-Zero´n, Denis Rodrigue. Oil displacement mechanisms of viscoelastic polymers in enhanced oil recovery (EOR): A review. J Petrol Explor Prod Technol (2014) 4:113–121 DOI 10.1007/s13202-013-0087-5.
2. Enick, R. M.; Olsen, D. K. Mobility and conformance control for carbon dioxide enhaced oil recovery (CO2-EOR) via thickeners, foams, and gels- adetailed literature review of 40 years of research, 2012.
3. Heller, J. P. Foams: Fundamentals and Applications in the Petroleum Industry 1994, 242 (Foams: Fundamentals and Applications in the Petroleum Industry), 201.
4. Holland, P. M.; Rubingh, D. N. In Cationic Surfactants: Physical Chemistry; Rubingh, D. N., Holland, P. M., Eds.; Marcel Dekker, Inc.: New York, USA, 1990, p 141.
5. Ke-xing Li, Xueqi Jing, Song He, Hao Ren, and Bing Wei. Laboratory Study Displacement Efficiency of Viscoelastic Surfactant Solution in EOR. Energy & Fuels is published by the American Chemical Society. 1155 Sixteenth Street N.W., Washington, DC 20036, 2016
6. M. Buchgraber, University of Leoben; T. Clemens, OMV E&P; and L.M. Castanier and A.R. Kovscek, Standford University. The displacement of viscous oil by associative polymer solutions. Society of Petroleum Engineers, SPE 122400, 2009, p 130 - 149.
7. Rosen, M. J. In Surfactants and Interfacial Phenomena; 3rd Edition ed.; Rosen, M. J., Ed.; John Wiley & Sons, Inc.: New York, 2004, p 34.
8. Siyang Xiao, Yongchao Zeng, Eric D. Vavra, Peng He, Maura C. Puerto, George J Hirasaki, and Sibani Lisa Biswal. Destabilization, Propagation, and Generation of Surfactant-Stabilized Foam during Crude Oil Displacement in Heterogeneous Model Porous Media. Langmuir is published by the American Chemical Society. 1155 Sixteenth Street N.W., Washington, DC 20036.
9. Y Hao, Y Xiang-An, Z Ren-Bao, Y Zhao-Peng A Laboratory Study on Durability of Foam Plug Created in Porous Media, Oilfield Chemistry Year 2009, Issue 3, Page 273-275,285.