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Analysis of an axially-symmetric problem of elasticity theory for a radially inhomogeneous cylinder of small thickness under mixed boundary conditions on lateral surfaces

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Abstract. *By means of the method of asymptotic integration of equations of elasticity theory, an axially-symmetric problem of elasticity theory is studied for a small thickness radially-inhomogeneous cylinder. We consider the case when elasticity module change radially according to linear law. It is assumed that mixed homogeneous boundary conditions are given on the lateral surfaces of the cylinder, the boundary conditions keeping the cylinder in equilibrium are given on the ends of the cylinder. The features of the stress-strain state in the radially-inhomogeneous cylinder are revealed based on the asymptotic analysis. Asymptotic formulas for displacements and stresses are structured. The analysis revealed two group of solutions. The solution corresponding to the first iterative process determines the penetrating stress-stain state of the cylinder. The stress-state determined by this solution is equivalent to the main vector of efforts applied in arbitrary section* $\xi = const$ *of the cylinder. The next iterative process determines the solutions of a boundary layer type character and localized at the end of the cylinder.*

Keywords. radially-inhomogeneous cylinder · asymptotic method · boundary layer · variation principle · main vector

Mathematics Subject Classification (2010): 74G10, 74H10

1 Introduction

Analysis of inhomogeneous shells based on three -dimensional equations of elasticity theory is a laborious task. Therefore, we have to handle different approximate methods allowing to simplify the calculation of shells. Complex nature of phenomena arising from deformation of inhomogeneous shells led to the creation of many applied theories each of which was constructed on the basis of a certain system of assumptions. To establish the range of applicability of the existing theories of inhomogeneous shells, it is required to analyze the stress-stain state of inhomogeneous shells from the standpoint of three-dimensional

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equations of elasticity theory . In the paper [7], a spatial problem of elasticity theory is studied for a small thickness isotropic cylinder and the asymptotic solution is compared with the solutions obtained from applied theories. Asymptotic theory of a small thickness transversally-isotropic cylinder is developed in [14].

In [3] an axially-symmetric problem of elasticity theory is studied for a radially-laminated cylinder with alternating hand and soft layers. The stress-strain state of a multilayer cylinder with the most comman form of cylindrical anisotropy is analyzed in [8]. In [12] a semi-analytic method is offered for solving the Almansi-Michelle problem for an inhomogeneous anisotropic cylinder. The influence of inhomogeneity of the material on the stress-stain state of the material is studied in [10,11]. In [6] a three –dimensional problem of elasticity theory is studied by the method of asymptotic integration of elasticity theory equations for a small thickness radially-inhomogeneous cylinder, when the lateral surface of the cylinder is free from efforts. Based on the analysis carried out, it is shown that the stress-stain state in a radially-inhomogeneous cylinder consists of three types inner stress state, simple edge effect and boundary layer.

2 Setting boundary value problems for a radially-inhomogeneous cylinder

We consider an axially-symmetric problem of elasticity theory for a small thickness radially -inhomogeneous isotropic hollow cylinder. In the cylindrical system of coordinates, we denote the area occupied by the cylinder by

$$\Gamma = \{r \in [r_1; r_2], \varphi \in [0, 2\pi], z \in [-L; L]\}.$$

Suppose that change in the elasticity modulus along the radius occurs according to the linear law

$$G(r) = G_* r, \lambda(r) = \lambda_* r,$$

where G_*, λ_* are some constant quantities.

Equilibrium equations in displacements have the form:

$$(L_0 + \partial_1 L_1 + \partial_1^2 L_2)\bar{u} = \bar{0}. \tag{2.1}$$

Here $\bar{u} = \bar{u}(\rho, \xi) = (u_\rho(\rho, \xi), u_\xi(\rho, \xi))^T, L_k$ are matrix differential operators of the form:

$$L_0 = \left\| \begin{matrix} (2G_0 + \lambda_0)(\partial^2 + \varepsilon\partial) - 2G_0\varepsilon^2 & 0 \\ 0 & G_0(\partial^2 + \varepsilon\partial) \end{matrix} \right\|,$$

$$L_1 = \left\| \begin{matrix} 0 & e^{\varepsilon\rho}\left[\varepsilon(G_0 + \lambda_0)\partial + \varepsilon^2\lambda_0\right] \\ e^{\varepsilon\rho}\left[\varepsilon^2(2G_0 + \lambda_0) + \varepsilon(G_0 + \lambda_0)\partial\right] & 0 \end{matrix} \right\|,$$

$$L_2 = \left\| \begin{matrix} \varepsilon^2 G_0 e^{\varepsilon\rho} & 0 \\ 0 & (2G_0 + \lambda_0)\varepsilon^2 e^{\varepsilon\rho} \end{matrix} \right\|,$$

$\partial_1 = \frac{\partial}{\partial\xi}$; $\partial_1^2 = \frac{\partial^2}{\partial\xi^2}$; $\partial = \frac{\partial}{\partial\rho}$; $\rho = \frac{1}{\varepsilon}\ln\left(\frac{r}{r_0}\right)$, $\xi = \frac{z}{r_0}$ are new pure variables; $\varepsilon = \frac{1}{2}\ln\left(\frac{r_2}{r_1}\right)$ is a small parameter characterizing the cylinder's thickness; $r_0 = \sqrt{r_1 r_2}$, $\xi \in [-l; l]$, $\rho \in [-1; 1]$, $l = \frac{L}{r_0}$; $\lambda_0 = \frac{\lambda_* r_0}{G_1}$, $G_0 = \frac{G_* r_0}{G_1}$ are pure variables, G_1 is some characteristic parameter with shear modulus dimension.

Suppose that on the lateral surface of the cylinder we are given the mixed homogeneous boundary conditions

$$\bar{\sigma}|_{\rho=\pm 1} = M\ \bar{u}|_{\rho=\pm 1} = \bar{0}, \tag{2.2}$$

where

$$\bar{\sigma} = (u_\rho, \sigma_{\rho\xi})^T, \quad M = M_0 + \partial_1 M_1,$$

$$M_0 = \left\| \begin{matrix} 1 & 0 \\ 0 & \frac{G_0}{\varepsilon}\partial \end{matrix} \right\|, M_1 = \left\| \begin{matrix} 0 & 0 \\ G_0 e^{\varepsilon\rho} & 0 \end{matrix} \right\|.$$

Assume that on the cylinder's ends we are given boundary conditions

$$\sigma_{\rho\xi}|_{\xi=\pm l} = f_{1s}(\rho), \sigma_{\xi\xi}|_{\xi=\pm l} = f_{2s}(\rho). \tag{2.3}$$

Here $f_{1s}(\rho)$, $f_{2s}(\rho)$ $(s = 1, 2)$ are rather smooth functions satisfying the equilibrium conditions.

3 Constructing homogeneous solutions for a small thickness radially-homogeneous cylinder

We look for the solution of (2.1), (2.2) in the form:

$$\bar{u}(\rho, \xi) = \bar{a}(\rho)e^{\alpha\xi}, \tag{3.1}$$

where

$$\bar{a}(\rho) = (u(\rho), w(\rho))^T.$$

Substituting (3.1) in (2.1), (2.2), we have:

$$\begin{cases} (L_0 + \alpha L_1 + \alpha^2 L_2)\bar{a} = \bar{0}, \\ (M_0 + \alpha M_1)\bar{a}|_{\rho=\pm 1} = \bar{0}. \end{cases} \tag{3.2}$$

Applying the method of asymptotic integration of elasticity theory equations [1,2,4,5,6,9], as $\varepsilon \to 0$ for (3.2) we get two groups of solutions:

$$1) u_\rho^{(1)} = E_0 g \left[\frac{sh(k_2 - \varepsilon)}{sh(\varepsilon t)} e^{k_1\rho} - \frac{sh(k_1 - \varepsilon)}{sh(\varepsilon t)} e^{k_2\rho} - e^{\varepsilon\rho} \right], \tag{3.3}$$

$$u_\xi^{(1)} = E_0 \xi. \tag{3.4}$$

where $g = \frac{\lambda_0}{2(G_0+\lambda_0)}$;$k_1 = \frac{-\varepsilon(1+t)}{2}$;$k_2 = \frac{\varepsilon(t-1)}{2}$;$t = \sqrt{\frac{10G_0+\lambda_0}{2G_0+\lambda_0}}$.
Double eigen values $\alpha = 0$. correspond to these solutions.
The stress corresponding to the solution (3.3),(3.4) is of the form:

$$\sigma_{\rho\rho}^{(1)} = E_0 \left\{ g \left[\lambda_0 - \frac{(2G_0 + \lambda_0)(t+1)}{2} \right] \cdot \frac{sh(k_2 - \varepsilon)}{sh(\varepsilon t)} e^{k_1\rho} - \right.$$

$$\left. -g \left[\lambda_0 + \frac{(2G_0 + \lambda_0)(t-1)}{2} \right] \cdot \frac{sh(k_1 - \varepsilon)}{sh(\varepsilon t)} e^{k_2\rho} \right\}; \tag{3.5}$$

$$\sigma_{\xi\xi}^{(1)} = E_0 \left\{ g\lambda_0 \frac{(1-t)}{2} \frac{sh(k_2 - \varepsilon)}{sh(\varepsilon t)} e^{k_1\rho} - \right.$$

$$\left. -g\lambda_0 \frac{(t+1)}{2} \frac{sh(k_1 - \varepsilon)}{sh(\varepsilon t)} e^{k_2\rho} + \frac{G_0(2G_0 + \lambda_0)}{G_0 + \lambda_0} e^{\varepsilon\rho} \right\} \tag{3.6}$$

$$\sigma_{\varphi\varphi}^{(1)} = E_0 \left\{ g \left[2G_0 + \lambda_0 \frac{(1-t)}{2} \right] \cdot \frac{sh(k_2 - \varepsilon)}{sh(\varepsilon t)} e^{k_1\rho} - \right.$$

$$-g\left[2G_0+\lambda_0\frac{(1+t)}{2}\right]\cdot\frac{sh(k_1-\varepsilon)}{sh(\varepsilon t)}e^{k_2\rho}\Bigg\} \quad (3.7)$$

$$\sigma^{(1)}_{\rho\xi}=0; \quad (3.8)$$

2)

a)

$$\alpha_k=\varepsilon^{-1}\left(\beta_{0k}+\varepsilon\beta_{1k}+...\right). \quad (3.9)$$

$$u^{(3,1)}_{\rho}(\rho;\xi)=\varepsilon\sum_{k=1}^{\infty}M_k\left[\beta_{0k}\cos\beta_{0k}\sin(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.10)$$

$$u^{(3,1)}_{\xi}(\rho;\xi)=-\varepsilon\sum_{k=1}^{\infty}M_k\left[\beta_{0k}\cos\beta_{0k}\cos(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.11)$$

$$\sigma^{(3,1)}_{\rho\rho}(\rho;\xi)=\sum_{k=1}^{\infty}M_k\left[2G_0\beta^2_{0k}\cos\beta_{0k}\cos(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.12)$$

$$\sigma^{(3,1)}_{\rho\xi}(\rho;\xi)=\sum_{k=1}^{\infty}M_k\left[2G_0\beta^2_{0k}\cos\beta_{0k}\sin(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.13)$$

$$\sigma^{(3,1)}_{\xi\xi}(\rho;\xi)=\sum_{k=1}^{\infty}M_k\left[-2G_0\beta^2_{0k}\cos\beta_{0k}\cos(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.14)$$

$$\sigma^{(3,1)}_{\varphi\varphi}(\rho;\xi)=O(\varepsilon); \quad (3.15)$$

where $\sin^2\beta_{0k}=0$.

b)

$$\alpha_k=\varepsilon^{-1}(\beta_{0k}+\varepsilon\beta_{1k}+...). \quad (3.16)$$

$$u^{(3,2)}_{\rho}(\rho;\xi)=\varepsilon\sum_{k=1}^{\infty}T_k\left[\beta_{0k}\sin\beta_{0k}\cos(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \quad (3.17)$$

$$u^{(3,2)}_{\xi}(\rho;\xi)=\varepsilon\sum_{k=1}^{\infty}T_k\left[\beta_{0k}\sin\beta_{0k}\sin(\beta_{0k}\rho)+\right.$$

$$+O(\varepsilon)]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \tag{3.18}$$

$$\sigma_{\varphi\varphi}^{(3,2)}=O(\varepsilon); \tag{3.19}$$

$$\sigma_{\rho\rho}^{(3,2)}=-\sum_{k=1}^{\infty}T_k\left[-2G_0\beta_{0k}^2\sin\beta_{0k}\sin(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \tag{3.20}$$

$$\sigma_{\rho\xi}^{(3,2)}=\sum_{k=1}^{\infty}T_k\left[2G_0\beta_{0k}^2\sin\beta_{0k}\cos(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \tag{3.21}$$

$$\sigma_{\xi\xi}^{(3,2)}=\sum_{k=1}^{\infty}T_k\left[2G_0\beta_{0k}^2\sin\beta_{0k}\sin(\beta_{0k}\rho)+\right.$$

$$\left.+O(\varepsilon)\right]\exp\left(\frac{1}{\varepsilon}(\beta_{0k}+\varepsilon\beta_{1k}+...)\xi\right); \tag{3.22}$$

where $\cos^2\beta_{0k}=0$.
The common solution (3.2) will be the sum of solutions (3.3), (3.4), (3.10), (3.11), (3.17), (3.18) :

$$\begin{array}{l} u_\rho(\rho,\xi)=u_\rho^{(1)}+u_\rho^{(3,1)}+u_\rho^{(3,2)}, \\ u_\xi(\rho,\xi)=u_\xi^{(1)}+u_\xi^{(3,1)}+u_\xi^{(3,2)}. \end{array} \tag{3.23}$$

For the stress tensor components we get:

$$\sigma_{\rho\xi}=\sigma_{\rho\xi}^{(3,1)}+\sigma_{\rho\xi}^{(3,2)},\ \ \sigma_{\xi\xi}=\sigma_{\xi\xi}^{(1)}+\sigma_{\xi\xi}^{(3,1)}+\sigma_{\xi\xi}^{(3,2)}, \tag{3.24}$$

$$\sigma_{\rho\rho}=\sigma_{\rho\rho}^{(1)}+\sigma_{\rho\rho}^{(3,1)}+\sigma_{\rho\rho}^{(3,2)},\ \ \sigma_{\varphi\varphi}=\sigma_{\varphi\varphi}^{(1)}+\sigma_{\varphi\varphi}^{(3,1)}+\sigma_{\varphi\varphi}^{(3,2)}. \tag{3.25}$$

4 Analysis of the stress-stain state and satisfying the boundary conditions on the ends of the cylinder

We represent the displacements in the form:

$$\begin{array}{l} u_\rho(\rho,\xi)=u_\rho^{(1)}+\sum\limits_{k=1}^{\infty}E_k u_k(\rho)e^{\alpha_k\xi}, \\ u_\xi(\rho,\xi)=u_\xi^{(1)}+\sum\limits_{k=1}^{\infty}E_k w_k(\rho)e^{\alpha_k\xi}. \end{array} \tag{4.1}$$

The second summand includes the displacements determined by the formulas (3.10), (3.11), (3.17), (3.18).

For the stresses we get

$$\sigma_{\xi\xi}=\sigma_{\xi\xi}^{(1)}+\sum_{k=1}^{\infty}E_k\sigma_{2k}(\rho)e^{\alpha_k\xi},\ \ \sigma_{\rho\xi}=\sum_{k=1}^{\infty}E_k\sigma_{1k}(\rho)e^{\alpha_k\xi}, \tag{4.2}$$

where

$$\sigma_{1k}(\rho) = \frac{G_0}{\varepsilon}\left(w_k'(\rho) + \varepsilon\alpha_k e^{\varepsilon\rho} u_k(\rho)\right),$$

$$\sigma_{2k}(\rho) = \frac{1}{\varepsilon}\left[(2G_0 + \lambda_0)\varepsilon\alpha_k e^{\varepsilon\rho} w_k(\rho) + \lambda_0(u_k'(\rho) + \varepsilon u_k(\rho))\right].$$

Let us consider the relation of the structured solutions with the main stress vector P acting in the section $\xi = const$. Note that

$$P = 2\pi\varepsilon \int_{-1}^{1} (\sigma_{\xi\xi} + \sigma_{\rho\xi})\, e^{2\varepsilon\rho} d\rho. \tag{4.3}$$

Substituting (4.2) in (4.3), we get:

$$P = 2\pi r_0^2 G_1 E_0 d_0 + 2\pi r_0^2 \varepsilon G_1 \sum_{k=1}^{\infty} E_k m_k e^{\alpha_k \xi}; \tag{4.4}$$

where

$$m_k = \int_{-1}^{1} (\sigma_{1k}(\rho) + \sigma_{2k}(\rho))\, e^{2\varepsilon\rho} d\rho.$$

$$d_0 = \frac{\varepsilon\lambda_0^2}{2(G_0 + \lambda_0) sh(\varepsilon t)}\left(\frac{(1-t)}{k_1 + 2\varepsilon} sh(k_2 - \varepsilon) sh(k_1 + 2\varepsilon) - \right.$$
$$\left. - \frac{(1+t)}{k_2 + 2\varepsilon} sh(k_1 - \varepsilon) sh(k_2 + 2\varepsilon)\right) + \frac{2G_0(2G_0 + \lambda_0)}{3(G_0 + \lambda_0)} sh(3\varepsilon);$$

Prove that all $m_k = 0 \quad (k = 1, 2...)$. For that we consider the following boundary value problem:

$$\sigma_{\rho\xi} = \sigma_{1k}(\rho) e^{\alpha_k \xi_j}, \quad \sigma_{\xi\xi} = \sigma_{2k}(\rho) e^{\alpha_k \xi_j} \text{ as } \xi = \xi_j \ (j = 1, 2). \tag{4.5}$$

The main vector corresponding to the stress state of the problem (4.5) in the section $\xi = const$, is led to the following form:

$$P_k = 2\pi\varepsilon m_k e^{\alpha_k \xi}. \tag{4.6}$$

By the condition of solvability of elasticity theory problem, P_k should not depend on the variable ξ. However, in (4.6) the right hand side depends on ξ. Hence it follows that $P_k = 0$, i.e. $m_k = 0$. From (4.4) we have:

$$P = 2\pi r_0^2 G_1 E_0 d_0 \tag{4.7}$$

The stress-state corresponding to the solutions (3.10),(3.11),(3.17),(3.18) is a self - equilibrium in any section $\xi = const$.

The solutions (3.3), (3.4) determine the inner stress-stain state of the cylinder. The solution (3.3), (3.4) corresponding to the first iterative process penetrates without damping. The solution (3.10), (3.11), (3.17), (3.18) is a boundary layer character. The first terms of its asymptotic expansion are equivalent to Saint-Venenant's edge effect of an inhomogeneous plate [15]. The stresses corresponding to these solutions are localized at the ends of the cylinder and when moving away from the ends, exponentially decrease.

As it was shown, the not self-equlibrium part of stresses may be relieved by means of the penetrating solution (3.3),(3.4), and the relation of the constant E_0 and the main vector P is given by the equality (4.7).

For determining the unknown constants T_k and M_k $(k = 1, 2, ...)$ we use the Legendre variational principle . Since the solution (4.1) satisfy the equilibrium equation and boundary conditions on the lateral surface, the variation principle takes the form [13,14]:

$$\sum_{s=1}^{2} \int_{-1}^{1} \left[(\sigma_{\rho\xi} - f_{1s})\, \delta u_\rho + (\sigma_{\xi\xi} - f_{2s})\, \delta u_\xi\right]\Big|_{\xi=\pm l} e^{2\varepsilon\rho} d\rho = 0. \tag{4.8}$$

Substituting (3.13),(3.14),(3.21),(3.22) in (4.8) and considering $\delta M_k, \delta F_k$ independent variables, we get the following system of linear algebraic equations:

1)

$$\sum_{k=1}^{\infty} F_{jk}^{(1)} M_{k0} = d_{0j}^{(1)}, \tag{4.9}$$

$$F_{jk}^{(1)} = 4G_0 \beta_{0j} \beta_{0k}^2 \cos\beta_{0k} \cos\beta_{0j} \frac{\sin(\beta_{0k} - \beta_{0j})}{\beta_{0k} - \beta_{0j}} \times$$

$$\times (e^{-(\beta_{0k}+\beta_{0j})l/\varepsilon} + e^{(\beta_{0k}+\beta_{0j})l/\varepsilon}); (k \neq j)$$

$$F_{jk}^{(1)} = 4G_0 \beta_{0k}^3 \cdot (e^{-(\beta_{0k}+\beta_{0j})l/\varepsilon} + e^{(\beta_{0k}+\beta_{0j})l/\varepsilon}); (k = j)$$

$$d_{0j}^{(1)} = \beta_{0j} \cos\beta_{0j} \left[\int_{-1}^{1} (f_{11}(\rho) \sin(\beta_{0j}\rho) - f_{21}^{*}(\rho) \cos(\beta_{0j}\rho))\, d\rho \cdot e^{-(\beta_{0j}l)/\varepsilon} + \right.$$

$$\left. + \int_{-1}^{1} (f_{12}(\rho) \sin(\beta_{0j}\rho) - f_{22}^{*}(\rho) \cos(\beta_{0j}\rho))\, d\rho \cdot e^{(\beta_{0j}l)/\varepsilon} \right],$$

$$f_{2s}^{*} = f_{2s}(\rho) - E_0 \left[\frac{\lambda_0^2}{4(G_0 + \lambda_0)} \times \right.$$

$$\left. \times \frac{((1-t) sh(k_2 - \varepsilon) e^{k_1\rho} - (1+t) sh(k_1 - \varepsilon) e^{k_2\rho})}{sh(\varepsilon t)} + \frac{G_0(2G_0 + \lambda_0)}{G_0 + \lambda_0} e^{\varepsilon\rho} \right],$$

where $\sin^2 \beta_{0j} = 0$,

$$M_k = M_{k0} + \varepsilon M_{k1} + ...$$

2)

$$\sum_{k=1}^{\infty} F_{jk}^{(2)} T_{k0} = d_{0j}^{(2)}, \tag{4.10}$$

$$F_{jk}^{(2)} = 4G_0 \beta_{0j} \beta_{0k}^2 \sin\beta_{0k} \sin\beta_{0j} \frac{\sin(\beta_{0k} - \beta_{0j})}{\beta_{0k} - \beta_{0j}} \times$$

$$\times (e^{-(\beta_{0k}+\beta_{0j})l/\varepsilon} + e^{(\beta_{0k}+\beta_{0j})l/\varepsilon}); (k \neq j)$$

$$F_{jk}^{(2)} = 4G_0 \beta_{0k}^3 \cdot (e^{-(\beta_{0k}+\beta_{0j})l/\varepsilon} + e^{(\beta_{0k}+\beta_{0j})l/\varepsilon}); (k = j)$$

$$d_{0j}^{(2)} = \beta_{0j} \sin\beta_{0j} \left[\int_{-1}^{1} (f_{11}(\rho) \cos(\beta_{0j}\rho) + f_{21}^{*}(\rho) \sin(\beta_{0j}\rho))\, d\rho \cdot e^{-(\beta_{0j}l)/\varepsilon} + \right.$$

$$\left. + \int_{-1}^{1} (f_{12}(\rho) \cos(\beta_{0j}\rho) + f_{22}^{*}(\rho) \sin(\beta_{0j}\rho))\, d\rho \cdot e^{(\beta_{0j}l)/\varepsilon} \right],$$

where $\cos^2\beta_{0j} = 0$,

$$T_k = T_{k0} + \varepsilon T_{k1} + ...$$

Determination of constants $T_{kp},\ M_{kp}\ (p = 1, 2, ...)$ invariably is led to the system whose matices coincide with the matrices of the systems (4.9),(4.10).

The system of infinite linear algebraic equations (4.9), (4.10) is always solvable under physically meaningful conditions imposed on the right hand side [7,15]. The solvability and convergence of the reduction method for (4.9), (4.10) was proved [15].

References

1. Akhmedov N.K. Mekhtiev M.F. Analysis of three-dimensional problem of the theory of elasticity for an inhomogeneous truncated hollow cone// Prikladnaya mathematika i mekhanika 57(5), (1993), 113-119. (in Russian)
2. Akhmedov N.K. Mekhtiev M.F., Axially-symmetric problem of elasiticity theory for a variable thickness inhomogeneous plate. *Russian AS. Applied mathematics and mechanics* , **59**(3), (1995), 518-523.
3. Akhmedov N.K. : Analysis of the boundary layer in the axisymmetric problem of the theory of elasticity for a radially multilayered cylinder and the propagation of axisymmetric wares, *Prikladnaya mathematika i mekhanika* **61**(5), 863-872 (1997). (in Russian).
4. Akhmedov, N.K., Akbarova S.B. Asymptotic analysis of a three-dimensional problem of elasticity theory for radially inhomogeneous transversally-isotropic hollow cylinder. // Izvestia Rossiyskoy AS Mekhanika tverdogo tela 2011. № 4, 170-180.
5. Akhmedov, N.K. and Sofiyev, A.H. Asymptotic analysis of three-dimensional problem of elasticity theory for radially inhomogeneous transversally-isotropic thin hollow spheres. Thin-Walled Structures, 2019; 139: 232-241.
6. Akhmedov, N.K., Akbarova, J.S., Ismailova: Analysis of axisymmetric problem from the theory of elasticity for an isotropic cylinder of small thickness with alternating elasticity modules *Eastern-European Journal of Enterprise Technologies,* 2/7 98, (2019) 13-19.
7. Bazarenko, N.A. Vorovich,I.I.: The asymptotic behavior of the solution of the problem of the theory of elasticity for a hollow cylinder of finite length. *J. Prikladnaya mathematika i mekhanika,* **29**(6), (1965) 1035-1052 (in Russian).
8. Chao Hsun Huang. Analysis of laminated circular cylinders of materials with the most general form of cylindrical anisotropy.: I. Axially symmetric deformations. *International Journal of Solids and Structures* **38** (34-35):6163-6182, (2001).
9. Goldenweiser A.L. Constructing approximate theory of shell bending by means of asymptotic integration of elasticity theory equations, *Prikladnaya mathematika i mekhanika* **27**(4), (1963) 593-608. (in Russian)
10. Horgan,C.O., Chan,A.M.: The pressurized hollow cylinder or disk problem for functionally graded isotropic linearly elastic materials. *Journal of Elasticity,* **55**(1), (1999) 43-59.
11. Jiann –Quo Tran, Hsi-Hung Chang. Torsion of cylindrically orthotropic elastic circular bars with radial inhomogeneity: some exact solutions and effects. *International Journal of Solids and Structures* **45** (2008) 303-319.
12. Lin,H.C., Stanley B. Dong: On the Almansi-Michell problems for an inhomogeneous, anisotropic cylinder *Journal of Mechanics* **22**(1), (2006) 51-57.
13. Lurie,A.l.: *Theory of Elasticity*. M.: Nauka, (1970) 939.
14. Mekhtiev M.F.: *Asymptotic analysis of spatial problems in elasticity.* Springer. 2019. p.241.

15. Ustinov,Yu. A.: *The mathematical theory of transversally inhomogeneous plates.* Rostov-na-Dona : OOOTsVVR; 2006 257. (in Russian).