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Stress-train state of a viscous–elastic liquid medium with regard to temperature and nanoparticles

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Abstract. *In the paper we explain the essence of the process of separation of inhomogeneous liquid form food system and based by the Maxwell model we study the stress strain state of viscous-elastic liquid with nanoparticles and exposed to the temperature action and determine that temperature and nanoparticles influence on elastic and viscous elements effects on change of the model's parameters. The character of change of these parameters and their values was described due to the assumption of interaction of atoms and nanoparticles exposed to the temperature action.*

Keywords. viscous-elastic liquid · filtration process · a system of heterogeneous food nanoparticle · Maxwell model.

Mathematics Subject Classification (2010): 82D80

1 Introduction

It is known that mechanical and hydromechanical processes occurring in machines and apparatus used in food industry are based on hydraulic rules and rheological regularities . Therefore , since the process of separation of liquid from heterogeneous food systems is a step by step hydromecanical process divided in the molecular form the study of the process at each stage is of great practical importance. Under the liquid form heterogeneous food medium one understands non-transparent, semi-dispensed systems (suspensions) consisting of colloidal substances, non-uniform, small dispersed particles. At first for these systems, the dilution process under the action of gravitation forces is implemented. In this case the mixture heterogeneous liquid is separated into fractions that differ by density of liquid mixture. In the next process called filtration the sediment is separated from suspension. Then the centrifiguration process is implemented. This time, the process of separation of suspension into fractions occurs under the action of centrifugal forces. In the separation process, the process of separation into fractions that differ by their density occurs. Each of these processes serves to separate the fractions that from a liquid from heterogeneous food

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system. It is known that the ratio of the centrifugal force to gravity force is taken as the main factor that affects the separation process of suspensions in machines and apparatus. Taking into account the types of sediments formed by crystal and amorhp particles in the process of separation of suspensions, the filtration process is carried out in 2 ways decreasing the filtration rate by giving constant pressure and keeping stable filtration rate and increasing the pressure drop. The pressure drop affecting on filtration process of suspension, thickness of sediment layer in the filter, structure and character of the sediment, composition viscosity and temperature of the suspension are taken as main factors. Since the flow rate of heterogeneous system filtered from the pores of the sediment layer or porous medium is slow, it is of laminar character [3, 4]. Since separation of suspensions with certain viscosity consists of sequential processes as a result of separation processes, the obtained new heterogeneous liquids (suspensions) will differ by their composition and viscosity. For increasing filtration rate of heterogeneous liquids in porous medium, in order to decrease is viscosity the existing methods and technologies are perfected and replaced by new ones. Chemical, heat effects, insertion of nanoparticles into heterogeneous nanoparticles and others are among them. The efficiency of methods and technologies are related to change of physical-mechanical features of high viscosity systems. Taking into account that such systems are described by viscous –elastic and viscous-plastic models, then allowing for interaction of molecules and atoms we encounter the description of model media. Study of the stress and strain state of such systems is one of the urgent problems [3 - 6].

2 Problem statement

As first we consider a viscous-elastic liquid model called the Maxwell model (Fig.1). This model consisting of sequentially-linked elastic elements with viscosity is described by the following relations [2, 5]:

$$\varepsilon = \varepsilon_1 + \varepsilon_2;\ \varepsilon_1 = \frac{\sigma}{E}; \dot{\varepsilon}_2 = \mu\sigma; \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \mu\sigma, \tag{2.1}$$

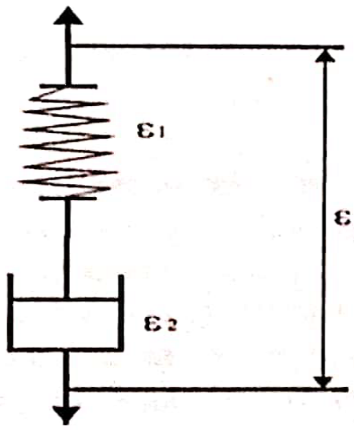

Fig. 1 Maxwell model.

where, ε is a total deformation, ε_i is deformation of each element $(i = 1, 2)$, σ is stress, E is a modules of elasticity, μ is an elasticity factor. The solution of equation (2.1) is in the following form:

$$\sigma(t) = E \cdot \left[\varepsilon(t) - \varepsilon(0)\exp(-E\mu t) - E\mu \int_0^t \exp(-E\mu(t-\tau))\varepsilon(\tau)d\tau \right] \tag{2.2}$$

Is was taken into accounts that there is no stress at initial moment. When appyling some techologies, there aries necessity to heat colloid substances and this change of the feature of viscous –elastic medium. Depending on the sequence of application of technologies assuming that the previous model does not change determine the change of its paameters. Here at first we study the stress-starin state of a viscous elastic medium with icreased temperature and formed when adding nanoparticles to this medium.

3 Problem solution

We imagine an elastic element of the Maxwell model in the form of one –dimensional “chain” of interacting atoms (Fig 2.) Such assumption is acceptable for higher molecular systems. Al first we study the stress-strain state of a model exposed to temperature. Assume that the “chain” consisting of atoms regardless of temperature and in the undeformed case has the following form (Fig 2.a)

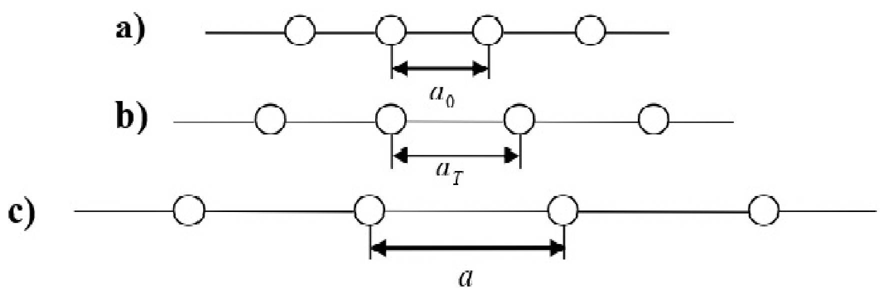

Fig. 2 Elastic element of the Maxwell model.
a) when temperature is not taken into account;
b) when temperature is taken into account;
c) influence of the pull on the heated “chain”.

The interaction force $F(a_0)$ between the atoms balances the “chain” at the expense of electromagnetic potential. Here a_0 is a distance between the atoms of the “chain” in the undeformed case and regardless of temperature (Fig 2,a).

Increase the temperature of the “chain” of atoms by $\Delta T \neq 0, \Delta T > 0$. Then taking into account the temperature, the “chain ” the atoms will as in Fig.2,b. Here $a_T > a_0$ is the distance between the atoms as a result of temperature effect . From the geometry of the “chain” this quantity is determined as follows:

$$\frac{a_T - a_0}{a_0} = \alpha \cdot \Delta T,\ a_T = a_0(1 + \alpha \cdot \Delta T) \tag{3.1}$$

here,α is a proportionality factor and $\alpha = const$. We introduce the denotation $\varepsilon_T = \frac{a_T - a_0}{a_0}$ and call it deformation resulting from temperature. In this case, the the force between atoms does not change, i.e. $F(a_T) = F(a_0)$.

Apply pull to the heated “chain”. Then distance between the atoms under the action of pull and temperature will be $a > a_T > a_0$. For the σ-stress in the “chain” we can write:

$$F(a) = R_x \cdot \sigma,$$

here the existing force between the atoms, R_x is characteristics area of the cut of the "chain" of atoms. Determine the stress in the "chain"

$$R_x \cdot \sigma = F(a) = F(a_T) + F'(a_T) \cdot (a - a_T) + ...$$

here, taking the linear part of the Taylor series, we get:

$$R_x \cdot \sigma = F(a_0) + F'(a_T) \cdot a_0 \cdot \frac{a - a_T}{a_0}. \tag{3.2}$$

Show the last multiplier of the second term at the right hand side of this equality as follows:

$$\frac{a - a_T}{a_0} = \frac{a - a_0 + a_0 - a_T}{a_0} = \varepsilon - \varepsilon_T$$

here, $\varepsilon = \frac{a - a_0}{a_0}$ is deformation created under the action of applied force and temperature. Then we can write equality (3.2) in the following form:

$$R_x \cdot \sigma = F(a_0) + F'(a_T) \cdot a_0 \cdot (\varepsilon - \varepsilon_T). \tag{3.3}$$

Assuming rigidity of the undeformed "chain" as $c = F'(a_0)$ we determine $F'(a_T)$:

$$F'(a_T) = F'(a_0) + F''(a_0)(a_T - a_0) = c + F''(a_0) \cdot a_0 \cdot \alpha \cdot \Delta T =$$
$$= c + F''(a_0) \cdot \varepsilon_T \cdot a_0.$$

Hence we get that $F'(a_T)$ is dependent on temperature and geometric parameters of the "chain" .

Assume that $c >> \varepsilon_T \cdot a_0 \cdot F''(a_0)$. Then according to relation (3.3) we can write:

$$R_x \cdot \sigma = F(a_0) + c \cdot a_0 \cdot (\varepsilon - \varepsilon_T)$$

Here, ignoring the quantity $F(a_0)$, we get

$$\sigma = \frac{c \cdot a_0}{R_x} \cdot (\varepsilon - \varepsilon_T) = \frac{c \cdot a_0}{R_x} \cdot (\varepsilon - \alpha \cdot \Delta T) = E \cdot (\varepsilon - \alpha \cdot \Delta T) \tag{3.4}$$

here $E = \frac{c \cdot a_0}{R_x}$ is an eclasticty modulus.

Note that since the quality c is dependent on ΔT, the E- elasticity modules is also dependent on ΔT.

Let us imagine the viscous element of the Maxwell model in the form of liquid –filled cylinder compressed by a piston. Assume that the temperature of liquid in the cylinder was increased and in this case the model does not change its form. This time, the temperature effect can show itself in different forms, for example in the form of increase of the friction force of liquid on the cylinder's wall, change of characteristics feature of liquid, etc, and in the form of combination of the effects. Allowing for these effects, we can reduce the temperature effect of the model under consideration to the change of viscosity factor, i.e. we can introduce the quantity μ_T. According to physics of the process, we conclude that $\mu_T \leq \mu$.

Since when the piston is tightened by the same force, the displacement rate decreases at the expense of friction, we can write the following relation:

$$\dot{\varepsilon}_T = \mu_T \sigma < \dot{\varepsilon} = \mu; \sigma; \;\; \mu_T \leq \mu.$$

Thus, based on (2.1) taking into account temperature effect in appropriate elements of the model, we can write the main equation of the Maxwell model as follows:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E_T} + \mu_T \sigma = \frac{\dot{\sigma}}{E \cdot e_T} + \mu \upsilon_T \sigma \tag{3.5}$$

here we have the following denotations:

$$e_T = \frac{E_T}{E} = c_T \cdot \frac{1}{c};\ \upsilon_T = \frac{\mu_T}{\mu};\ (\upsilon_T \leq 1)$$

The solution of the problem is found from equality , (3.5) in the following form:

$$\sigma(t) = E_T \cdot [\varepsilon(t) - \varepsilon(0)\exp(-E_T\mu\upsilon_T t) -$$
$$E_T\mu\upsilon_T \int_0^t \exp(-E_T\mu\upsilon_T(t-\tau))\varepsilon(\tau)d\tau \Bigg] \quad (3.6)$$

For $\sigma = const = \sigma_0$ we get:

$$\varepsilon(t) = \sigma_0\left(\frac{1}{E_T} + \mu_T t\right) = \frac{\sigma_0}{E \cdot e_T} + \mu\upsilon_T\sigma_0 t.$$

It was determined that temperature effect on elastic and viscous elements of the Maxwell model leads to change in the modul's parameters.

Character of change of these parameters and their values were described within the assumption of interaction of atoms exposed to temperature effect.

Now we consider a model of viscous-elastic liquid with regard to temperature effect. This model consisting of sequentially linked, elastic and viscous elements exposed to temperature effect, based on the Maxwell model is mathematically expressed by the following relations:

$$\begin{aligned} &\varepsilon = \varepsilon_{1T} + \varepsilon_{2T};\ \ \varepsilon_{1T} = \tfrac{\sigma}{E_T};\ \ \dot\varepsilon_{2T} = \mu\sigma; \\ &\dot\varepsilon = \tfrac{\dot\sigma}{E_T} + \mu_T\sigma = \tfrac{\dot\sigma}{E \cdot e_T} + \mu\upsilon_T\sigma; \end{aligned} \quad (3.7)$$

$$e_T = \frac{E_T}{E} = c_T \cdot \frac{1}{c}; \varepsilon_T = \frac{\mu_T}{\mu}; (\varepsilon_T \leq 1),$$

here ε is total deformation, ε_{iT} is deformation dependent on each element E_T, and μ_T is temperature dependent stress, elasticity modulus and viscosity factor, respectively. The solution of equation (3.7) is in the form of (3.6):

Introducing nanoparticles to the viscous elastic liquid medium exposed to temperature, when assuming the assumption that the initial structure of the model does not change, we determine the change of its characteristics. For that we structure a new model that enables to describe approximately the changes. We imagine the elastic element of the Maxwell model exposed to temperature in the form of one-dimensional “chain” of interacting atoms (Fig 2,b).

Introduce nanoparticles to the heated viscous elastic medium Fig 3. Assume that every nanoparticle introduced in this case occupies intermediate place (may be several nanoparticles).

This time the structure of the elastic element consisting of rectilinear “chain” does not change. This case is obtained as a result from the assumption that when nanoparticles are introduced the existing model of the viscous-elastic medium is retained. Assume that the nanoparticles interact only with neighboring particles. Then for the heated chain with no nanoparticles, we can write:

$$R_x \cdot \sigma \approx F(a) = F(a_T) + F'(a_0) \cdot a_0 \cdot (\varepsilon - \varepsilon_T), F(a_T) = F(a_0),$$

$$\sigma = \frac{c \cdot a_0}{R_x} \cdot (\varepsilon - \varepsilon_T) = \frac{c \cdot a_0}{R_x} \cdot (\varepsilon - \alpha \cdot \varDelta T) = E \cdot (\varepsilon - \alpha \cdot \varDelta T), c = F'(a_0)$$

here a_0, a_T and a are the distance between atoms at the initial moment, in the case when the temperature effect, the pull and temperature effect are taken into account, $F(a_0)$ is interacting force between atoms at the initial moment, σ is stress in the "chain" $F(a)$ is the force existing between the atoms, R_x is a characteristic area of the cut of the "chain" of atoms ε_T, ε is deformation formed by the temperature effect and under the action of applied force and temperature. The force between the atoms does not change under the action of temperature, i.e. $F(a_T) = F(a_0)$,$c = F'(a_0)$ is rigidity of the undeformable "chain", $E = \frac{c \cdot a_0}{R_x}$ is elasticity modulus, α is a proportionality factor $\Delta T > 0$ is temperature.

Fig. 3 A linear heated "chain" with introduced nanoparticles.

If after introducing nanoparticals to the heated viscous elastic medium we apply the pull, for the stress in the "chain" we can write the following equality:

$$R_x \cdot \sigma = F_n \left(\frac{a}{2} \right),$$

here n shows that the given quality belongs to the nanoparticle. Taking into account the linear part of the Taylor series, we can write the following approximate equality:

$$R_x \cdot \sigma = F_n \left(\frac{a}{2} \right) \approx F_n \left(\frac{a_T}{2} \right) + F_n' \left(\frac{a_T}{2} \right) \cdot \left(\frac{a}{2} - \frac{a_T}{2} \right) + \cdots$$

We denote by $F_n' \left(\frac{a_T}{2} \right) = c_n$ the rigidity of the undeformed "chain" . Then

$$R_x \cdot \sigma = F \left(\frac{a_0}{2} \right) + c_n \cdot \left(\frac{a}{2} - \frac{a_T}{2} \right) \approx c_n \cdot \frac{a_0}{2} \left(\varepsilon - \varepsilon_T \right) = c_n \cdot \frac{a_0}{2} \left(\varepsilon - \alpha \cdot \Delta T \right).$$

Thus, we obtain $\sigma = \frac{c_n \cdot a_0}{2 \cdot R_x} (\varepsilon - \varepsilon_T) = E_n \cdot (\varepsilon - \alpha \cdot \Delta T)$, $E_n = \frac{c_n \cdot a_0}{2 \cdot R_x}$.

We imagine the viscous element of the heated viscous-elastic medium in the form of a liquid –filled cylinder compressed by a piston. Assume that the introduced nanoparticles do not change the model of the liquid exposed to temperature. In this case, the influences of nanoparticles in the heated liquid will similar to the influence of hard particles. The influence of nanoparticles can be different. The choice of effect depends on the interaction between physics-chemical properties of the heated liquid and nanoparticles. The influence of nanoparticles may be reduced to the change of viscosity factor for the model under consideration, i.e. we can introduce the quantity that differs from the viscosity of the heated liquid medium. According to the physics of the process we can write $\mu_n \leq \mu_n$.

Assume that introduction of nanoparticles to the heated liquid (suspension) reduces to increase in the friction form of liquid. Applying the same pull to the piston, the displacement rate will decrease at the expense of friction

$$\dot{\varepsilon}_n = \mu_n \cdot \sigma < \dot{\varepsilon} = \mu_T \cdot \sigma.$$

Thus based on equation (3.7) allowing for nanoparticles, for the model of viscous-elastic liquid (suspension) exposed to temperature, we get:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E_n} + \mu_n\sigma = \frac{\dot{\sigma}}{E_T \cdot e_n} + \mu_T \upsilon_n \sigma, \tag{3.8}$$

here $e_n = \frac{E_n}{E_T} = c_n \cdot \frac{1}{2 \cdot c_T}; \upsilon_n = \frac{\mu_n}{\mu_T}; (\upsilon_n \leq 1)$.

The solution of equation (3.8) is the following form:

$$\sigma(t) = E_T e_n \cdot [\varepsilon(t) - \varepsilon(0)\exp(-E_T e_n \mu_T \upsilon_n t) - \\ -E_T e_n \mu_T \upsilon_n \int_0^t \exp(-E_T e_n \mu_T \upsilon_n (t-\tau))\varepsilon(\tau) d\tau \Bigg] \tag{3.9}$$

For $\sigma = const = \sigma_0$ we get:

$$\varepsilon(t) = \sigma_0 \left(\frac{1}{E_n} + \mu_n t \right) = \frac{\sigma_0}{E_T \cdot e_T} + \mu_T \upsilon_n \sigma_0 t.$$

On the example of the Maxwell model it was shown that taking into account temperature effect in the model's elements, introduction of nanoparticles influences on change of its parameters. Character of change of these parameters and their estimations was described within the assumption of interaction of nanoparticles with atoms of liquid medium exposed to temperature.

References

1. Abdullayev S., Nagiyev F. Nanohydrodinamics Research center of high technologies Baku, 2011.-158 p.: il.ISBN 5-86874-164-1.
2. Christenson R. Introduction to theory of viscous-elasticity. M. Mir:1974, 338 p.
3. Farzaliyev M.H., Aslanov M.S. Apllication of seperation process of liquid form heterogemeous food systems. The III Repeblican scientific-partical conference "Food and textile industry's development prospectives and its duties" May 21, 2019, Azerbaijan State University of Economics (UNEC), Baku, p.186-188.
4. Machines and apparatus of food production.In 2 books Book 1. Textbook for Higer institutes. S.T. Antipov, I.T. Kretov, A.N. Ostrikov and others Editored by acad. V.A. Panfilov, M. Visshaya shkola, 2001, 703 p.
5. Ramazanov M.A., Aslanov M.s. Synthesis of toe Maxwell model based on nanoparticles. Digest Journal of Nanomaterials and Biostructures, ISSN 1842-3582, 7(2), Romania, April-June 2012, p.817-822.
6. Sergeev G. B. Nanochemisty: textbook / G.B. Sergeyev. – second edition. -M.: KSU , 2007.-336 p.: il. ISBN 978-5-98227-288-1