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Influence of heat transfer conditions on the flow regimes of an anomalously thermoviscous liquid

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Abstract. *Traditional models of viscous liquid flow assume constant viscosity, independent of pressure, temperature, and deformation rate. However, experiments show that incorporating variable viscosity and non-Newtonian properties better captures dynamic flow regimes and physical effects, such as oscillations in flow rate or shear stress. Temperature-dependent viscosity plays a crucial role in non-isothermal flows, influencing parameter distribution, critical Reynolds number, and flow rate. Many industrial liquids exhibit complex, non-monotonic viscosity–temperature behavior, forming high-viscosity zones ("viscous barriers") during flow with wall heat exchange. This study presents numerical results on how heat exchange conditions affect flow regimes and induce self-oscillations in an annular channel with anomalously thermoviscous liquid.*

Keywords. anomalously thermoviscous liquid · annular channel · viscous barrier · flow rate self-oscillations

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1 Introduction

Inhomogeneities in flows formed during their interaction with external fields, caused by the peculiarities of the physical properties of liquids, often lead to the emergence of remarkable

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hydrodynamic effects. The formation of various types of flow anomalies is characteristic of non-Newtonian liquids, which are widely used in technological processes for the hydrocarbon raw materials extraction and in the chemical industry.

A detailed description of the regularities of non-Newtonian systems dynamics is given in the well-known monograph [5]. The monograph describes the phenomenon of the occurrence of the polyacrylamide solution flow rate oscillation during its flow in a porous medium, which is caused by the effect of a constant pressure drop, the blocking of the filtration flow by polymer molecular clews and the presence of a finite relaxation time of the polymer system.

The paper [6] describes the occurrence of shear stresses oscillations when measuring the thixotropic liquid viscosity in a rotational viscometer. In this case, the mechanism of the occurrence of self-oscillations can be described using of the bond's destruction-restoration kinetics in a structured medium [5].

Flow rate oscillations can also occur not only in non-Newtonian systems, but also during filtration of liquid containing gas bubble nuclei formed in a porous medium near the saturation pressure [1, 5].

The given examples indicate that for the flow of liquids with variable physical properties dynamic systems with feedback are formed, which lead to oscillatory flow regimes.

The temperature dependence of viscosity plays a decisive role when calculating heat exchange devices. Thus, in the paper [7] the influence of the temperature dependence of viscosity on the hydraulic resistance value and, accordingly, on the liquid flow rate in the heat exchanger channel was demonstrated.

The temperature dependence of the viscosity for most liquids is satisfactorily approximated by an exponentially decreasing function. However, the viscosity of some liquids with a complex molecular structure (for example, polymer solutions and melts) has a non-monotonic dependence on temperature, which is determined by molecules polymerization processes at the stage of increasing viscosity and depolymerization when viscosity decreases.

The flow of a liquid with a non-monotonic dependence of viscosity on temperature, later called anomalously thermoviscous liquids, was first described in the paper [8]. It was found that the flow hydrodynamic parameters are significantly affected by the non-uniform distribution of the viscosity field in the channel, the parameters of which depend on the heat exchange conditions specified on the channel walls. For example, using first kind boundary conditions for temperature on the channel walls, the highly viscous region ("viscous barrier") formed in the channel has the shape of an elongated arc [8].

In the paper [3], oscillatory modes of the flow rate change of an anomalously thermoviscous liquid were described during flow in an annular channel, on the walls of which heat exchange conditions were implemented that changed abruptly along the channel length. It was shown that oscillatory flow regimes are caused by continuous cyclic movements of the "viscous barrier", which determines the hydraulic resistance magnitude and, consequently, the liquid flow rate. Thus, a jump in the heat exchange intensity leads to the formation of a dynamic system with feedback.

The aim of the present work is to establish the influence of heat exchange intensity on the nature of the flow rate periodic changes during the flow of liquid with a non-monotonic dependence of viscosity on temperature.

2 Problem statement

Let us consider the flow of an incompressible anomalously thermoviscous liquid in an annular channel formed by two coaxial cylinders of radii r_0 and R ($r_0 < R$), under the action of a pressure gradient $\Delta p/L$ (Fig. 1).

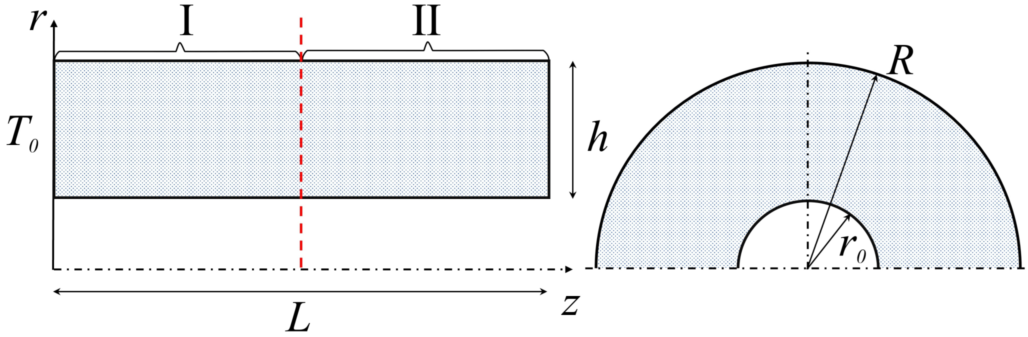

Fig. 1 Scheme of the annular channel

Hot liquid with a constant temperature T_0 enters the channel. On the inner and outer walls of the annular channel, convective heat exchange with the environment occurs, and the intensity of heat exchange has different constant values in the intervals [0, L_1] and [L_1, L], L_1 = 0,45 (zones I and II in Fig. 1). The temperature of the liquid entering the channel is higher than the ambient temperature, so the liquid cools as it passes through the channel and a non-uniform temperature field is formed in the channel.

The mathematical model, consisting of the continuity equation, generalized Navier-Stokes equations and the temperature equation, in cylindrical coordinates in dimensionless variables and taking into account axial symmetry, has the form [4]:

$$\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{u_r}{r}=0,$$

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+u_z\frac{\partial u_r}{\partial z}=$$

$$=-\frac{\partial p}{\partial r}+\frac{1}{Re}\left[2\frac{\partial}{\partial r}\left(\mu\frac{\partial u_r}{\partial r}\right)+\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)\right)+\frac{2\mu}{r}\left(\frac{\partial u_r}{\partial r}-\frac{u_r}{r}\right)\right],$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}=$$

$$=-\frac{\partial p}{\partial z}+\frac{1}{Re}\left[\frac{\partial}{\partial r}\left(\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)\right)+2\frac{\partial}{\partial z}\left(\mu\frac{\partial u_z}{\partial z}\right)+\frac{\mu}{r}\left(\frac{\partial u_z}{\partial r}-\frac{\partial u_r}{\partial z}\right)\right],$$

$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+u_z\frac{\partial T}{\partial z}=\frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right),$$

where u_r and u_z are the radial and axial components of the velocity vector, p is the pressure, T is the temperature, $\mu = \mu(T)$ – is the dynamic variable viscosity of liquid, $Re=\frac{\rho u_0 R}{\mu_{\max}}$, $\mathrm{Pe}=\frac{u_0 R}{\chi}$ are the dimensionless Reynolds and Peclet criteria, ρ is the liquid density, u_0 is the characteristic velocity, μ_{max} is the maximum liquid dynamic viscosity in the temperature range under consideration, χ is the thermal diffusivity coefficient.

Dimensionless initial and boundary conditions have the form:

$$u_r(r,\ z,\ 0)=u_z(r,\ z,\ 0)=0, p(r,\ z,\ 0)=0, T(r,\ z,\ 0)=0,$$

$$p(r,\ 0,\ t)=1,\quad p(r,\ \Lambda,\ t)=0,$$

$$u_r(\delta_o,\ z,\ t)=u_r(1,\ z,\ t)=u_z(\delta_o,\ z,\ t)=u_z(1,\ z,\ t)=0,$$

$$T(r,\ 0,\ t)=1,$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=\delta_o} = \left.\frac{\partial T}{\partial r}\right|_{r=1} = \begin{cases} \alpha_I T, & \text{if } 0 \leq z < \Lambda_1 \\ \alpha_{II} T, & \text{if } \Lambda_1 < z \leq \Lambda \end{cases}$$

where α_I and α_{II} are the dimensionless heat transfer coefficients in zone I and zone II, respectively, $\Lambda = \frac{L}{R}$, $\Lambda_1 = \frac{L_1}{R}$, $\delta_o = \frac{r_o}{R}$ – are the dimensionless geometric parameters of the annular channel.

The non-monotonic dependence of liquid viscosity on temperature has the form (Fig. 2):

$$\mu\left(T\right) = e^{-\beta(T-1/2)^2},$$

where $\beta > 0$ is a parameter describing the character of the viscosity change.

It should be noted that the parameter β is related to the value of the full width at half maximum of the Gaussian curve (FWHM) by the expression:

$$FWHM = 2,355\sqrt{\frac{1}{2\beta}} \approx \frac{1,665}{\sqrt{\beta}}.$$

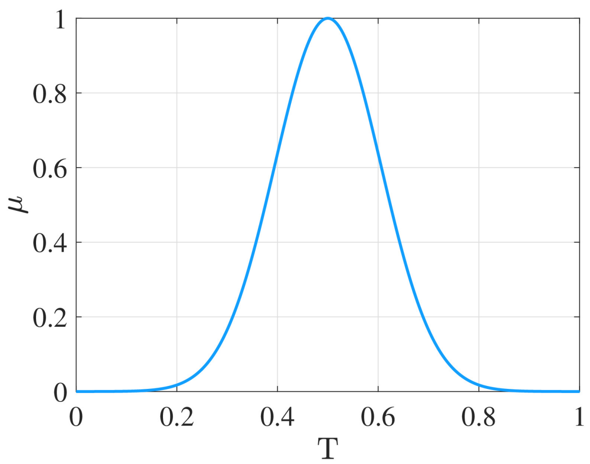

Fig. 2. Dependence of liquid viscosity on temperature (β = 45, FWHM = 0,248)

3 Results

The finite volume method and the SIMPLE (Semi-Implicit Method for Pressure-Linked Equation) algorithm [9], which was modified to take into account the variable viscosity coefficient, were used to numerically solve the equations of the mathematical model. The original computer code was written in the C++ programming language in the Qt Creator development environment.

Fig. 3 shows the viscosity distribution patterns at successive moments in time during the flow of anomalously thermoviscous liquid in an annular channel, on the walls of which in zone I heat exchange is specified with a heat transfer coefficient of α_I = 20, and the walls in zone II are thermally insulated (α_{II} = 0).

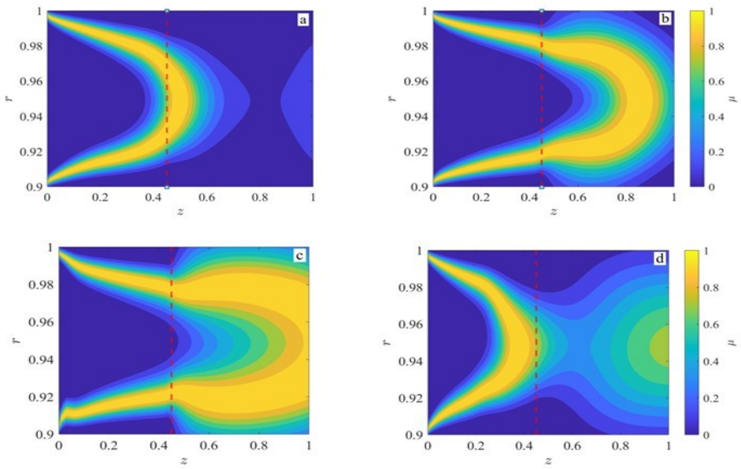

Fig. 3. Viscosity distribution during the anomalously thermoviscous liquid flow at α_I = 20 and α_{II} = 0 at different successive moments of time. The red dotted line indicates the boundary of the abrupt change in heat exchange conditions

Immediately after the fluid flow start at time t = 0, under the action of the pressure difference, the fluid flow rate increases linearly, reaching a certain maximum value, which is limited by resistance forces due to the "viscous barrier" formation (Fig. 3a). As a result of the "viscous barrier" movement following the anomalously thermoviscous liquid flow, the heated liquid following it is introduced into the heat-insulated zone of the channel, which leads to the formation of a secondary "viscous barrier", the isolines of which intersect the channel walls and, thus, determine the growth of resistance to the flow (Fig. 3b). For some time, the secondary "viscous barrier" expands, being in a common contour with the primary one (Fig. 3c). In this case, the liquid flow slows down and heat exchange becomes the determining factor in the formation of highly viscous zones, leading to the separation of the primary and secondary "viscous barriers". In the next step, the primary "viscous barrier" takes on its original form, and the secondary one leaves the channel (Fig. 3d). The process then repeats, acquiring a cyclical nature.

Figures 4 and 5 show the dependences of the anomalously thermoviscous liquid flow rate on time for different values of the dimensionless heat transfer coefficient and the corresponding phase portraits.

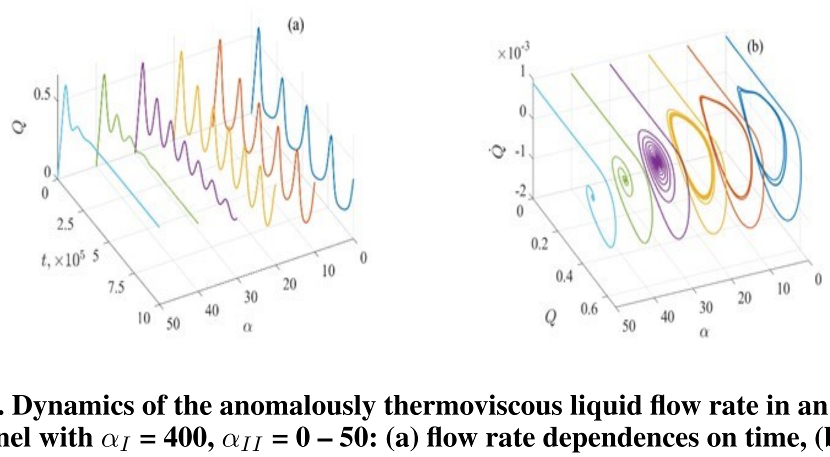

Fig. 4. Dynamics of the anomalously thermoviscous liquid flow rate in an annular channel with α_I = 400, α_{II} = 0 – 50: (a) flow rate dependences on time, (b) phase trajectories

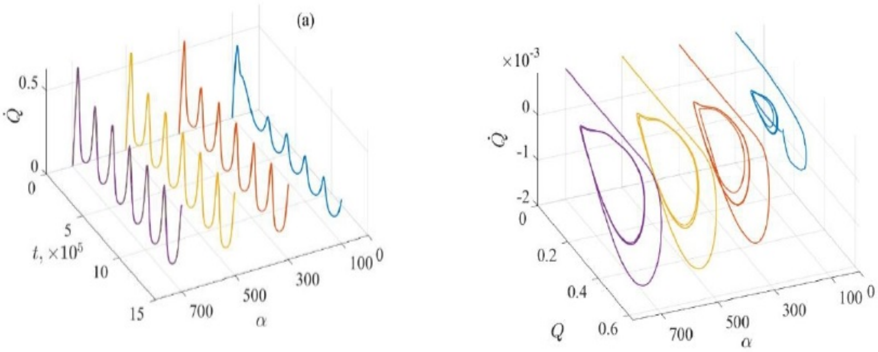

Fig. 5. Dynamics of the anomalously thermoviscous liquid flow rate in an annular channel with α_I = 90 – 700, α_{II} = 0: (a) flow rate dependences on time, (b) phase trajectories

The results shown in Fig. 4 were obtained for a channel on the walls of which in zone I (located near the entrance to the channel, see Fig. 1) a fairly intense heat exchange occurs (α_I = 400), while in zone II the intensity of heat exchange was much less (α_{II} = 0 – 50).

The results shown in Fig. 5 were obtained for a channel whose walls in zone II were thermally insulated (α_{II} = 0), while in zone I heat exchange conditions of varying intensity were set (α_I = 90-700).

It is evident that, depending on the heat exchange conditions, both damped and undamped flow rate self-oscillations can be observed. In the case of damped oscillations, the system tends to some stable equilibrium state and the fluid flow rate becomes constant over time. For the case of undamped flow rate oscillations, the phase trajectory tends to a certain closed curve. The fact that any initially given point on the phase plane, regardless of whether it is outside or inside the curve, tends towards it, indicates that a stable limit cycle is being formed. In this case, the liquid flow will occur in the mode of undamped oscillations. It should be noted that with a slight decrease in the magnitude of the pressure difference due to which the flow occurs, flow rate fluctuations are not observed.

4 Conclusion

In the present paper, the influence of heat exchange intensity on the anomalously thermoviscous liquid flow rate dynamics in an annular channel was studied. Using numerical modeling, it was found that when a liquid with a non-monotonic Gaussian-type viscosity-temperature dependence flows in an annular channel with piecewise constant heat exchange on the walls, the flow rate self-oscillations may occur.

It is shown that the flow rate self-oscillations are a consequence of the continuous cyclic movement of the “viscous barrier” – a highly viscous region of the flow, which is a significant component of the total hydraulic resistance to the liquid flow in the channel. An increase in the “viscous barrier” size leads to the flow slowdown, which in turn leads to an increase in the 0influence of heat exchange and, accordingly, to a decrease in hydraulic resistance.

Various modes of the flow rate self-oscillations were discovered, time dependences of flow rate and corresponding phase portraits were constructed for different values of the parameter characterizing the heat exchange intensity. It has been established that if a sufficiently high value of heat exchange intensity is set on the walls of the channel’s first half, then, depending on the heat exchange intensity on the walls of the channel’s second half, various modes of the flow rate self-oscillations arise: undamped self-oscillations in the complete absence of heat exchange and damped self-oscillations at finite heat exchange intensities. If the walls of the channel's second half are thermally insulated (there is no heat exchange), then at different heat exchange intensities on the walls of the channel's first half either undamped the flow rate self-oscillations may occur, or there will be no oscillatory modes of the flow rate change.

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References

1. Buevich, Yu.A., Leonov, A.I.: Natural oscillations in Couette flows of an incompressible Maxwellian fluid, J. Appl. Mech. Tech. Phys. 7, 68-69 (1966).
2. Kalur, G.C., Frounfelker, B.D., Cipriano, B.H., Norman, A.I., Raghavan, S.R.: Viscosity increase with temperature in cationic surfactant solutions due to the growth of wormlike micelles, Langmuir 21 (24), 10998-11004 (2005).
3. Kireev, V.N., Mukhutdinova, A.A., Urmancheev, S.F.: Self-oscillation mode during the flow of an anomalous thermoviscous liquid: Reports of the Russian Academy of Sciences. Physics, technical sciences 514 (1), 63-69 (2024). (in Russian)
4. Kochin, N.E., Kibel, I.A., Roze, N.V.: Theoretical hydromechanics, John Wiley & Sons Ltd., Chichester (1964).
5. Mirzadzhanzade, A.Kh., Khasanov, M.M., Bakhtizin, R.N.: Modeling of oil and gas production processes. Nonlinearity, nonequilibrium, uncertainty, Moscow–Izhevsk: Institute of Computer Research (2004). (in Russian)
6. Myerholtz, R.W.: Oscillating flow behavior of high-density polyethylene melts, J. Appl. Polimer Sci. 2 (5), 300–307 (1967).
7. Petukhov, B.S.: Heat transfer and friction in turbulent pipe flow with variable physical properties, Adv. Heat Trans. 6, 503-564 (1970).
8. Patankar, S.V.: Numerical Heat Transfer and Fluid Flow, McGraw Hill, New York (1980).
9. Urmancheev, S.F., Kireev, V.N.: Steady flow of a fluid with an anomalous temperature dependence of viscosity, Dokl. Phys. 49 (5), 328-331 (2004).