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Formulation and solution of long-line equations with distributed parameters and nonlinear conductivity

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Abstract. *High-voltage overhead power lines are used in power systems to transmit high power over long distances. The characteristics of ultra-high-voltage overhead lines 500÷1150 kV require accounting for active power losses due to alternating current corona. Modern synchronized vector measurement systems for active power and voltage have accuracy classes of 0,2 and 0,1, respectively. Typically, when solving the long-line equation, active power losses due to corona are represented as conductivity. This is equivalent to accounting for specific corona power loss characteristics as a 2nd-degree power function. It has been established that the actual characteristics of specific corona losses as a function of voltage, depending on weather conditions, are a 4 ÷ 8th-degree power function. Solving differential equations for a long line with nonlinear conductivity is impossible. This paper proposes the formation of nonlinear equations for a long line with distributed parameters by representing specific corona losses as nonlinear conductivity. An approach is proposed for solving nonlinear equations of length with distributed parameters by representing specific losses due to corona by nonlinear conductivity.*

Keywords. long line equations · distributed parameters · overhead power lines · corona losses · nonlinear conductivity

Mathematics Subject Classification (2010): 35L50, 94C05

1 Introduction

To date, the most accurate and standard method for modeling steady-state overhead line modes has been the long line equations (LLE) with distributed parameters in hyperbolic functions (Fig. 1). When deriving the LLE, an assumption was made about the quadratic dependence of active power losses due to corona on voltage. Numerous measurements of active power losses due to corona in ultra-high voltage (UHV) overhead lines (OHL) and the processing of experimental results have established that the dependences of active power

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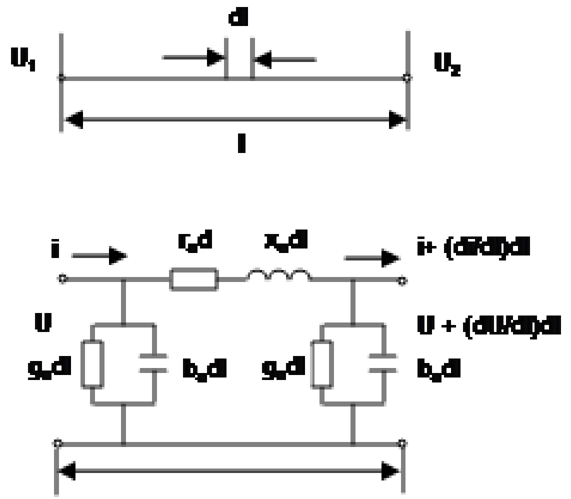

Fig. 1.1 Representation of a line with distributed parameters

losses due to corona are a function of voltage of 4th to 8th degree. In this regard, this article proposes a methodology, algorithm, and software implementation for modeling active power losses due to corona as a function of voltage, a real dependence with an arbitrary degree, taking into account the voltage distribution along the line. It is difficult to obtain analytical equations for a line with distributed parameters in hyperbolic functions, taking into account the function of voltage, with dependencies that differ from the second degree by mathematical transformations. Currently, UHV transmission lines 220 kV and higher are widely used in electric power systems for long-distance power transmission.

These lines are long, and their specific features require that power losses due to corona be taken into account when modeling. Therefore, adequate modeling of UHV transmission line modes is of great importance.

2 Modeling of overhead power line (OHL) using long line equations

The equations of a long line for steady-state conditions can be represented as a chain with distributed parameters and have the following form [1]-[3]:

$$\begin{aligned} \dot{U}_1 &= \dot{U}_2 ch\dot{\gamma}_0\ell + \sqrt{3}\cdot \dot{I}_2\dot{Z}_2 sh\dot{\gamma}_0\ell\ ; \\ \dot{I}_1 &= \dot{I}_2 ch\dot{\gamma}_0\ell + \frac{\dot{U}_2}{\sqrt{3}\cdot\dot{Z}_2} sh\dot{\gamma}_0\ell\ , \end{aligned} \tag{2.1}$$

where Z_{in} is the line's wave impedance, $\gamma_0 = \beta_0 + j\alpha_0$ is the wave propagation coefficient per unit length, $Z_0 = r_0 + jx_0$ is the line's specific resistance, $Y_0 = g_0 + jb_0$ is the line's specific conductivity, g_0 is the line's specific active conductivity, and b_0 is the line's specific capacitive conductivity. Also, $\dot{\gamma}_0 = \sqrt{\dot{Z}_0\dot{Y}_0}$, α_0 is the attenuation coefficient of the electromagnetic wave per unit length of the line, and β_0 is the coefficient of change in the phase of the electromagnetic wave per unit length of the line.

In long-distance and ultra-high-voltage transmission lines, it becomes necessary to account for the wave nature of energy propagation. In this case, the analysis of transmission line operating modes is based on a representation of the line with distributed parameters (Fig. 1.1).

Table 2.1 Characteristics of specific corona losses as a function of voltage

Overhead-line voltage, kV	Wire brand	Splitting pitch, cm	Good weather		Dry snow		Rain		Granular frost		Annual average	
			ΔP_{k0}, W/m	ρ	ΔP_{k0}, W/m	ρ	ΔP_{k0}, W/m	ρ	ΔP_{k0}, W/m	ρ	ΔP_{k0}, W/m	ρ
500	3AC330/43	40	2.39	7.0	10.9	7.3	33.7	5.5	92.4	4.7	11.0	5.6
	3AC400/51	40	1.73	6.4	7.73	6.7	26.7	6.1	77.9	5.2	8.7	5.8
	3AC500/64	40	1.32	6.0	5.75	6.3	20.5	6.5	64.2	5.4	6.9	5.8

The characteristic impedance of a line is a function of the line parameters

$$\dot{Z}_2 = \sqrt{\dot{Z}_0/\dot{Y}_0}, \tag{2.2}$$

In the long line equations, corona losses are represented as a specific conductivity:

$$\Delta P_{k_0} = g_0 \cdot U_{nom}^2,$$

where ΔP_{k_0} is the specific power loss due to corona at the nominal line voltage U_{nom}.

When calculating the overhead power line mode, the following main weather groups are usually taken into account [8 - 11], each of which is characterized by its own average level of losses: good weather; dry snow; rain; frost, etc.

Based on the approximation of the specific corona loss characteristics (SCLC) obtained using the generalized characteristics method versus voltage, it was established that the corona loss dependence is a function of voltage of the 4th to 8th power. For example, for some 500 kV overhead lines, the specific corona loss characteristics obtained using the generalized characteristics method are presented in Table 2.

For the SCLC, under different weather conditions, the indicators of the stresses have fractional values in the range from 4 to 8. For weather conditions: good weather and dry snow, the degree of SCLC varies around 7. For frost, around 4. For average annual losses, the degree of SCLC is around 5.

Typically, during modeling, the specific losses due to the corona of the power line are specified for different weather conditions in the power line [4 - 6]. To calculate the specific characteristics of the power losses due to the corona, generalized characteristics methods are used [8].

In a distributed-parameter cable, corona power losses are taken into account by specific conductivity. This corresponds to corona power losses in overhead lines, taking into account the voltage distribution along the line, which can be represented as

$$\Delta P_{k_2} = \Delta P_{k_0} \cdot \int_0^L \left(\frac{U_l}{U_{nom}}\right)^2 dl, \tag{2.3}$$

Where U_{nom} is the nominal voltage of the line, U_1 is the voltage distribution along the line, and L is the length of the line in *km*.

The active power losses due to the corona of the overhead power line, obtained by integrating the voltage along the line taking into account the characteristic of the corona losses from the voltage at an arbitrary ρ-th degree, have the form

$$\Delta P_{kVL\rho} = \Delta P_{k_0} \cdot \int_0^L \left(\frac{U_l}{U_{nom}}\right)^\rho dl\ . \tag{2.4}$$

Since analytical integration of (2.4) for an arbitrary ρ-th degree is not possible, numerical methods must be used for integration.

Methodological errors, depending on the model and modeling method used, can have values comparable to the errors in measuring the mode parameters [16]. In this regard, requirements for permissible methodological errors in modeling the overhead power line mode become relevant.

Total error for parameter ΔU can be approximately represented as the sum of its components [7].

Total error of the OS for ΔU **in** [7] is approximately presented as

$$\Delta U = \Delta_{mod} + \Delta_{meas} \tag{2.5}$$

where Δ_{mod} **is** a deviation caused by the inadequacy of mathematical VL model **used** to calculate the optimal values, and Δ_{meas} is a component caused by the inaccuracy of measurements of the current state of the regime.

The required accuracy of Δ_{mod} can be determined from (2.5) if Δ_{mod} is required to be a statistically insignificant factor among all factors.

Moving on to (2.5) in the norms, we have

$$\|\Delta U\| \leq \|\Delta_{mod}\| + \|\Delta_{meas}\| .$$

Hence, provided that the norm $\|\Delta_{mod}\|$ constitutes a specified fraction of $\|\Delta U\|$, i.e., $\|\Delta_{mod}\| = \varepsilon \|\Delta U\|$, we obtain

$$\|\Delta_{mod}\| \leq \frac{\varepsilon}{1-\varepsilon} \left(\|\Delta_{change}\|\right) . \tag{2.6}$$

Using the optimal overhead-line model, one can proceed to requirements for the accuracy of the controlled operating parameters; $\|\Delta_{meas}\|$ determines the required accuracy of the mathematical model in state estimation. For example, with a measurement error of 0.2%, the methodological error should not exceed 0.05%.

Calculations of steady-state modes of high-voltage power transmission lines were performed. A 500 kV overhead power line with a length of 350 km was used in the modeling.

3 Methodology for modeling the modes of high-voltage power transmission lines based on the results of the LLE with representation of equivalent conductivity.

This article considers the issues of increasing the accuracy of modeling AC power lines in steady-state conditions based on the equations of a line with distributed parameters in hyperbolic functions.

Taking into account the corona losses of the ρ-th power, the power losses due to the corona of a transmission line of length L, and the voltage distribution, we have

$$\Delta P_{kVL\rho} = \Delta P_{k_0} \cdot \int_0^L \left(\frac{U_l}{U_{nom}}\right)^\rho dl. \tag{3.1}$$

The calculation of voltage along the overhead power line is carried out using the equation of a long line with distributed parameters (2.1).

Expression (2.4) can be rewritten as

$$\Delta \mathrm{P}_{\mathrm{kVL}\rho} = \Delta \mathrm{P}_{k_0} \cdot L_{eq\rho} = \Delta \mathrm{P}_{k_0} \cdot L \cdot k_{int\rho}, \tag{3.2}$$

where the component

$$L_{eq\rho} = \int_0^L \left(\frac{U_l}{U_{nom}}\right)^\rho dl$$

can be interpreted as the equivalent length of the transmission line in terms of active power losses due to corona, taking into account the dependence of power losses due to corona on the voltage of the ρ-th degree.

In (3.2), $k_{int\rho}$ is the relative value of the voltage integral along the line:

$$k_{int\rho} = \frac{1}{L} \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{\rho} dl = \frac{L_{eq\rho}}{L}.$$

For overhead power transmission lines with transmitted power less than the natural power, the values of $k_{int\rho}$ are greater than one.

Modeling the voltage distribution along the line at the 5th degree, the results according to equations (2.1) having an exponent of 2 will have an error of the order of 1,11/1,04=1,067. The case of real characteristics of corona losses when modeling the modes of overhead power lines has been considered in [17 - 18].

In this work, to model overhead power lines with nonlinear corona conductivity and real characteristics according to the LLE (2.1), we propose using the reduced equivalent corona conductivity $g_{0,\text{eq},kv,\rho}$.

The values of the stress integral along the line in accordance with equations (2.1) for the 2nd, 3rd and 5th degrees of stress for the LLE under the above conditions can be presented in the following simplified form:

$$\int_0^L \left(\frac{U_l}{U_{nom}} \right)^{\rho} dl \cong \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{2} dl \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl \tag{3.3}$$

or

$$\Delta P_{k\rho} = \Delta P_{k0} \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{\rho} dl \cong \Delta P_{k0} \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{2} dl \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl \tag{3.4}$$

For the equivalent nonlinear conductivity of the line in terms of active power losses due to corona, taking into account the dependence of power losses due to corona on voltage of the ρ-th degree in equations (2.1), we have

$$g_{eq(\rho-2)} = \frac{\Delta \mathrm{P}_{k_0}}{\mathrm{U}^2_{nom}} \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl = g_0 \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl, \tag{3.5}$$

$$\Delta P_{kVL\rho} \cong \frac{\Delta \mathrm{P}_{k_0}}{\mathrm{U}^2_{nom}} \cdot \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl \tag{3.6}$$

At the first stage, the specific equivalent conductivity is calculated as a nonlinear function of the voltage level, with correction of the equivalent conductivity corresponding to weather conditions.

In accordance with (3.5 - 3.6), a method has been developed for calculating corona losses by specific equivalent active conductivity,

$$g_{eq(\rho-2)} = \frac{\Delta \mathrm{P}_{k_0}}{\mathrm{U}^2_{nom}} \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl = g_0 \cdot \int_0^L \left(\frac{U_l}{U_{nom}} \right)^{(\rho-2)} dl. \tag{3.7}$$

In (3.6), the following integral must be evaluated:

$$\int_0^L \left(\frac{U_l}{U_{nom}}\right)^{\rho-2} dl.$$

If $\rho = 5$, then $\rho - 2 = 3$, and the exponent in the integral is equal to 3.

However, the influence of the magnitude of specific corona losses on the voltage distribution along the line and, accordingly, the inverse influence on the value of the integral on the equivalent conductivity is not taken into account here. To account for mutual influence, iterative refinement is required for $\int_0^L \left(\frac{U_l}{U_{nom}}\right)^3 dl$ taking into account

$$g_{eq(\rho-2)} = g_0 \cdot \int_0^L \left(\frac{U_l}{U_{nom}}\right)^{(\rho-2)} dl.$$

The second stage. Using the LLE (2.1) and expressions (3.5 - 3.7).

The block diagram for modeling an overhead power line using the LLE with nonlinear corona conductivity and voltage-dependent power losses of arbitrary degree is shown in Fig. 3.

1 Calculation of the overhead line mode according to equations (2.1) corresponding to the voltage exponent ρ=2.
2 Calculation of the voltage integral of the (ρ-2)-th degree using the expression

$$k_{int(\rho-2)} = \frac{1}{L} \cdot \int_0^L \left(\frac{U_l}{U_{nom}}\right)^{(\rho-2)} dl.$$

3 Comparison of the corona-loss values obtained by direct integration of voltage along the overhead line to the ρ-th power with the values obtained from (2.1), using the equivalent conductivity $g_{eq(\rho-2)}$. This maintains equality between the losses obtained by the two methods.

When the desired accuracy is achieved, the iterative process 2.1 - 2.3 is terminated.

In accordance with the methodology and algorithm 1-3, the Mathcad program was developed.

4 Calculation example.

Calculation results for a 500 kV power transmission line with 3*AC330/43 wires and a length of 350 km. The specific parameters of the circuit are taken to be r_0 = 0,029 Ohm/km, x_0 = 0,299 Ohm/km, and b_0 = 3,74·10^{-6} S/km. Specific corona losses for a 500 kV power transmission line may vary within the range of 5÷100 kW / km or more. Calculations were performed for corona loss levels of 5, 20, 40, 96 kW / km. For hoar frost, ΔP_{k_0} = 96 kW/km was assumed. Specific corona conductivity for hoar frost was supposed to be g_0 = 3,84·10^{-7} S/km.

The calculations were carried out with the following parameters of the overhead line end mode: overhead line end voltage U_2 = 500 kV, active power P_2 = 900 and reactive power Q_2 = 7,327 at the end of the overhead line. For the equivalent conductivity corresponding to

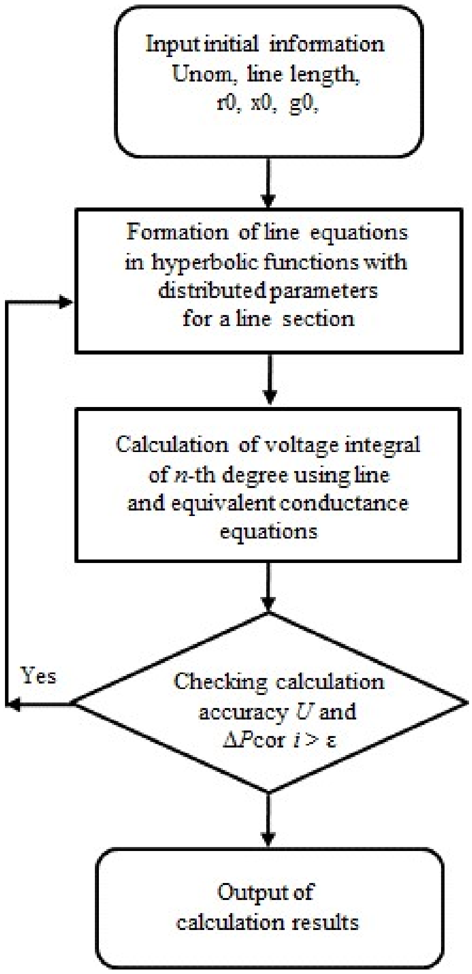

Fig. 4.1 Block diagram for modeling an overhead power line using the LLE with nonlinear conductivity and voltage-dependent power losses of arbitrary degree

Table 4.1 Results of simplified modeling of the mode integral based on the initial LLE (2.1)

No.	Specific conductivity	Voltage integral for exponent ρ		
	g_0, 10^{-7} S/km	2	3	5
1	3.84	1.04204	1.0639	1.1096
2	4.08608	1.042129	1.06408	1.1098

the corona losses, calculated using the voltage integral of the third degree, we have g_{eq03}= 4,08607$\cdot 10^{-7}$ cm/km.

The simplified integral calculations in Table 4 show that the error in determining the equivalent specific corona conductivity is within 0.8%.

The results of the second iteration of the calculation of the components of line power losses are presented in the form of a printout below.

For voltage and current according to the data at the end of the OL based on LLE

$$I_2 := \frac{P_2 - jQ_2}{\sqrt{3}U_2} = 1.039230 - 0.008460i$$

$$U_1 := U_2 \cosh(\gamma L) + \sqrt{3} I_2 Z_c \sinh(\gamma L) = 484.965507 + 190.993624i$$

$$I_1 := \frac{1}{\sqrt{3}Z_c} U_2 \sinh(\gamma L) + I_2 \cosh(\gamma L) = 1.008281 + 0.376045i$$

$$U_{11}(y) := U_2 \cosh(\gamma y) + \sqrt{3} I_2 Z_c \sinh(\gamma y)$$

$$I_{11}(y) := \frac{1}{\sqrt{3}Z_c} U_2 \sinh(\gamma y) + I_2 \cosh(\gamma y)$$

$$U_{\text{mod}} := |U_1| = 5.212198 \times 10^2$$

$$I_{\text{mod}} := |I_1| = 1.076123$$

U(y) := |U_{11}(y)|

I(y) := |I_{11}(y)|

Voltage angle at the beginning of the line along the LLE

$$\tan\varphi_U := \frac{Im(U_1)}{Re(U_1)} = 0.393829$$

$$\varphi_U := \arctan(\tan\varphi_U) = 375.175512 \times 10^{-3}\ \text{rad}$$

$$\varphi_{\text{grad}\,U} := \varphi_U \cdot \frac{180}{\pi} = 21.495973$$

Power at the beginning and the end of the line according to the LLE

$$S_{12} := \sqrt{3}(U_1 \overline{I_1}) = 722.541367 + 649.421871i$$

$$S_{21} := \sqrt{3}(U_2 \overline{I_2}) = 900.000000 - 7.327000i$$

$$S_{12\,\text{mod}} := |S_{12}| = 971.501309$$

$$S_{21\,\text{mod}} := |S_{21}| = 900.029824$$

$$P_{12} := \sqrt{3} Re(U_1 \overline{I_1}) = 9.713404 \times 10^2$$

$$P_{21} := \sqrt{3} Re(U_2 \overline{I_2}) = 9.000000 \times 10^2$$

$$\Delta P_{\Sigma UDL} := P_{12} - P_{21} = 71.340449$$

$$\Delta P_{\text{kor}UDL} := \Delta P_{\Sigma UDL} - \Delta P_{N2} = 37.259328$$

Voltage integration in Mathcad for $\boldsymbol{\rho} = 5$ and $\boldsymbol{\rho} - 2 = 3$

$\text{IntU}_2 := \int_0^L \left(\frac{U(y)}{U_2}\right)^2 dy = 3.6372 \times 10^2$	$K_{\text{IntU}_2} := \frac{\text{IntU}_2}{L} = 1.0392 \times 10^0$
$\text{IntU}_3 := \int_0^L \left(\frac{U(y)}{U_2}\right)^3 dy = 3.7085 \times 10^2$	$K_{\text{IntU}_3} := \frac{\text{IntU}_3}{L} = 1.0596 \times 10^0$
$\text{IntU}_4 := \int_0^L \left(\frac{U(y)}{U_2}\right)^4 dy = 3.7816 \times 10^2$	$K_{\text{IntU}_4} := \frac{\text{IntU}_4}{L} = 1.0805 \times 10^0$
$\text{IntU}_5 := \int_0^L \left(\frac{U(y)}{U_2}\right)^5 dy = 3.8567 \times 10^2$	$K_{\text{IntU}_5} := \frac{\text{IntU}_5}{L} = 1.1019 \times 10^0$
$\text{IntU}_7 := \int_0^L \left(\frac{U(y)}{U_2}\right)^7 dy = 4.0130 \times 10^2$	$K_{\text{IntU}_7} := \frac{\text{IntU}_7}{L} = 1.1466 \times 10^0$

Table 4.2 Iterative modeling results based on the initial LLE (2.1)

Iteration No.	Specific conductivity	Voltage integral for exponent ρ			Simplified-integral error	
	g_0, 10^{-7} S/km	2	3	5	$k_{int2}k_{int3}$	$k_{int5} - k_{int2}k_{int3}$, %
1	3.84	1.0420	1.0639	1.1096	1.1086	0.0877
2	4.0861	1.0421	1.0641	1.1098	1.1089	0.0791

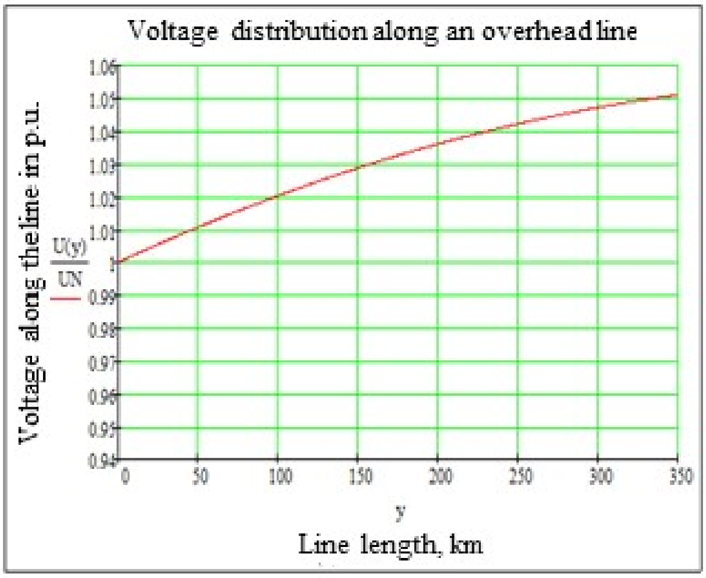

Fig. 4.1 Voltage distribution along a 500 kV power transmission line

After two iterations, the refined values of the voltage integral along the 3rd degree line and the equivalent specific active conductivity of the overhead line have high accuracy.

The results of modeling the 500 kV overhead-line mode using the LLE with iterative refinement of the equivalent specific corona conductivity for a fifth-degree voltage characteristic are shown in Table 4.

The specific equivalent conductivity of the overhead line, reduced to the third-degree integral, is $g_{eq03} = 4.0861 \times 10^{-7}$ S/km, which differs from the value at U_{nom} by 6.4%. The VL-ETALON software was developed to account for nonlinear corona conductivity.

Figures 4 and 4 show the voltage and current distributions along the UHV transmission line based on the LLE, taking into account nonlinear conductivity for a transmitted power of 700 MW, $U_2 = 500$ kV, and $P_{cor} = 96$ kW/km.

The adequacy and efficiency of the method and the assessment of methodological errors of mathematical modeling are analyzed using the example of a 500 kV power transmission line with a length of 350 km at a voltage of 500 kV at the end of the power transmission line.

The proposed method for modeling high-voltage power transmission lines based on the LLE fully reflects the dependence and influence of the distributed nature of the parameters of the power transmission line and the mutual influence of the real characteristics of specific corona losses on voltage and the quadratic dependence of wire heating losses on voltage along the line.

The reliability of the proposed methodology has been verified and confirmed by comparing the results of modeling a 350 km high voltage power transmission line.

1 According to the obtained results, the LLE was used to model the corona losses of the SCLC by.

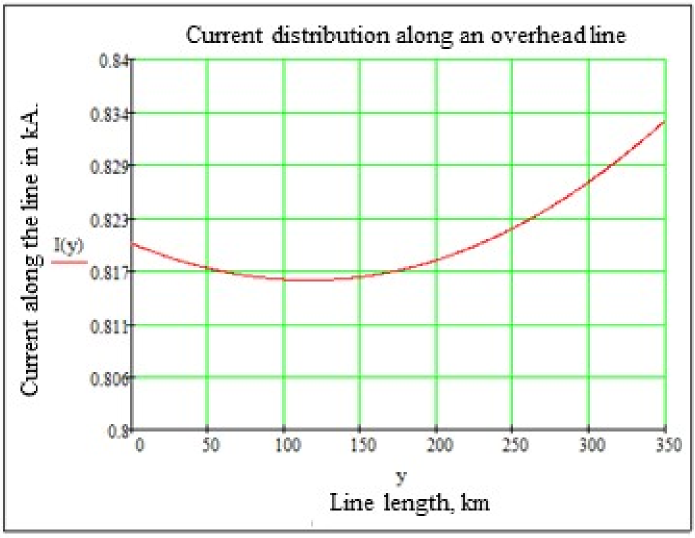

Fig. 4.2 Current distribution along a 500 kV power transmission line

Table 4.3 Chain-modeling results and comparison with the proposed method

	No. of sections	No. of nodes	Section length, km	Operating parameters of the UHV line					Relative modeling error, %	
				U_{con}	δ_{con}	ΔP_{cor}	ΔP_n	ΔP_{sum}	ΔP_{sum}	U_1
1	1	2	350	496.413	22.159	38.847	35.398	74.245	4.055	-0.717
2	2	3	175	499.140	21.657	38.000	34.731	72.732	1.934	-0.172
3	3	4	116.66	499.619	21.567	37.753	34.505	72.258	1.271	-0.076
4	10	11	35	499.962	21.503	37.407	34.203	71.610	0.363	-0.008
5	14	15	25	499.978	21.500	37.364	34.168	71.532	0.252	-0.004
6	25	26	14	499.990	21.497	37.316	34.130	71.445	0.131	-0.002
7	35	36	10	499.997	21.496	37.299	34.115	71.414	0.088	-0.001
8	Equivalent-conductivity method			500.000	21.4959	37.2917	34.0809	71.3727	0	0

2 Modeling of high-voltage power transmission lines (HVH) using chain diagrams with lumped parameters of varying lengths up to 2 km was performed using the program, and steady-state equations were solved using Newton's method using a program developed in the MatLab environment, with automated specification of the number and lengths of sections. The SCLC was represented by equivalent conductivity of the ρ-th order at the nodes.

The validity of the proposed approach was tested using a 500 kV, 350 km power transmission line. Results for chain schemes with section lengths from 3.535 to 350 km are given in Table 4.

For 5 km sections, the number of iterations is 49, and for 3,535 sections, the number of iterations increases to 200.

Methodological errors in modeling the modes of high-voltage power transmission lines, depending on the number and length of sections of chain circuits, can reach 0.7% for the node voltage.

Thus, the methodological errors of modeling the modes of the transmission line with high voltage are greater than the measurement error of the active power measuring complex.

Sufficient and acceptable accuracy of modeling 500 kV high-voltage power lines are achieved with sections less than 50 km long.

For transmission lines with sections 10 km or less in length, the dimension and the cost of solving the problem (number of iterations) using the Newton-Raphson method increases significantly, which requires calculations to be carried out with double precision and leads to a sharp increase in the complexity of the solution. For sections of chain circuits with lengths less than 50 km, practically permissible errors are achieved.

If the accuracy class of the measurement complex in p.u. is 0,2%, then when measuring a power of 700 MW, the methodical error based on the LLE (in hyperbolic functions) can reach 3 MW or more, i.e., it is 0,42%, which is unacceptable. The developed computer program allows for adequate modeling of steady-state modes of corona-generating overhead power lines.

Equations with distributed parameters in hyperbolic functions allow for more complete and accurate flow rate calculation results. For operational flow rate modeling, where accelerated solution generation is required, the use of π from the lumped parameter scheme is recommended.

When solving problems of operational measurement by intelligent devices, modeling and identifying the components of active power losses of high-voltage power transmission lines where high modeling accuracy is required, it is proposed to use more accurate models of high-voltage power transmission lines.

A comparative analysis of the results of modeling the transmission modes of 500, 700, 900 MW using the proposed methodology is presented using a test example of 500 km. Potential methodological errors in modeling the high-voltage power transmission line (HVH) regime were analyzed for different weather group levels and specific corona losses. Calculations were performed for corona loss levels of 5 - 100 kW/km.

A methodology, algorithm, and software implementation for modeling active power losses due to corona are proposed based on the LLE in hyperbolic functions by introducing an equivalent nonlinear specific conductivity corresponding to the (ρ-2)-th power of voltage.

5 Conclusion

1. The equations for a long line with distributed parameters for high-voltage power transmission lines are based on the representation of corona losses by conductivity. The dependences of active power losses due to corona are a function of voltage of 4th to 8th power. In fact, this is a function of nonlinear conductivity. Modeling of modes using the long line equations has methodological errors.
2. A methodology, algorithm and software implementation for modeling active power losses due to corona with an arbitrary degree dependence are proposed, taking into account the voltage distribution along the line based on the LLE in hyperbolic functions and the equivalent specific conductivity of the corresponding $(\rho - 2)$-th degree of voltage, with iterative refinement.
3. It has been established that the methodological errors in the mode parameters when modeling the mode of the high-voltage power transmission line using simplified line equations are comparable with the accuracy of measurements obtained using modern intelligent measurement systems.
4. The adequacy and efficiency of the method and the assessment of methodological errors of mathematical modeling are analyzed using examples of 500, 750, 1150 kV power transmission lines.
5. By conducting steady-state calculations using 500, 750, and 1150 kV overhead power lines of different lengths, it was established that two iterations of the distributed-parameter line equations in hyperbolic functions with refinement of the nonlinear conductivity achieve the benchmark modeling accuracy. The difference between the approximate and exact values quickly. The resulting formula becomes more precise with each iteration. Although the study is theoretical in nature, its significance extends beyond pure mathematics.

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