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Qualitative property of solution nondivergent degenerated elliptic equations

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Abstract. *In the paper of solutions nondivergent degenerated elliptic equations is considered. Weight function from at classes of Makenhaupt. We have been proved some estimates of solutions.*

Keywords. elliptic equations · initial-boundary problems · invariant solutions · degenerate equation · qualitative property

Mathematics Subject Classification (2010): 35J70

1 Introduction

The fact is that such equations arise in numerous problems of modern physics, mechanics, chemistry, biology, biochemistry, biophysics, ecology and many other areas of science and applied issues. Main areas of research: regularity of solutions, solvability of boundary value problems. The studies of Leray, Schauder, Morray, de Giorgi, Nash, Serren, Moser, Ladyzhenskaya, Uraltseva and many other authors led not only to the solution of Hilbert problems, but also to the creation of many methods that play a fundamental role in various branches of mathematics.

At present, a complete theory of boundary and initial-boundary problems for elliptic, parabolic and hyperbolic equations in domains with a smooth boundary has been constructed. One of the central results of this theory is that if the coefficients of the equation and boundary conditions, their right-hand sides, and the boundary of the domain are sufficiently smooth, then the solution to the problem is a correspondingly smooth function. If the above conditions are violated, then this leads to the appearance of singularities in the solutions. For example, the coefficients of the equation are discontinuous, the boundary smoothness conditions are violated, there are degenerations, and so on. The development of the work of Krylov and Safonov, who proposed a probabilistic method for obtaining results for differential equations. We use stochastic differential equations are used in the dissertation. The goal is to obtain Hölder estimates for a degenerate non-linear non-divergence elliptic equation.

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Krylov and Safonov obtained Hölder estimates for a linear nondivergent elliptic equation of the second order. The results obtained for non-divergence equations can be considered as an analog of the De Giorgi and Nash estimates established earlier for solutions of equations of divergent form.

2 Formulation of the problem

Now we consider Harnak problem for degenerate linear non-divergent elliptic equations.

Corresponding the problems for problems of mechanics considers in work [2, 7]. From regularity problem we can show following results [1, 5, 6].

Let Ω be a bounded domain with nonsmath boundary from spaces $\mathrm{R}^n,\ n \geq 3$. Following equation is considered

$$\sum_{i,j=1}^{n} \mathrm{a}_{ij}(x)\, u_{x_i x_j} + \sum_{i=1}^{n} b_i(x)\, u_{x_i} + c(x)\, u(x) = 0, \tag{2.1}$$

where for $x \in \mathit{\Omega}$ the matrix $\mathrm{a}_{ij}(x)$ satisty conditions: $\mathrm{a}_{ij}(x) = \mathrm{a}_{ji}(x)\,,\ \ i,j = \overline{1,n}$ and for any $x \in n$ and $\xi \in R^n$

$$c\sum_{i=1}^{n} \omega(x)\, \xi_i^2 \leq \sum_{i,j=1}^{n} a_{ji}(x)\, \xi_i \xi_j \leq c^{-1} \sum_{i=1}^{n} \omega(x)\, \xi_i^2, \tag{2.2}$$

where $\omega(x) \in A_p$ class of Makenheupt, $1 < p < \infty$. The coefficients $b_i(x)\,,\ \ i = \overline{1,n}$, $c(x)$ are bounded, and $c(x) \leq 0$.

We introduce ellipsoids E_{r,x_0} by standard. Let's mes $\mathit{\Omega}$ Lebesgue measure of the domain $\mathit{\Omega}$.

Firstly, we proved lemma on increasing of positive solutions.

Lemma. Let u – of generalized solution from weighted Sobolev spacer of equation (2.1). Then there exist the positive constants depending only on known parameters, such the

$$mes\left(E_{r,x_0} \cap \{u \geq 0\}\right) \geq \varepsilon\, mes E_{r,x_0},$$

then

$$\inf_{E_r} u \ \geq c_1,$$

c_1 – some const.

Theorem. Let in the E_{r,x_0} the positive solution $u(x)$ of the equation be defined and all conditions elated of coefficients are hold. If $r \leq 1$ then satisfy inequality

$$\sup_{E_{r,x_0}} u(x) \leq c_2 \ \mathop{inf}_{E_{r,x_0}} u(x)\,.$$

Proof. Without loss of generality, let

$$\sup_{E_{r,x_0}} u(x) = 1.$$

Let

$$\frac{1}{r} \leq K < \frac{1}{2r}.$$

Assume $A = \sup_{E_{r,x_0}} u(x)\,.$

By the definition we have

$$\sup_{E_{r,x_0}} u(x) \leq 2^n \cdot c_3$$

Applying lemma to the function $u\,(x)$.
We will obtain

$$\underset{E_{r,x_0}}{inf}\ u\,(x) \geq 2^n \cdot c_4$$

It is easy to see that

$$\underset{E_{r,x_0}}{inf}\ u\,(x) \geq \underset{E_{2r,x_0}}{inf}\ u\,(x) > 0$$

Let's choosing constant, we have been proved of theorem.

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