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Optimization of a batch sequential pumping scheme for commercial oil blends taking into account rheological properties of co-pumped oils

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Abstract. *The article addresses the optimization of sequential pumping of two oils with different viscosities through a technological section of a main oil pipeline. Changes in pumping flow rate caused by variations in product viscosity are taken into account. Under the adopted assumptions, the problem reduces to choosing between pumping the oils as a single blend and pumping them sequentially. It is shown that sequential pumping becomes more energy-efficient as the viscosity of the binary mixture approaches linear additivity. A selection criterion is proposed based on laboratory measurements of the viscosities and densities of the original oils and the viscosity of their 1:1 mixture. Constraints are formulated to ensure that the planned oil volumes are pumped within a specified time. When sequential pumping is selected, the less viscous oil is pumped mainly in pure form, while part of it is mixed with the more viscous oil to form the second batch. A methodology for determining the optimal pumping strategy is developed.*

Keywords. sequential pumping · compounding · oil mixing · multi-grade oil blend · rational blending · high-viscosity oil · energy efficiency

Mathematics Subject Classification (2010): 35K55, 35B44, 76S05

1 Introduction

According to the Forecast of Scientific and Technological Development of the Sectors of the Fuel and Energy Complex of Russia for the Period up to 2035 [6], one of the priorities in oil production in the long term may be the production of heavy and viscous oil, the resources of which in our country are enormous. They are estimated at 7 billion tonnes, and by this indicator Russia ranks third in the world after Canada and Venezuela. Moreover,

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these resources are located in areas with developed infrastructure created for the production of conventional oil (in the European part of the country, Western Siberia and Sakhalin). In principle, the development of such fields is an economically more attractive task that can be solved in a relatively short time than the creation of technologies for the safe production of hydrocarbon raw materials on the shelf of the Arctic seas under complex ice conditions or automated complexes for subsea (under-ice) production with competitive economic indicators. According to the same document [6], the most promising method for transporting heavy and viscous oil is its mixing with light oil grades. The authors of this article have repeatedly proposed methods for the optimal formation of oil cargo flows with differing rheological properties in order to obtain minimum total pumping costs [1, 12, 13], essentially proposing new compounding approaches based on viscosity indicators. These approaches have already begun to be implemented during oil acceptance at the Ukhta-1 oil pumping station of the Ukhta-Yaroslavl main oil pipeline for oil from the Yarega field, where a mixing unit and all the infrastructure necessary for pumping this oil (thermally insulated tanks, heating points, a pumping station with screw pumps, etc.) were built [5]. Oil mixing at the Ukhta-1 oil pumping station is carried out by density and viscosity.

The work [6] aroused great interest among the authors of the present article, where, as an alternative to dilution and mixing of oils with different viscosities, it was shown that in a number of cases it is more advantageous or economical to pump the oils sequentially or to form special batches of mixtures from them, which should also be pumped sequentially. To prove this proposition, the assumption of constant pumping capacity was used. However, a change in the viscosity of the pumped product causes a significant change in the flow rate for a technological section of a main oil pipeline [2–4, 14]. At the same time, it should be noted that even if it is not advantageous to pump oils or their specially selected mixtures with differing viscosities sequentially within one technological section, then, within the framework of solving the more general problem of forming oil cargo flows for the main oil pipeline system as a whole, stated in [10], a situation may well arise when it will be most efficient to deliver the viscous oil components without mixing at one mixing node, that is, to pump it as a separate batch to the next downstream oil mixing node and already there to dilute it with low-viscosity components. Thus, direct delivery of viscous components will be carried out to that part of the branched main oil pipeline system which is less sensitive to an increase in viscosity from the point of view of energy consumption. Therefore, in order to supplement the stated problem of forming oil cargo flows with the possibility of sequential pumping of viscous oil components, it is necessary to solve the problem of the optimal selection of mixing concentrations of specially prepared oil batches for their sequential pumping within one technological section of a main oil pipeline, taking into account the change in pumping flow rate for each batch.

2 Theory

2.1 Problem statement

It is noted in [6] that the problem of optimizing the composition of an oil mixture in batches formed for sequential pumping through a technological section of an oil pipeline may have four possible solutions:

- formation of one batch of an oil mixture in concentrations corresponding to the supply share in the planned pumping assignment;
- formation of two separate “pure” batches consisting entirely of the oil of each supplier (the oils are not mixed);

- formation of two batches, one by admixing part of the less viscous oil into the batch of the more viscous oil, and the other from the remainder of the less viscous oil after mixing;
- admixing part of the more viscous oil into the batch of the less viscous oil, followed by pumping the remaining more viscous oil in a separate batch.

We consider it necessary to note that a more general mixing option should also be considered and added, when not only one of the initial oils, but both are admixed. Thus, two oil batches are formed with viscosities differing from those of the initial oils. All four cases described above are, in essence, particular cases of the proposed more general mixing option.

For convenience of reasoning, the more viscous oil will be denoted by index 1, and the less viscous oil by index 2. As the concentration determining the composition of the mixture, we shall take the mass content of the less viscous oil in it. Thus, the initial oil No. 1 has a concentration of 0, and oil No. 2 has a concentration of 1. The mixture concentration corresponding to the supply share in the planned pumping assignment is determined by the formula:

$$x_{\text{mix}} = \frac{M_2}{M_1 + M_2} \tag{2.1}$$

where M_1, M_2 are the initial masses of the more viscous and less viscous oils, respectively, which must be pumped through the technological section according to the planned assignment over a fixed period of time.

Then the mass balance when pumping oil from two suppliers [3] should be supplemented as follows:

$$\sum M = M_1 + M_2 = (M_1 + \varDelta M_2 - \varDelta M_1) + (M_2 + \varDelta M_1 - \varDelta M_2) \tag{2.2}$$

where $\varDelta M_1, \varDelta M_2$ are the portions of the masses of the more viscous and less viscous oils, respectively, required for diluting the initial oils and forming new batches of mixtures for their sequential pumping through the technological section of the main oil pipeline;

$M_1 + \varDelta M_2 - \varDelta M_1$- mass of the newly formed batch No. 1;

$M_2 + \varDelta M_1 - \varDelta M_2$ - mass of the newly formed batch No. 2.

The concentrations of the newly formed batches No. 1 and No. 2, respectively:

$$x_1 = \frac{\varDelta M_2}{M_1 + \varDelta M_2 - \varDelta M_1} \tag{2.3}$$

$$x_2 = \frac{M_2 - \varDelta M_2}{M_2 + \varDelta M_1 - \varDelta M_2} \tag{2.4}$$

The optimization problem consists in finding such mixing concentrations of the newly formed oil batches x_1 and x_2 that, when they are pumped sequentially through one technological section of a main oil pipeline, their total energy consumption for pumping will be the lowest.

2.2 Influence of the composition of a batch formed from oils differing in viscosity on energy consumption during its pumping

The energy consumption of pumping units operating according to the pump-to-pump scheme, required for pumping oil with mass is determined by formula [2–4, 14]:

$$E = \sum_{i=1}^{n} N_i \tau = \sum_{i=1}^{n} \rho g \frac{H_i Q}{\eta_i} \tau = \sum_{i=1}^{n} \rho g \frac{H_i V}{\eta_i} = gM \sum_{i=1}^{n} \frac{H_i}{\eta_i} \quad (2.5)$$

where Q is the steady-state oil flow rate in the technological section of the main oil pipeline;

N_i, H_i, η_i- consumed power, head and efficiency of the i-th pumping unit for flow rate Q;

τ- pumping time;

ρ- density of the pumped oil;

n- number of operating pumping units,

A change in the viscosity of the pumped product directly affects the pumping flow rate. The steady-state oil flow rate in the technological section of the main oil pipeline must be determined from the equation of the balance of heads [2,4,14] developed by the pumping units and the head consumed by the oil pipeline, which can be determined by the well-known Leibenzon formula [2–4,14]:

$$\sum_{i=1}^{n} H_i = \sum_{i=1}^{n} (a_i - b_i Q^2) = \beta \frac{Q^{2-m} \nu^m}{D^{5-m}} L \quad (2.6)$$

where β, m are the coefficients of the Leibenzon equation [2], depending on the corresponding turbulent-flow zone;

ν- kinematic viscosity of the pumped product;

D- internal diameter of the pipeline;

L is the equivalent (calculated) length of the technological section;

a_i, b_i - approximation coefficients of the head characteristic of the i-th centrifugal pump.

Equation (2.6) determines the dependence of the change in flow rate in the technological section of the main oil pipeline on the viscosity of the pumped oil batch. To calculate energy consumption by formula (2.5), the dependence of the pumping-unit efficiency is represented by formula [2]:

$$\eta_i = c_i Q + d_i Q^2 + e_i Q^3 \quad (2.7)$$

c_i, d_i, e_i - approximation coefficients from the characteristic of the i-th centrifugal pump;

Thus, having determined the flow rate Q from equation (2.6) depending on the change in the viscosity of the oil batch, the energy required for pumping the mass M should be calculated by the formula:

$$E = gM \sum_{i=1}^{n} \frac{a_i - b_i Q^2}{c_i Q + d_i Q^2 + e_i Q^3} = gMH_\eta \quad (2.8)$$

H_η- the energy, reduced to a unit weight of the pumped liquid, required for pumping at flow rate Q:

$$H_\eta = \sum_{i=1}^{n} \frac{a_i - b_i Q^2}{c_i Q + d_i Q^2 + e_i Q^3} \quad (2.9)$$

The influence of the mixing concentration on the mixture viscosity is approximated sufficiently well by formula [10–14]:

$$\lg\big(\lg(\nu + c)\big) = x \lg\big(\lg(\nu_1 + c)\big) + (1 - x) \lg\big(\lg(\nu_2 + c)\big) \quad (2.10)$$

x - oil concentration in the mixture.

It should be noted that, according to the results of previous studies [7–9, 11], the viscosity of the mixture is not additive and has different curvature depending on the mixing concentration of the initial oils. The curvature of the dependence of the viscosity of a binary mixture can be estimated by the following formula:

$$Y = 2\frac{\nu_{1/2} - \nu_2}{\nu_1 - \nu_2} \tag{2.11}$$

where $\nu_1, \nu_2, \nu_{1/2}$ are the viscosity values of the initial oils and their binary mixture at a mixing ratio of 1:1 (the mixing concentration is 0.5).

The closer the value Y is to unity, the more strongly the linear additivity of the mixture viscosity is manifested.

2.3 Solution of the problem

For further analysis of the total energy consumption during pumping of two specially formed oil batches, a quantity that we called the mass imbalance share will be required:

$$\mathrm{Dis} = \frac{\Delta M_2 - \Delta M_1}{M_1 + M_2} \tag{2.12}$$

Taking into account formulas (2.3) - (2.4), we obtain:

$$\mathrm{Dis} = \frac{x_1(x_2 - x_{\mathrm{mix}}) - (1 - x_2)(x_{\mathrm{mix}} - x_1)}{x_2 - x_1} = x_{\mathrm{mix}} + \frac{x_1 - x_{\mathrm{mix}}}{x_2 - x_1} \tag{2.13}$$

In essence, the mass imbalance share shows by how much we artificially, by admixing the initial oils, shift the mass ratio of the specially formed oil batches planned for pumping.

The value of the difference between the mixture concentration corresponding to the supply share in the planned pumping assignment and the mass imbalance share should be called the shifted mass fraction of batch No. 2 relative to the entire mass of the initial oils.

$$x'_{\mathrm{mix}} = x_{\mathrm{mix}} - \mathrm{Dis} = \frac{x_{\mathrm{mix}} - x_1}{x_2 - x_1} = \frac{M_2 + \Delta M_1 - \Delta M_2}{M_1 + M_2} \tag{2.14}$$

When planning pumping in batches, it is necessary to take into account the circumstance that changes in the pumping flow rates of the batches relative to the pumping flow rate in a single mixture affect the time required to fulfill the planned assignment. The search should be limited to such mixing concentrations for the newly formed oil batches, the implementation of whose sequential pumping fits within the time for fulfilling the planned assignment:

$$\frac{M_1 + \Delta M_2 - \Delta M_1}{\rho_1 Q_1} + \frac{M_2 + \Delta M_1 - \Delta M_2}{\rho_2 Q_2} \le \tau_{\mathrm{run}} \le \frac{M_1 + M_2}{\rho_{\mathrm{mix}} Q_{\mathrm{mix}}} k \tag{2.15}$$

where ρ_1, ρ_2 are the densities of the formed batches No. 1 and No. 2, respectively.

ρ_{mix} - density of the mixture whose concentration corresponds to the supply share in the planned pumping assignment;

Q_1, Q_2 - steady-state flow rates for pumping the formed batches No. 1 and No. 2, respectively, determined by equation (2.15).

Q_{mix}- steady-state flow rate when pumping the mixture whose concentration corresponds to the supply share in the planned pumping assignment;

k- time-reserve coefficient for fulfilling the planned assignment relative to pumping the initial oils in a single mixture (greater than unity).

From (2.15), we obtain the following constraint conditions for the search for optimal concentrations:

$$x'_{\text{mix}} = \frac{x_{\text{mix}} - x_1}{x_2 - x_1} \leq \frac{\dfrac{\rho_{\text{mix}} Q_{\text{mix}}}{\rho_1 Q_1} - k}{\dfrac{\rho_{\text{mix}} Q_{\text{mix}}}{\rho_1 Q_1} - \dfrac{\rho_{\text{mix}} Q_{\text{mix}}}{\rho_2 Q_2}} \quad (2.16)$$

To estimate the total energy consumption for pumping two oil batches, we shall use the equivalent energy required for pumping all oil batches, reduced to a unit weight of the pumped liquid $H^{\eta'}_{\text{mix}}$, which can be found from the following relation:

$$gM_1 H_1^{\eta'} + gM_2 H_2^{\eta'} + gM_1(H_2^{\eta'} - H_1^{\eta'}) + gM_2(H_1^{\eta'} - H_2^{\eta'}) = H^{\eta'}_{\text{mix}}(M_1 + M_2) \quad (2.17)$$

Taking into account (2.13), we obtain:

$$H^{\eta'}_{\text{mix}} = H_1^{\eta'} x'_{\text{mix}} + H_2^{\eta'}(1 - x'_{\text{mix}}) = H_1^{\eta'} \frac{x_{\text{mix}} - x_1}{x_2 - x_1} + H_2^{\eta'} \left(1 - \frac{x_{\text{mix}} - x_1}{x_2 - x_1}\right) \quad (2.18)$$

Since the formula is linearly additive with respect to x'_{mix}, and this quantity according to formula (2.14) is, in essence, the fraction of the shift of the concentration of the selected batch № 1 x_1 relative to x_{mix} over the entire range $x_2 - x_1$, the following results from geometrical considerations. The point on the straight line connecting the values $H_1^{\eta'}$ and $H_2^{\eta'}$ corresponding to x_1 and x_2 for the dependence $H_\eta = f(x)$ determined by formulas (2.6), (2.9) and (2.10), which is located above the value x'_{mix}, will correspond to $H^{\eta'}_{\text{mix}}$. Fig. 2.1 illustrates an example of finding $H^{\eta'}_{\text{mix}}$.

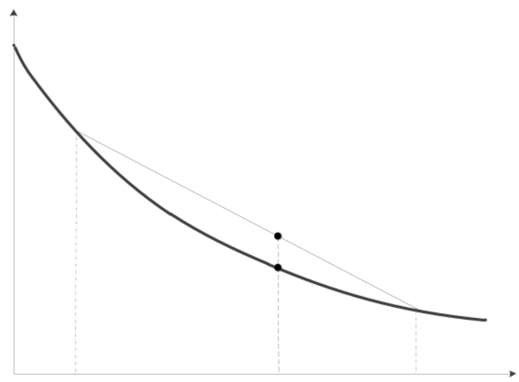

Fig 2.1. Method for determining the equivalent reduced energy for pumping oils in batches

From Fig. 2.1 it becomes obvious that the solution of the problem of searching for optimal mixing concentrations of specially formed oil batches depends on the convexity or concavity of the dependence $H_\eta = f(x)$. With fixed parameters of the technological section of the main oil pipeline and characteristics of the pumping equipment, as well as known viscosity values of the initial mixtures, the form of the function $H_\eta = f(x)$ depends on the parameter Y.

The form of the function $H_\eta = f(x)$ determined by formulas (2.6), (2.9) and (2.10), depending on the parameter Y, is presented in Fig. 2.2. When approaching a linear dependence, the function $H_\eta = f(x)$ has two inflection points. In the other cases, the function is either concave ($Y \ll 1$) or convex ($Y \approx 1$). Since the deviation of energy consumption for the case with two inflection points from the linear dependence is small, this can be neglected, because the effect of selecting measures for optimal mixing will be small. It is possible that these inflection points are caused by the empirical functions (2.6) and (2.10) and do not occur in reality. However, if a solution for this case is nevertheless found purely mathematically, it is most advantageous to form two batches for this case: to leave one batch as the untouched low-viscosity oil № 2, and to dilute the other, more viscous oil № 1 to a concentration close to 0.25. But the effect, if it exists, as noted, will be small.

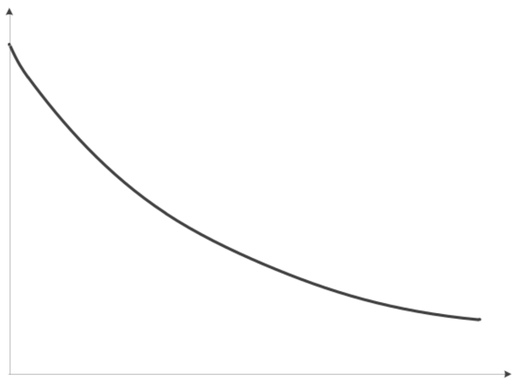

Fig. 2.2. Dependence of the function $H_\eta = f(x)$ on Y

In the other cases, the answer to the above-stated problem reduces either to pumping the oils as a single mixture or to pumping them in separate "pure" batches, that is, without performing admixing operations.

Thus, a criterion is required by which it can be determined which way of carrying out pumping is more energy-efficient: sequentially or as a single mixture.

The criterion $Y_{1/2}$ can be determined by solving the equation in which a 1:1 mixture gives the following value $H_\eta(0.5)$ according to equations (2.6), (2.9) and (2.10).

$$H_\eta(0.5) = \frac{H^{\eta'}_{\text{oil}\,1} + H^{\eta'}_{\text{oil}\,2}}{2}$$

In order to solve the above-stated problem, it is necessary to:

1. measure the viscosities of the initial oils and the viscosity of the mixture with a mixing ratio of 1:1 ($x = 0.5$); calculate the value $Y_{1/2}$;
2. calculate, from the measured viscosity values of the initial oils and the parameters of the technological section with the installed pumping equipment, the $H^{\eta'}_{\text{oil}\,1}$, $H^{\eta'}_{\text{oil}\,2}$ and $H_\eta(0.5)$;
3. determine the criterion $Y'_{1/2}$ and compare it with $Y_{1/2}$. If $Y'_{1/2}$ is greater than $Y_{1/2}$, pumping as a single mixture is more advantageous; if it is smaller, pumping in different batches is more advantageous. If the values are close, the energy consumption when pumping as a single mixture or sequentially will be approximately the same.
4. if sequential pumping is selected, it is necessary to check constraints (2.16). If such a constraint exists, it is necessary to select the option of admixing the more viscous oil until the condition of planned pumping within the specified time is fulfilled.

To verify the validity of the theoretical substantiations presented, numerical calculations were carried out to determine the function $H(\eta)$ of x_1 and x_2 for various NM-type pumps, the characteristics of which are presented in [2]. The calculation results coincided with the theoretical reasoning above. Examples of calculations are presented in Figs. 2.3 - 2.5.

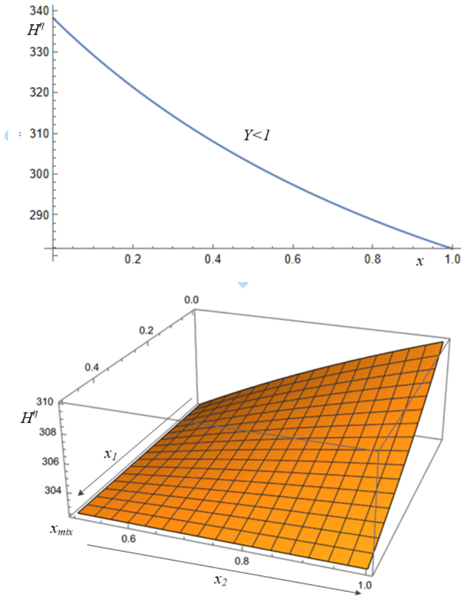

Fig.2.3. Example of the numerical solution $H(\eta)$ for the entire range at $Y \ll 1$

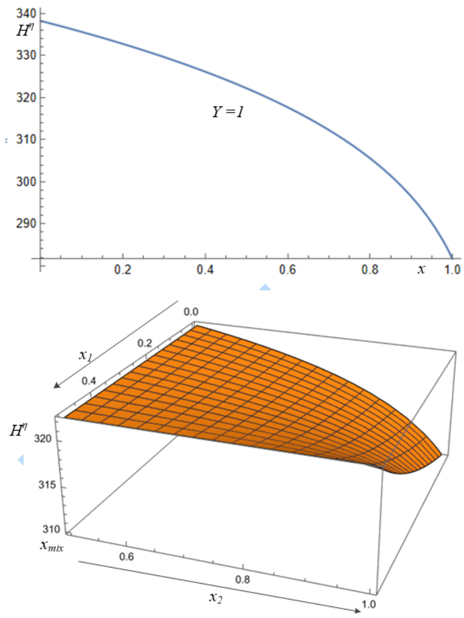

Fig.2.4. Example of the numerical solution $H(\eta)$ for the entire range at $Y \approx 1$

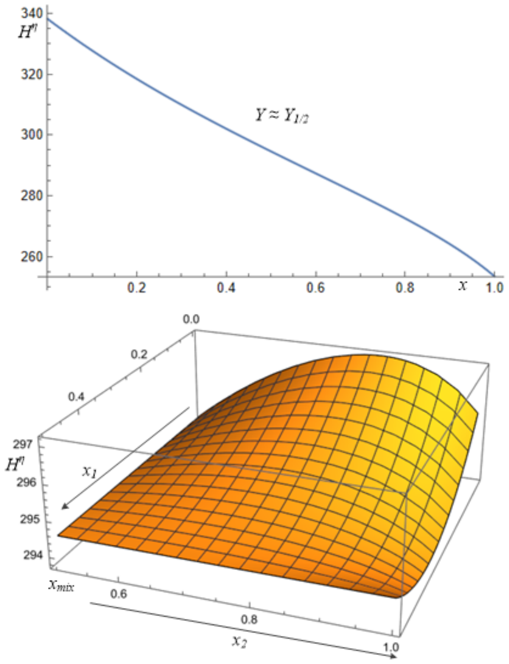

Fig.2.5. Example of the numerical solution $H(\eta)$ for the entire range at $Y \approx Y_{1/2}$

3 Discussion

The solution of the stated problem is based on the assumption that the pumping modes do not change during fulfillment of the planned assignment. However, the conclusions obtained do not contradict changing pumping modes either. It is simply necessary to follow the recommendation that, in the case of sequential pumping, low-viscosity oil should be pumped at reduced modes, and high-viscosity oil at increased modes. The technical capability of the tank farm to accumulate certain batches was also not considered. However, we believe that the share of additional energy costs caused by switching pumping between batches is small compared with the total energy costs, and even relatively frequent switching in the case of a small tank-farm volume should not affect the result of choosing the pumping strategy.

Based on the equations obtained, the formation of a third and further batches for the initial two oils will always be inexpedient. However, when mixtures are formed not from two but from three or more initial oils, the answer is not obvious and requires additional study.

4 Conclusion

1 Thus, it was theoretically proved that the solution of the problem of optimal pumping of oil batches with differing viscosities reduces to choosing either pumping as a single mixture or as separate pure batches, depending on the proposed criterion.
2 A criterion is proposed by which the expediency of sequential pumping of specially formed oil batches with different viscosities can be determined. For this purpose, it is necessary to carry out laboratory measurements of the viscosities of the initial oils and of a mixture with a mixing ratio of 1:1.

The obtained solution of the above-stated problem is planned to supplement the more general optimization problem of distributing cargo flows of oils with differing viscosities with the lowest total energy consumption [10].

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