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Analysis and visualization of the error generated during the tracking of moving objects coordinates in robotic systems

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Abstract. *The issue of modeling the method of determining the coordinates of a moving mechanical or robotic object, scanning and tracking its position in a multimedia simulator was considered. Based on the results of metrological analysis of deviations (errors) from the surface of the scanned object and measurement results, the coordinates of the points of intersection of the scanning beam and the object's surface are determined with high accuracy, thus enabling the simulation of adaptive control of the tracking and adjustment process in robotic and automated systems.*

Keywords. scanning · adaptive control · robotic systems · tracking devices · error estimation · automated machinery

1 Introduction

Accurate error analysis during the tracking of moving objects' coordinates is essential for precise determination of the trajectories of tools or robotic end-effectors [1]. This ensures high-quality operation in automated manufacturing processes. Furthermore, high sensitivity and accuracy are required in controlling industrial robots, CNC machines, automated assembly lines, and other mechanical systems. Therefore, error estimation and minimization play a critical role in improving the performance and reliability of automated tracking and control systems.

2 Analysis of Movement Patterns

The input data for the simulation system during the execution of a virtual mechanical operation are the coordinates of the moving tool or robotic end-effector and the inclination angles recorded by the coordinate tracking device. The obtained results are monitored by the operator through display systems, and complex calculations are performed based on these measurements, followed by a metrological analysis of the system. In this analysis, the

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starting point is the coordinates of the moving tool. A specific error threshold is defined, and by simulating movements along different trajectories, we identify spatial regions where the error in determining the Cartesian coordinates of the reference points does not exceed the acceptable level.

Let us consider the transformation of the system's reference points into Cartesian coordinates. We define the following notations: $R_{i,j}$ is the i-th spatial coordinate of the j-th reference point; $r_{k,j}$ is the k-th coordinate of the projection of the j-th reference point in the camera or sensor matrix; L_{mn} is the distance between the m-th and n-th reference points, fixed by the design of the measurement system.

The relationship between the projected coordinates r (measured by the sensor or camera) and the actual Cartesian coordinates R of the reference points in the robotic or mechanical system can be described by the following system of equations:

$$\begin{aligned}
\frac{r_{k,j}}{R_{i,j}} = \frac{f}{R_{3,j}}, \quad k \in \{1,2\}, \quad j \in \{1,2,3\}, \\
\sum_{i=1}^{3} (R_{i,1} - R_{i,2})^2 = L_{12}^2, \\
\sum_{i=1}^{3} (R_{i,1} - R_{i,3})^2 = L_{13}^2, \\
\sum_{i=1}^{3} (R_{i,2} - R_{i,3})^2 = L_{23}^2.
\end{aligned} \tag{2.1}$$

Here, $R_{i,j}$ denotes the i-th Cartesian coordinate of the j-th reference point in the mechanical or robotic system, $r_{k,j}$ is the k-th coordinate of the projected reference point in the sensor/camera plane, f is the focal length of the camera or the scaling factor in the sensor measurement, and L_{mn} is the known fixed distance between the m-th and n-th reference points, ensuring the rigid structure of the system.

Let the image of the j-th reference point fall on the sensor pixel P_j, with coordinates of the upper-left corner $(r_{1,j}, r_{2,j})$ expressed in row and column units. The coordinates of the reference point image at the corners of this pixel are then

$$(r_{1,j}, r_{2,j}), \quad (r_{1,j}, r_{2,j+1}), \quad (r_{1,j+1}, r_{2,j}), \quad (r_{1,j+1}, r_{2,j+1}).$$

This mapping allows us to approximate the continuous position of each reference point using the discrete sensor measurements, which is essential for calculating spatial errors in robotic systems.

If we consider the center of the sensor pixel as the estimated position of the reference point in the image plane, the absolute measurement errors are known and equal to 0.5 pixel units. Since the pixels are geometrically small, the unknown nonlinear mapping from the sensor coordinates r to the Cartesian coordinates R of the reference point can be approximated linearly in the neighborhood of the point:

$$R \approx (r_{1,j+0.5}, r_{2,j+0.5})\,.$$

This linear approximation is commonly used in robotic and mechanical systems to simplify error estimation and spatial localization from discrete sensor measurements.

When determining the spatial coordinates of the reference points in the mechanical or robotic system, the corresponding measurement errors can be estimated. The possible positions of the reference points projected onto the known sensor pixels P_i form a hypercube C in a six-dimensional space defined by the sensor coordinates $r_{i,j}$. This hypercube is then

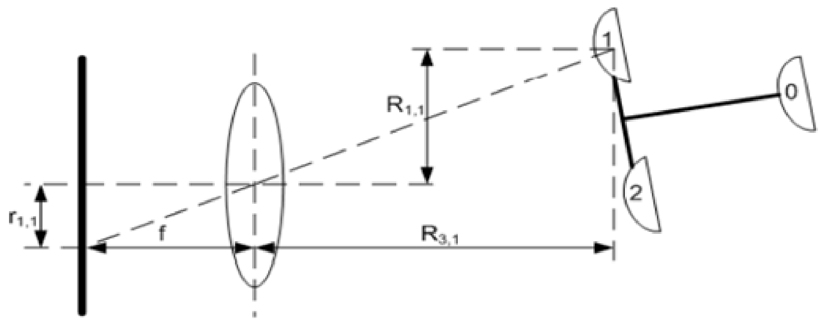

Fig. 2.1 Projection of a reference point onto the sensor pixel grid.

mapped into a higher-dimensional space representing the Cartesian coordinates $R_{i,j}$ of the reference points. The mapping between the sensor space r and the Cartesian space R is one-to-one within the linear approximation.

According to the main theorem of linear programming, the maximum error along any Cartesian axis is attained at the vertices of this polyhedron. Each vertex corresponds to a combination of extreme sensor pixel positions, i.e., the corners of the sensor pixels, providing the bounds of the possible positional error in the mechanical system.

Since each reference point projection corresponds to one sensor pixel and each pixel has four corners, the hypercube C contains $4^3 = 64$ vertices. For each vertex, the Cartesian coordinates of the corresponding reference points in the mechanical system are computed, and the measurement error is estimated using the minimum and maximum values.

To construct an error surface that represents the positional uncertainty, it is necessary to estimate the error for all possible positions of the reference points in the workspace of the robotic or mechanical system. Rays that are reflected at extreme angles may not reach the sensor and are therefore excluded from the calculations.

The error for each Cartesian coordinate $R_{i,j}$ must be determined. This involves a large number of calculations. To reduce computational effort, the problem is divided into subproblems. For instance, consider the z-coordinate of the first distant reference point in the system, corresponding to $R_{3,1}$ in the model. First, we identify at least one point along the ray passing through the sensor center and normal to the sensor plane where the coordinate error matches a specified level ε_0. This reduces to solving the equation

$$\varepsilon\left(R_{3,1}\right) - \varepsilon_0 = 0. \tag{2.2}$$

This equation is solved using the bisection method. The search interval is gradually refined to achieve a positional accuracy of $\varepsilon_0 = 1$ mm. In the first step, the search region is defined based on the mechanical system or robotic manipulator specifications: the reference points should be located between 60 and 90 cm from the sensor.

In the second step, a new projection position r is selected, and system (2.1) is solved for all vertices of the hypercube C corresponding to this position. As a result, the absolute errors of determining the Cartesian coordinates R of the reference points are obtained. To reduce computational effort, practical constraints are applied. For instance, in robotic welding or machining, the end-effector follows a predetermined trajectory, so the points of interest are likely near the workpiece surfaces. Therefore, the scanning or measurement process is repeated until the errors are estimated for all relevant points in the workspace.

3 Visualization of Error Estimation

The calculations were carried out in the MATLAB package, and the results show the error limits for the coordinates of the distant reference point according to the designed algorithm, assuming that the robotic end-effector is located near the target position [1–4]. Experiments have shown that this method is sufficient for adequate modeling of precise manipulator motion. Based on the presented areas, it was determined that the amplitude of displacements along the X axis should not exceed 10 cm at a distance of 70 cm from reference point 1 along the Z axis.

The MATLAB code optimizes the distances between the three reference points and calculates the resulting positional errors. In the presented code, the coordinates for reference point 1 are fixed. For reference points 2 and 3, changes along the X and Z axes are taken into account. The purpose of the calculations is to compute the error caused by the assumed distances between the reference points and to graphically visualize the resulting error distribution.

The error estimation surface for determining the coordinates of the robotic tracking system is shown in Fig. 3.

Generally, the MATLAB code calculates the positional error values corresponding to the assumed distances between the reference points of the robotic system. These error values are visualized in an error surface plot for varying coordinates along the X and Z axes. This method can be used to determine the minimum error and the optimal placement of reference points in the manipulator or robotic structure. Additionally, it is possible to find the minimum of the error function computationally: the smallest value from the matrix of error values is selected, which gives the best coordinates for the reference point. The `min` function in MATLAB is used for this calculation, treating all matrix elements as a single column vector and finding the smallest value in this vector.

For the given system configuration and parameters, the minimum value of the error function is 1.10 mm.

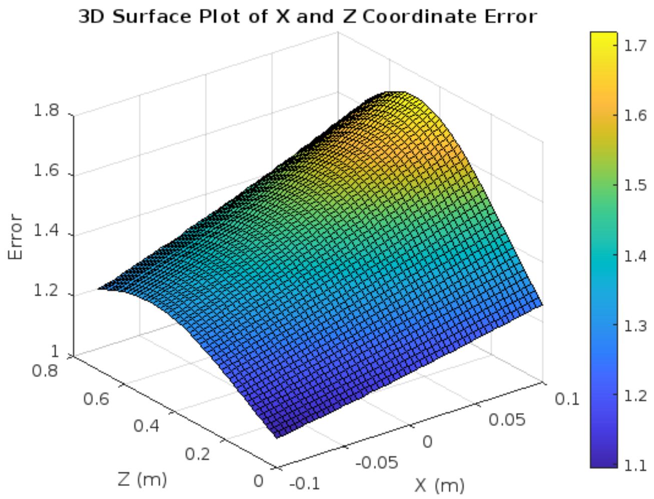

Fig. 3.1 Error estimation in determining the coordinates of the robotic tracking system.

4 Conclusion

As a result of this study, it was determined that the error estimation algorithm can identify the most optimal coordinates of the reference points in a robotic or mechanical system.

Therefore, it can be effectively used for coordinate registration and calibration during the modeling and simulation of robotic manipulators or automated production processes.

The method ensures high precision in the tracking of moving components, which is critical for manufacturing accuracy and automated control.

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