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# LOSS OF SMOOTHNESS OF SOLUTIONS OF A HYPERBOLIC-PARABOLIC SYSTEM WITH SINGULAR COEFFICIENTS

**Abstract**

*In the paper, the Cauchy problem for a hyperbolic -parabolic system with non-smooth coefficient at the higher derivative in the hyperbolic part is studied. It is proved that if this coefficients satisfies the logarithmic Lipschitz condition, the loss of smooothness of solutions happens.*

**1. Introduction.** Various problems of thermoelasticity are reduced to the Cauchy problem for hyperbolic-parabolic system of equations. At rather smoothness of coefficients these problems were studied by different authors [5,7-15,17-19]. In the mentioned papers the well-posedness of the appropriate Cauchy problem or mixed problem and also behaviour of solutions was investigated.

In this paper, in the domain $[0,T]\times R^n$ we consider the Cauchy problem:

$$\left.\begin{array}{c}\ddot{u}+\alpha(t)\Delta^2u-\Delta v=f(t,x),\\ \dot{v}-\Delta v+\Delta\dot{u}=g(t,x)\end{array}\right\}\tag{1}$$

with initial conditions

$$u(0,x)=u_0(x),\ \dot{u}(0,x)=u_1(x),\ v(0,x)=v_0(x),\tag{2}$$

where $\alpha(t)$ is a real functions determined on $[0,T]$, and $f(t,x)$, $g(t,x)$ are some function determined on $[0,T]\times\Omega$, $\dot{u}=\dfrac{\partial u}{\partial t}$, $\ddot{u}=\dfrac{\partial^2 u}{\partial t^2}$, $\dot{v}=\dfrac{\partial v}{\partial t}$.

Denote by $Lip[0,T]$ a class of scalar functions determined on $[0,T]$ and satisfying the Lipschitz condition.

Using the semigroup method, we can prove that if the conditions

$$\alpha(t)\in Lip[0,T],\tag{3}$$

$$\alpha(t)\geq\alpha_0>0,\tag{4}$$

$$f(t,x)\in L_2(0,T;H^s),g(t,x)\in L_2(0,T;H^{s-1}),\tag{5}$$

$$u_0(.)\in H^s,u_1(.)\in H^{s-1},v_0(.)\in H^{s-1},s\geq 0,\tag{6}$$

are fulfilled, then problem (1), (2) has a unique solution

$$u_0(.)\in([0,T];H^s)\cap C^1\in\left([0,T];H^{s-1}\right),$$

$$v_0(.)\in L_\infty(0,T;H^{s-1})\cap L_2(0,T;H^s).$$

Therewith, for the appropriate solutions $u\,(t,x)$, $v\,(t,x)$ it is valid the energetic estimation

$$\left\|\dot{u}\,(t,.)\right\|^2_{H^{s-1}}+\left\|u\,(t,.)\right\|^2_{H^{s+1}}+\left\|v\,(t,.)\right\|^2_{H^{s-1}}+\int\limits_0^t\left\|v\,(t,.)\right\|^2_{H^s}\leq$$

$$\leq c[\|u_1(.)\|^2_{H^{s-1}} + \|u_0(.)\|^2_{H^{s+1}} + \|v_0(.)\|^2_{H^{s-1}} + $$
$$+\int_0^t (\|f(\tau,.)\|_{H^{s+1}} + \|g(\tau,.)\|_{H^{s-1}})]. \tag{7}$$

If condition (3) is not fulfilled, it is impossible to prove the well-posednes of problem (1), (2). In this case, similar to hyperbolic equations, the loss of smoothness of solutiouns happens.

In the work we consider problem (1), (2), when instead of condition (3) it is assumed that $a(t)$ satisfies the logarithmic Lipschitz condition.

**2. Problem statement and the main result.** Denote by $LL_\omega[0,T]$ a class of functions $a(t)$ satisfying the following condition

$$|a(t+\tau) - a(t)| \leq M_\alpha |\tau| \cdot |\log|\tau|| \, \omega(\tau), \tag{8}$$

where $M_\alpha > 0$, $t, t+\tau \in [0,T]$,

$$\left.\begin{array}{c} \omega(\tau) \text{ is a bounded function on } [0,T] \\ \text{monotonically decreasing tends to zero as } \tau \to 0 \end{array}\right\} \tag{9}$$

Obviously, if $a(t) \in LL_\omega[0,T]$, the Lipschitz coefficient $L_\alpha = M_\alpha |\log|\tau|| \, \omega(\tau)$ may increase unboundedly.

If $a(t) \in LL_\omega[0,T]$, these coefficients are said to be singular. For linear hyperbolic equations the similar problems were studied in detail [see e.i. [1-4,16]).

In the paper we get the following result.

**Theorem 1**. *Let conditions (4)-(6),(8) and (9) be fulfilled. Then for the solution of equation (1) the following estimation is valid:*

$$E_\alpha(t) \leq c_\delta \left[ E_{\alpha+\delta}(0) + \int_0^t (\|f(s,.)\|_{H^{\alpha+\delta}} + \|g(s,.)\|_{H^{\alpha-1+\delta}}) \, ds \right], \tag{10}$$

*where* $\alpha \geq 0$, $\delta > 0$,

$$E_\alpha(t) = \|\dot{u}(t,.)\|^2_{H^{\alpha-1}} + \|u(t,.)\|_{H^{\alpha+1}} + \|v(t,.)\|_{H^{\alpha-1}} + \int_0^t \|v(t,.)\|_{H^\alpha} \, d\tau \tag{11}$$

In the same way we study the following problem

$$\left.\begin{array}{c} \ddot{u} - a(t)\Delta u - \nabla v = f(t,x), \\ \dot{v} - \Delta v + \operatorname{div} \dot{u} = g(t,x), \end{array}\right\} \tag{12}$$

$$u(0,x) = u_0(x), \ \dot{u}(0,x) = u_1(x), \ v(0,x) = v_0(x), \tag{13}$$

$u = (u_1(t,x), ..., u_n(t,x))$, $\nabla v = \left(\frac{\partial v}{\partial x_1}, ..., \frac{\partial v}{\partial_n}\right)$, $\operatorname{div} \dot{u} = \left(\frac{\partial \dot{u}}{\partial x_1}, ..., \frac{\partial \dot{u}}{\partial x_n}\right)$, are the given functions.**

**Theorem 2**. *Let conditions (4)-(6), (8) and (9) be fulfilled. Then for the solution of problem (12),(13) the following estimation is valid :*

$$\widetilde{E}_\alpha(t) \le c_\delta \left[ \widetilde{E}_{\alpha+\delta}(0) + \int_0^t \left( \|f(s,.)\|_{H^{\alpha+\delta}} + \|g(s,.)\|_{H^{\alpha-1+\delta}} \right) ds \right],$$

*where* $\alpha \ge 0,\ \delta > 0,$

$$\widetilde{E}_\alpha(t) = \left\| \dot{u}(t,.) \right\|^2_{H^{\alpha-1}} + \|u(t,.)\|_{H^\alpha} + \|v(t,.)\|_{H^{\alpha-1}} + \int_0^t \|v(t,.)\|_{H^\alpha} d\tau.$$

We give only the proof of theorem 1. The proof of theorem 2 is conducted in the similar way.

**3. Proof of theorem 1.** Let the function $u(t,x)$,$v(t,x)$ be the solution of problem (1), (2).Then $u(t,x)$,$v(t,x)$ will be the solution of the following Cauchy problem:

$$\left. \begin{array}{l} \ddot{\widehat{u}}(t,\xi) + \alpha(t)\,|\xi|^4\,\widehat{u}(t,\xi) + |\xi|^2\,\widehat{v}(t,\xi) = \widehat{f}(t,\lambda) \\ \dot{\widehat{v}}(t,\xi) + |\xi|^2\,v(t,\xi) - |\xi|^2\,\dot{\widehat{u}}(t,\xi) = \widehat{g}(t,\xi) \end{array} \right\} \tag{14}$$

$$\widehat{u}(0,\xi) = \widehat{u}_0(\xi), \ddot{\widehat{u}}(0,\xi) = \widehat{u}_1(\xi), \widehat{v}(0,\xi) = \widehat{v}_0(\xi) \tag{15}$$

where $\widehat{u} = Fu$, $\widehat{v} = Fv$, $\widehat{u}_0 = Fu_0$, $\widehat{v}_0 = Fv_0$, $\widehat{f} = Ff$, $\widehat{g} = fg$,and $F$ is the Fourier transformation.

Let

$$\alpha_\varepsilon(t) = \frac{1}{\varepsilon} \int \widetilde{\alpha}(t+\tau)\,\rho\left(\frac{\tau}{\varepsilon}\right) d\tau,\ \varepsilon > 0,$$

where $\rho \in C_0^\infty(-1;1)$, $0 \le \rho \le 1$, $\int \rho(\tau) d\tau = 1$, $\int \left|\rho'(\tau)\right| d\tau \le 4$, $\widetilde{\alpha}(t) = \alpha(t)$, $0 \le t \le T$, $\widetilde{\alpha}(t) = \alpha(0)$, $t \le 0$ and $\widetilde{\alpha}(t) = \alpha(T)$, $t \le T$.

Using the definitions of $\alpha_\varepsilon(t)$ and properties of $\alpha(t)$, we get

**Lemma 1.** *For any* $\varepsilon > 0$ *the following estimation is valid*

$$|\alpha_\varepsilon(t) - \alpha(t)| \le M_\alpha \varepsilon \cdot |\log \varepsilon|\,\omega(\varepsilon) \tag{16}$$

**Lemma 2.** *For any* $\varepsilon > 0$ *the following estimations are valid*

$$\left|\dot{\alpha}_\varepsilon(t)\right| \le 4M_\alpha \cdot |\log \varepsilon|\,\omega(\varepsilon) \tag{17}$$

$$\alpha_\varepsilon(t) \ge \alpha_0. \tag{18}$$

Determine "the regularized monatomic energy "

$$E_a^\varepsilon(t,\xi) = \left| \ddot{\widehat{u}}(t,\xi) \right|^2 + |\xi|^4\,\alpha_\varepsilon(t)\,|\widehat{u}(t,\xi)|^2 + |\widehat{v}(t,\xi)|^2 + 2\,|\xi|^2 \int_0^t |v(t,\xi)|^2\,d\tau.$$

Calculate the derivative $E_a^\varepsilon(t)$ :

$$\frac{dE_a^\varepsilon(t,\xi)}{dt} = 2\,\mathrm{Re}\,\ddot{\widehat{u}}(t,\xi)\dot{\bar{\widehat{u}}}(t,\xi) + 2a_\varepsilon(t)\,|\xi|^4\,\mathrm{Re}\,\widehat{u}(t,\xi)\dot{\bar{\widehat{u}}}(t,\xi) +$$

$$+\alpha_\varepsilon(t)\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|^2+2\left|\xi\right|^2\operatorname{Re}\widehat{v}(t,\xi)\dot{\bar{\widehat{v}}}(t,\xi)+2\left|\xi\right|^2\left|\widehat{v}(t,\xi)\right|^2.$$

Using (14), hence we get

$$\frac{dE_a^\varepsilon(t,\xi)}{dt}=2\operatorname{Re}\left(\widehat{f}(t,\xi)+(\alpha(t)-\alpha_\varepsilon(t))\left|\xi\right|^4\widehat{u}(t,\xi)\right)\dot{\bar{\widehat{u}}}(t,\xi)+$$

$$+\dot{\alpha}_\varepsilon(t)\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|^2+2\operatorname{Re}g(t,\xi)\left|\widehat{v}(t,\xi)\right|^2.$$

Hence we have the following estimation :

$$\frac{dE_a^\varepsilon(t,\xi)}{dt}=2\left(\left|\widehat{f}(t,\xi)\right|+\left|\alpha(t)-\alpha_\varepsilon(t)\right|\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|\right)\left|\dot{\widehat{u}}(t,\xi)\right|+$$

$$+\left|\dot{\alpha}_\varepsilon(t)\right|\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|^2+2\left|g(t,\xi)\right|\left|\widehat{v}(t,\xi)\right|^2.$$

Applying the Schwartz inequality we upper estimate the right hand side. As a result we have:

$$\frac{dE_a^\varepsilon(t,\xi)}{dt}\le\left|\widehat{f}(t,\xi)\right|^2+\left|\widehat{g}(,\xi)\right|^2+2\left|\alpha(t)-\alpha_\varepsilon(t)\right|\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|\left|\dot{\widehat{u}}(t,\xi)\right|+$$

$$+\left|\dot{\alpha}_\varepsilon(t)\right|\left|\xi\right|^4\left|\widehat{u}(t,\xi)\right|^2+\left|\dot{\widehat{u}}(t,\xi)\right|^2+\left|\widehat{v}(t,\xi)\right|^2.$$

In what follows, using estimations (16)-(18), hence we get

$$\frac{dE_\alpha^\varepsilon(t,\xi)}{dt}\le\left|\widehat{f}(t,\xi)\right|^2+\left|\widehat{g}(t,\xi)\right|^2+\bar{c}M\varepsilon\left|\log\varepsilon\right|\overline{\omega}(\varepsilon)\left|\xi\right|^2\left[\left|\dot{\widehat{u}}(t,\xi)\right|^2+\left|\widehat{v}(t,\xi)\right|^2\right],$$

where $\bar{c}=\max\left\{\frac{a_0+1}{a_0},2\right\}$.

Integrating the both hand sides on the interval $[0,t]$, we get

$$E_\alpha^\varepsilon(t,\xi)\le E_\alpha^\varepsilon(0,\xi)+\int_0^t\left[\left|\widehat{f}(t,\xi)\right|^2+\left|\widehat{g}(t,\xi)\right|^2\right]d\tau+$$

$$+\int_0^t\bar{c}M\left|\xi\right|^2\varepsilon\left|\log\varepsilon\right|\omega(\varepsilon)E_\alpha^\varepsilon(\tau,\xi)d\tau.$$

Introduce the denotation :

$$E_a(t,\xi)=\left|\dot{\widehat{u}}(t,\xi)\right|^2+\left|\xi\right|^4a\left|\widehat{u}(t,\xi)\right|^2+\left|\widehat{v}(t,\xi)\right|^2+2\left|\xi\right|^2\int_0^t\left|v(t,\xi)\right|^2d\tau.$$

Obviously

$$E_\alpha^\varepsilon(t,\xi)\ge\left|\dot{\widehat{u}}(t,\xi)\right|^2+\left|\xi\right|_0^4\alpha\left|\widehat{u}(t,\xi)\right|^2+\left|\widehat{v}(t,\xi)\right|^2+$$

$$+2\left|\xi\right|^2\int_0^t\left|v(t,\xi)\right|^2d\tau\ge M_0E_a(t,\xi),$$

and

$$E_a^\varepsilon(t,\xi) \le \left|\dot{\widehat{u}}(t,\xi)\right|^2 + |\xi|^4 a_1 \left|\widehat{u}(t,\xi)\right|^2 + |\widehat{v}(t,\xi)|^2 +$$

$$+2|\xi|^2 \int_0^t |\widehat{v}(t,\xi)|^2 \, d\tau \le M_1 E_\alpha(t,\xi), \tag{19}$$

where $M_0 = \min(1, a_0)$, $\alpha_1 = \max \alpha(t)$ , $M_1 = \max(1, a_1)$.

From (17)-(19) it follows that

$$E_\alpha^\varepsilon(t,\xi) \le \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau +$$

$$+ \frac{M_1 \bar{c} \, |\xi|^2 \, \varepsilon \, |\log \varepsilon| \, \omega(\varepsilon)}{M_0} \int_0^t E_\alpha(\tau,\xi) d\tau.$$

Applying the Gronwall lemma, hence we have the inequality:

$$E_\alpha^\varepsilon(t,\xi) \le \left\{ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right\} \times$$

$$\times \exp \left[ \frac{M_1 \bar{c} \, |\xi|^2 \, \varepsilon \, |\log \varepsilon| \, \omega(\varepsilon)}{M_0} t \right].$$

Choosing $\varepsilon = \frac{1}{|\xi|^2}$ from the last one we have.

$$E_\alpha^\varepsilon(t,\xi) \le \left\{ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right\} \times$$

$$\times \exp \left[ \frac{M_1 \bar{c} \log \frac{1}{|\xi|^2} \omega \frac{1}{|\xi|^2}}{M_0} t \right] =$$

$$= \left\{ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right\} |\xi|^{2M_1 \bar{c} \omega \left(|\xi|^{-2}\right) T}.$$

On the other hand, as $\tau \to 0$ $\omega(\tau)$ monotically decreasing tends to zero, therefore

$$\lim_{|\xi| \to \infty} \omega\left(|\xi|^{-2}\right) = 0.$$

Then there exists $\lambda_1 > 0$ such that for $|\xi| \ge \lambda_1$ for the following inequality is fulfilled

$$\omega\left(|\xi|^{-2}\right) \le \frac{M_0 \delta}{\bar{c} M_1 T}.$$

Thus for $|\xi| \geq \lambda_1$ we have the inequality

$$E_\alpha(t,\xi) \leq \lambda^{2\delta} \left[ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right]. \qquad (20)$$

Multiply the both hand sides of (20) by $|\xi|^\alpha$ and integrate on the interval $(\lambda_1, +\infty)$

$$\int_{|\xi|\geq\lambda_1} |\xi|^\alpha E_\alpha(t,\xi) d\xi \leq$$

$$\leq \int_{|\xi|\geq\lambda_1} |\xi|^{\alpha+2\delta} \left[ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right] d\xi. \qquad (21)$$

If $\lambda_1 \leq 1$, then

$$\int_{|\xi|<\lambda_1} |\xi|^\alpha E_\alpha(t,\xi) d\xi \leq$$

$$\leq \int_{|\xi|<\lambda_1} |\xi|^{\alpha+2\delta} \left[ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right] d\xi. \qquad (22)$$

If $\lambda_1 > 1$, then

$$\int_{|\xi|<\lambda_1} |\xi|^\alpha E_\alpha(t,\xi) d\xi = \int_{|\xi|<\lambda_1} |\xi|^\alpha E_\alpha(t,\xi) d\xi + \int_{1\leq|\xi|<\lambda_1} |\xi|^\alpha E_\alpha(t,\xi) d\xi \leq$$

$$\leq M_2 \int_{|\xi|<\lambda_1} |\xi|^{\alpha+2\delta} \left[ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right] d\xi \qquad (23)$$

where $M_2 = \max\left(1, \lambda_1^{2\delta}\right)$.

Thus, according to (21)-(23) we have

$$\int_{R^N} |\xi|^\alpha E_\alpha(t,\xi) d\xi \leq$$

$$\leq \int_{R^N} |\xi|^{\alpha+2\delta\alpha+2\delta} \left[ \frac{M_1}{M_0} E_\alpha(0,\xi) + \frac{1}{M_0} \int_0^t \left[ |f(t,\xi)|^2 + |g(t,\xi)|^2 \right] d\tau \right] d\xi,$$

from which the validity of the assertion of theorem 1 follows.

**4. Proof of the lemma.** Using definition of $\alpha_\varepsilon(t)$, we get

$$|\alpha_\varepsilon(t) - \alpha(t)| = \left| \frac{1}{\varepsilon} \int \widetilde{\alpha}(t+\tau) \rho\left(\frac{\tau}{\varepsilon}\right) d\tau - \frac{1}{\varepsilon} \int \widetilde{\alpha}(t) \rho\left(\frac{\tau}{\varepsilon}\right) d\tau \right| =$$

$$= \frac{1}{\varepsilon}\left|\int (\widetilde{\alpha}(t+\tau)-\widetilde{\alpha}(t))\rho\left(\frac{\tau}{\varepsilon}\right)d\tau\right| \le M_\alpha \int\limits_{|\tau|<\varepsilon} |\tau|\,|\log|\,|\tau|\,w(|\tau|)\,\rho(s)ds \le$$

$$\le M_\alpha\varepsilon\cdot|\log\varepsilon|\,\omega(\varepsilon)\int\limits_{|\tau|<\varepsilon}\rho(s)ds = M_\alpha\varepsilon\cdot|\log\varepsilon|\,\omega(\varepsilon),$$

i.e. Lemma 1 is valid.

In the similar way, using the definition of $\alpha_\varepsilon(t)$, we get

$$\left|\dot{\alpha}_\varepsilon(t)\right| = \left|\frac{1}{\varepsilon^2}\int \widetilde{\alpha}(t+\tau)\,\dot{\rho}\left(\frac{\tau}{\varepsilon}\right)d\tau\right| =$$

$$= \left|\frac{1}{\varepsilon^2}\int \widetilde{\alpha}(t+\tau)\dot{\rho}\left(\frac{\tau}{\varepsilon}\right)d\tau - \frac{1}{\varepsilon^2}\int \widetilde{\alpha}(t)\dot{\rho}\left(\frac{\tau}{\varepsilon}\right)d\tau\right| \le$$

$$\le M\cdot|\log\varepsilon|\,\omega(\varepsilon).\frac{1}{\varepsilon}\int\dot{\rho}\left(\frac{\tau}{\varepsilon}\right)d\tau = 4M\cdot|\log\varepsilon|\,\omega(\varepsilon),$$

i.e.

$$\left|\dot{\alpha}_\varepsilon(t)\right| \le 4M\cdot|\log\varepsilon|\,\omega(\varepsilon),$$

On the other hand, taking into account (4), we have

$$\alpha_\varepsilon(t) \ge \frac{\alpha_0}{\varepsilon}\int\left(\frac{\tau}{\varepsilon}\right)d\tau = \frac{\alpha_0}{\varepsilon}\cdot\varepsilon = \alpha_0.$$

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