Veli M. KURBANOV, Yasemen I. HUSEYNOVA

ON CONVERGENCE OF SPECTRAL EXPANSION OF ABSOLUTELY CONTINUOUS VECTOR-FUNCTION IN EIGEN VECTOR-FUNCTIONS OF FOURTH ORDER DIFFERENTIAL OPERATOR

Abstract

In the paper, a fourth order ordinary differential operator with matrix coefficients is considered, absolute and uniform convergence of orthogonal expansion of an absolutely continuous vector-function in eigen vector-functions of the given operator is studied, and the rate of uniform convergence of this expansion is established.

Consider on the interval $G=(0,1)$ the operator

$$L\psi=\psi^{(4)}+U_2(x)\,\psi^{(2)}+U_3(x)\,\psi^{(1)}+U_4(x)\,\psi$$

with matrix coefficients $U_\ell(x)=(u_{\ell ij}(x))_{i,j=1}^m$, $\ell=\overline{2;4}$, where $u_{\ell ij}(x)\in L_1(G)$ are real functions $u_{\ell ij}(x)=u_{\ell ji}(x)$.

Denote by $D(G)$ the class of m-component vector-functions absolutely continuous together with own derivatives to the third order inclusively on the closed interval $\overline{G}=[0,1]$ $\left(D(G)=W_{1,m}^4(G)\right)$.

Under the eigen vector-function of the operator L responding to the eigen value λ we'll understand any vector-function $\psi(x)=(\psi_1(x),\psi_2(x),...,\psi_m(x))^T\in D(G)$ identically not equal to zero and satisfying almost everywhere in G the equation (see [1])

$$L\psi+\lambda\psi=0.$$

Let $L_p^m(G)$, $p\geq 1$, be the space of m-component vector-functions $f(x)=(f_1(x),f_2(x),...,f_m(x))^T$ with the norm

$$\|f\|_{p,m}=\left\{\int_G|f(x)|^p\,dx\right\}^{1/p}=\left\{\int_G\left(\sum_{i=1}^m|f_i(x)|^2\right)^{p/2}dx\right\}^{1/p}.$$

Suppose that $\{\psi_k(x)\}_{k=1}^\infty$ is a complete, orthornomalized system in $L_2^m(G)$ consisting of eigen-functions of the operator L. Denote by $\{\lambda_k\}_{k=1}^\infty$, $\lambda_k\leq 0$ the appropriate system of eigen values.

Denoting $\mu_k=\sqrt[4]{-\lambda_k}$ introduce into consideration the partial sum of the orthogonal expansion of the vector-function $f(x)\in W_{1,m}^1(G)$ in the system $\{\psi_k(x)\}_{k=1}^\infty$

$$\sigma_\nu(x,f)=\sum_{\mu_k\leq\nu}f_k\psi_k(x),\quad \nu>0,$$

where

$$f_k=(f,\psi_k)=\int_0^1\langle f(x),\psi_k(x)\rangle\,dx=\int_0^1\sum_{j=1}^m f_j(x)\,\psi_{kj}(x)\,dx,$$

$$\psi(x) = \left(\psi_{k1}(x), \psi_{k2}(x), ..., \psi_{km}(x)\right)^T.$$

In the paper the following theorem is proved.

Theorem. *Let the vector-function* $f(x) \in W^1_{1,m}(G)$*, the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *be uniformly bounded, and the following conditions be fulfilled*

$$\left| \left\langle f(x), \psi_k^{(3)}(x) \right\rangle \Big|_0^1 \right| \le C_1(f)\, \mu_k^{\alpha}, \quad 0 \le \alpha < 3, \quad \mu_k \ge 4\pi; \tag{1}$$

$$\sum_{n=2}^{\infty} n^{-1} \omega_{1,m}\left(f', n^{-1}\right) < \infty. \tag{2}$$

Then the expansion of the vector-function $f(x)$ *in the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *converges absolutely and uniformly on* $\overline{G} = [0,1]$*, and it is valid the estimation*

$$\sup_{x \in \overline{G}} |\sigma_\nu(x,f) - f(x)| \le const \left\{ C_1(f)\, \nu^{\alpha-3} + \sum_{n=[\nu]}^{\infty} \omega_{1,m}\left(f', n^{-1}\right) n^{-1} + \right.$$

$$\left. + \left(\|f\|_{\infty,m} + \left\|f'\right\|_{1,m} \right) \sum_{r=2}^{4} \nu^{1-r} \, \| \| U_r \| \|_1 + \nu^{-1} \left\|f'\right\|_{1,m} \right\}, \nu \ge 2, \tag{3}$$

where $\omega_{1,m}(g,\delta)$ *is an integral modulus of continuity of the vector-function* $g(x) = (g_1(x), g_2(x), ..., g_m(x))^T \in L_1^m(G)$*;* $\| \| U_r \| \|_1 = \sum_{i,j=1}^{m} \|U_{rij}\|$*,* $r = \overline{2,4}$*; const is independent of* $f(x)$*.*

Note that such theorems for a second order operator were proved in [2]-[4].

For proving the theorem we estimate the Fourier coefficient f_k of the vector-function $f \in W^1_{1,m}(G)$.

Lemma. *For the Fourier coefficients* f_k *of the vector-function* $f(x) \in W^1_{1,m}(G)$ *satisfying condition (1) the estimation* $(\mu_k \ge 4\pi)$

$$|f_k| \le const \left\{ C_1(f)\, \mu_k^{\alpha-4} + \frac{\omega_{1,m}\left(f', \mu_k^{-1}\right)}{\mu_k} + \right.$$

$$\left. + \frac{\|f'\|_{1,m}}{\mu_k^2} + \frac{\|f\|_{\infty,m} + \|f'\|_{1,m}}{\mu_k^2} \sum_{r=2}^{4} \mu_k^{2-r} \, \| \| U_r \| \|_1 \right\} \tag{4}$$

is valid.

Proof of the lemma. For the eigen vector-function $\psi_k(t)$ the following formula is valid (see [5], [6])

$$\mu_k^{-\ell} \psi_k^{(\ell)}(t) =$$

$$= \sum_{j=1}^{3} X_{kj}(0) (-i\omega_j)^{\ell} \exp(-i\omega_j \mu_k t) + (-i\omega_4)^{\ell} B_{k4}(0) \exp(i\omega_4 \mu_k (1-t)) +$$

$$+ \sum_{j=1}^{3} (-1)^{\ell} \omega_j^{\ell+1} \int_0^t M(\xi, \psi_k) \exp(i\omega_j \mu_k (\xi - t))\, d\xi -$$

$$-(-i)^{\ell}\,\omega^{\ell+1}\int_t^1 M(\xi,\psi_k)\exp(i\omega_4\mu_k(\xi-t))\,d\xi, \tag{5}$$

where

$$\omega_1=-\omega_2=-1,\quad \omega_4=-\omega_3=i,\quad \ell=\overline{0,3},\quad \mu_k\neq 0;$$

$$X_{kj}(x)=\frac{1}{4}\sum_{s=0}^{3}\omega_j^{s+1}(-i\mu_k)^{s-3}\,\psi_k^{(3-s)}(x);$$

$$B_{k4}(x)=\exp(i\omega_4\mu_k(1-x))\times$$

$$\times\left\{X_{k4}(0)+\omega_4\int_x^1 M(\xi,\psi_k)\exp(i\omega_j\mu_k(\xi-x))\,d\xi\right\};$$

$$M(\xi,\psi_k)=\frac{1}{4\mu_k^3}\sum_{r=2}^{4}U_r(\xi)\,\psi_k^{(4-r)}(\xi).$$

With regard to definition of the eigen function $\psi_k(x)$ calculate the Fourier coefficients f_k for $\mu_k\geq 1$:

$$f_k=(f,\psi_k)=\frac{1}{\mu_k^4}(f,L\psi_k)=\frac{1}{\mu_k^4}\left(f,U_2\psi_k^{(2)}+U_3\psi_k^{(1)}+U_4\psi_k\right)+\frac{1}{\mu_k^4}\left(f,\psi_k^{(4)}\right)=$$

$$=\frac{1}{\mu_k^4}\left\langle f,\psi_k^{(3)}\right\rangle\Big|_0^1-\frac{1}{\mu_k^4}\left(f',\psi_k^{(3)}\right)+\frac{1}{\mu_k^4}\left(f,U_2\psi_k^{(2)}\right)+\frac{1}{\mu_k^4}\left(f,U_3\psi_k^{(1)}\right)+\frac{1}{\mu_k^4}(f,U_4\psi_k). \tag{6}$$

Taking into account the estimations (see [7])

$$\left\|\psi_k^{(s)}\right\|_{\infty,m}\leq const\,(1+\mu_k)^{s+\frac{1}{p}}\,\|\psi_k\|_{p,m}\,,p\geq 1, \tag{7}$$

of uniform boundedness of the system $\{\psi_k(x)\}_{k=1}^{\infty}$, we find

$$\frac{1}{\mu_k^4}\left|\left(f,U_2\psi_k^{(2)}\right)\right|\leq\frac{const}{\mu_k^4}\|f\|_{\infty,m}\,\|\|U_2\|\|_1\,\|\psi_k\|_{\infty,m}\,\mu_k^2\leq\frac{const}{\mu_k^2}\|f\|_{\infty,m}\,\|\|U_2\|\|_1\,; \tag{7'}$$

$$\frac{1}{\mu_k^4}\left|\left(f,U_3\psi_k^{(1)}\right)\right|\leq\frac{const}{\mu_k^3}\|f\|_{\infty,m}\,\|\|U_3\|\|_1\,;$$

$$\frac{1}{\mu_k^4}\left|(f,U_4\psi_k)\right|\leq\frac{const}{\mu_k^4}\|f\|_{\infty,m}\,\|\|U_4\|\|_1\,.$$

From condition (1)

$$\frac{1}{\mu_k^4}\left|\left\langle f,\psi_k^{(3)}\right\rangle\Big|_0^1\right|\leq C_1(f)\,\mu_4^{\alpha-4}. \tag{8}$$

For estimating the second addend in the right side of equality (6), use formula (5) for $\ell=3$

$$\frac{1}{\mu_k^4}\left(f',\psi_k^{(3)}\right)=\frac{1}{\mu_k}\sum_{j=1}^{3}\left(f',X_{kj}(0)\exp(-i\omega_j\mu_k t)\right)(-i\omega_j)^3+$$

$$+\frac{1}{\mu_k^4}\left(-i\omega_4\right)^3\left(f', B_{k4}\left(0\right)\exp\left(i\omega_4\mu_k\left(1-t\right)\right)\right)+$$

$$+\frac{1}{\mu_k}\sum_{j=1}^{3}\left(-1\right)^3\omega_j^4\left(f', \int_0^t M\left(\xi,\psi_k\right)\exp\left(i\omega_j\mu_k\left(\xi-t\right)\right)d\xi\right)+$$

$$+\frac{1}{\mu_k}i^3\omega_4^4\left(f', \int_t^1 M\left(\xi,\psi_k\right)\right)\exp\left(i\omega_4\mu_k\left(\xi-t\right)d\xi\right). \tag{9}$$

Estimate each term in the given equality. Passing to coordinates, we get

$$\left(f', X_{kj}\left(0\right)\exp\left(-i\omega_j\mu_k t\right)\right)=\sum_{\ell=1}^{m}\int_0^1 f_l'\left(t\right)X_{kj}^{\ell}\left(0\right)\exp\left(-i\omega_j\mu_k t\right)dt=$$

$$=\sum_{\ell=1}^{m}X_{kj}^{\ell}\left(0\right)\int_0^1 f_l'\left(t\right)\exp\left(-i\mu_k\omega_j t\right)dt, \quad j=\overline{1,3}.$$

Here, taking into account estimation (7) for $p=\infty$, from uniform boundedness of the system $\{\psi_k\left(x\right)\}_{k=1}^{\infty}$ and estimation (see [5], [6])

$$\left|\int_0^1 f_\ell'\left(t\right)\exp\left(-i\omega_j\mu_k t\right)dt\right|\le const\left\{\omega_1\left(f_\ell', \frac{1}{\mu_k}\right)+\frac{1}{\mu_k}\left\|f_\ell'\right\|_1\right\}, \quad \ell=\overline{1,m}$$

we find

$$\left(f', X_{kj}\left(0\right)\exp\left(-i\omega_j\mu_k t\right)\right)=const\left\{\omega_{1,m}\left(f', \frac{1}{\mu_k}\right)+\frac{1}{\mu_k}\left\|f'\right\|_{1,m}\right\}. \tag{10}$$

Now estimate the coefficients $B_{k4}\left(0\right)$. Having written formula (5) for $\ell=0$, for $B_{k4}\left(0\right)$ we find.

$$\left|B_{k4}\left(0\right)\right|\le C\left\|\exp\left(i\omega_4\mu_4\left(1-\cdot\right)\right)\right\|_{\infty}^{-1}\times$$

$$\times\left\{\left\|\psi_k\right\|_{\infty,m}+\sum_{j=1}^{3}\left|X_{kj}\left(0\right)\right|+\sum_{j=1}^{3}\left\|\int_0^{\cdot}\left|M\left(\xi,\psi_k\right)\right|d\xi\right\|_{\infty}+\left\|\int_{\cdot}^{1}\left|M\left(\xi,\psi_k\right)\right|d\xi\right\|_{\infty}\right\}.$$

Taking into account

$$\left|M\left(\xi,\psi_k\right)\right|\le\left|\frac{1}{4\mu_k^3}\sum_{r=2}^{4}U_r\left(\xi\right)\psi_k^{(4-r)}\left(\xi\right)\right|\le\frac{1}{4\mu_k^3}\sum_{r=2}^{4}\left\|U_r\left(\xi\right)\right\|\left\|\psi_k^{(4-r)}\right\|_{\infty,m}\le$$

$$\le\frac{const}{\mu_k}\left[\sum_{r=2}^{4}\left\|U_r\left(\xi\right)\right\|\mu_k^{r-2}\right]\left\|\psi_k\right\|_{\infty,m}\le\frac{const}{\mu_k}\left[\sum_{r=2}^{4}\left\|U_r\left(\xi\right)\right\|\mu_k^{r-2}\right]; \tag{11}$$

$$\left|X_{kj}\left(0\right)\right|\le const\left\|\psi_k\right\|_{\infty,m}\le const, \quad j=\overline{1,3},$$

we get

$$\left|B_{k4}\left(0\right)\right|\le const\left\|\psi_k\right\|_{\infty,m}\le const, \quad k\in N.$$

Using the last estimation in the second addend of equality (9), we have

$$\left|\left(f', B_{k4}(0)\exp\left(i\omega_4\mu_k(1-t)\right)\right)\right| \leq \left|\sum_{\ell=0}^{m}\int_0^1 f_l'(t)\exp\left(i\omega_4\mu_k(1-t)\right)dtB_{k4}^{\ell}(0)\right| \leq$$

$$\leq const\left\{\omega_{1,m}\left(f', \frac{1}{\mu_k}\right)+\frac{1}{\mu_k}\left\|f'\right\|_{1,m}\right\}. \tag{12}$$

By virtue of (11), the third and fourth addends are estimated as follows

$$\left|\left(f', \int_0^t M(\xi, \psi_k)\exp\left(i\omega_j\mu_k(\xi-t)\right)d\xi\right)\right| \leq$$

$$\leq \frac{const}{\mu_k}\sum_{r=2}^{4}\left\|\left\|U_r\right\|\right\|_1\mu_k^{2-r}\left\|f'\right\|_{1,m}, \tag{13}$$

$$\left|\left(f', \int_t^1 M(\xi, \psi_k)\exp\left(i\omega_4\mu_k(\xi-t)\right)d\xi\right)\right| \leq$$

$$\leq \frac{const}{\mu_k}\sum_{r=2}^{4}\left\|\left\|U_r\right\|\right\|_1\mu_k^{2-r}\left\|f'\right\|_{1,m}. \tag{14}$$

By virtue of inequalities (10), (12)-(14), from equality (9) we find

$$\frac{1}{\mu_k^4}\left|\left(f', \psi_k^{(3)}\right)\right| \leq$$

$$\leq \frac{const}{\mu_k}\left\{\omega_{1,m}\left(f', \frac{1}{\mu_k}\right)+\frac{1}{\mu_k}\left\|f'\right\|_{1,m}+\frac{1}{\mu_k}\left\|f\right\|_{1,m}\sum_{r=2}^{4}\left\|U_r\right\|_1\mu_k^{2-r}\right\}. \tag{15}$$

Taking into account estimations (7'), (8) and (15), in equality (6) we get

$$|f_k| \leq const\left\{C_1(f)\mu_k^{\alpha-4}+\frac{\omega_{1,m}\left(f', \frac{1}{\mu_k}\right)}{\mu_k}+\right.$$

$$\left.+\frac{\left\|f'\right\|_{1,m}}{\mu_k^2}+\frac{\left\|f\right\|_{\infty,m}+\left\|f'\right\|_{1,m}}{\mu_k^2}\sum_{r=2}^{4}\mu_k^{2-r}\left\|\left\|U_r\right\|\right\|_1\right\}.$$

The lemma is proved.

Proof of the theorem. Represent the series $\sum_{k=1}^{\infty}|f_k|\,|\psi_k(x)|$ in the form

$$\sum_{k=1}^{\infty}|f_k|\,|\psi_k(x)| = \sum_{0\leq\mu_k<4\pi}|f_k|\,|\psi_k(x)|+\sum_{\mu\geq4\pi}|f_k|\,|\psi_k(x)|.$$

From the condition of "Sum of units" (see [7])

$$\sum_{\tau\le\mu_k\le\tau+1} 1 \le const, \quad \forall\tau\ge 0 \tag{16}$$

and uniform boundedness of the system $\{\psi_k(x)\}_{k=1}^{\infty}$

$$\sum_{\tau\le\mu_k<4\pi} |f_k|\,|\psi_k(x)| \le const \sum_{\tau\le\mu_k<4\pi} |(f,\psi_k)| \le$$

$$\le const\,\|f\|_{1,m} \sum_{\tau\le\mu_k<4\pi} 1 \le const\,\|f\|_{1,m}\,.$$

From the conditions of the theorem, condition (16) and statement of the lemma, we get

$$\sum_{\mu_k\ge 4\pi} |f_k|\,|\psi_k(x)| \le const \sum_{\mu_k\ge 4\pi} |f_k| \le \Bigg\{ C_1(f) \sum_{\mu_k\ge 4\pi}^{\infty} \mu_k^{\alpha-4} +$$

$$+\sum_{\mu_k\ge 4\pi}^{\infty} \frac{\omega_{1,m}\left(f',\mu_k^{-1}\right)}{\mu_k} + \left\|f'\right\|_{1,m} \sum_{\mu_k\ge 4\pi}^{\infty} \mu_k^{-2} +$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{4} \|\|U_r\|\|_1 \left(\sum_{\mu_k\ge 4\pi}^{\infty} \mu_k^{-r}\right)\Bigg\} \le$$

$$\le const\Bigg\{ C_1(f) \sum_{n=[4\pi]}^{\infty}\left(\sum_{n\le\mu_k\le n+1} \mu_k^{\alpha-4}\right) +$$

$$+\sum_{n=[4\pi]}^{\infty}\left(\sum_{n\le\mu_k\le n+1} \mu_k^{-1}\omega_{1,m}\left(f',\frac{1}{\mu_k}\right)\right) + \left\|f'\right\|_{1,m}\sum_{n=[4\pi]}^{\infty}\left(\sum_{n\le\mu_k\le n+1}\mu_k^{-2}\right) +$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{4} \|\|U_r\|\|_1 \sum_{n=[4\pi]}^{\infty}\left(\sum_{n\le\mu_k\le n+1}\mu_k^{-r}\right)\Bigg\} \le$$

$$\le const\Bigg\{ C_1(f)\sum_{n=[4\pi]}^{\infty} n^{\alpha-4}\left(\sum_{n\le\mu_k\le n+1} 1\right) +$$

$$+\sum_{n=[4\pi]}^{\infty}\frac{1}{n}\omega_{1,m}\left(f',\frac{1}{n}\right)\left(\sum_{n\le\mu_k\le n+1} 1\right) + \left\|f'\right\|_{1,m}\sum_{n=[4\pi]}^{\infty} n^{-2}\left(\sum_{n\le\mu_k\le n+1} 1\right) +$$

$$+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{4}\|\|u_r\|\|_1\sum_{n=[4\pi]}^{\infty} n^{-2}\left(\sum_{n\le\mu_k\le n+1} 1\right)\Bigg\} \le$$

$$\le const\Bigg\{ C_1(f)\,[4\pi]^{\alpha-3} + \sum_{n=[4\pi]}^{\infty} n^{-1}\omega_{1,m}\left(f',\frac{1}{n}\right) +$$

$$+\left\|f'\right\|_{1,m}[4\pi]^{-1}+\left(\left\|f'\right\|_{1,m}+\|f\|_{\infty,m}\right)\sum_{r=2}^{4}\||U_r\||_1[4\pi]^{1-r}\right\}<\infty.$$

Thus, the expansion $\sum\limits_{k=1}^{\infty} f_k\psi_k(x)$ of the function $f(x)$ converges absolutely and uniformly on $\overline{G}$.

From the basicity of the system $\{\psi_k(x)\}_{k=1}^{\infty}$ in $L_{2(G)}^m$ this expansion converges uniformly to the function $f(x)$. Consequently,

$$f(x)=\sum_{k=1}^{\infty} f_k\psi_k(x), \quad x\in\overline{G}. \tag{17}$$

Now, establish estimation (3). From equality (17) uniform boundedness of the system $\{\psi_k(x)\}_{k=1}^{\infty}$, conditions (16), (2) and the lemma

$$\sup_{x\in\overline{G}}|\sigma_\nu(x,f)-f(x)|=\sup_{x\in\overline{G}}\left|\sum_{\mu_k\le\nu} f_k\psi_k(x)-\sum_{k=1}^{\infty} f_k\psi_k(x)\right|=$$

$$=\sup_{x\in\overline{G}}\left|\sum_{\mu_k>\nu} f_k\psi_k(x)\right|=\sum_{\mu_k\ge\nu}|f_k|\,|\psi_k|_{\infty,m}\le C\sum_{\mu_k\ge\nu}|f_k|\le C\sum_{n=[\nu]}^{\infty}\left(\sum_{n\le\mu_k\le n+1}|f_k|\right)\le$$

$$\le C\sum_{n=[\nu]}^{\infty}\left(\sum_{n\le\mu_k\le n+1}\left\{C_1(f)\,\mu_k^{\alpha-4}+\frac{\omega_{1,m}\left(f',\mu_k^{-1}\right)}{\mu_k}+\mu_k^{-2}\left\|f'\right\|_{1,m}+\right.\right.$$

$$\left.\left.+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{4}\mu_k^{-r}\||U_r\||_1\right\}\right)\le$$

$$\le const\left\{C_1(f)\,\nu^{\alpha-3}+\sum_{n=[\nu]}^{\infty} n^{-1}\omega_{1,m}\left(f',n^{-1}\right)+\right.$$

$$\left.+\nu^{-1}\left\|f'\right\|_{1,m}+\left(\|f\|_{\infty,m}+\left\|f'\right\|_{1,m}\right)\sum_{r=2}^{4}\mu^{1-r}\||U_r\||_1\right\}.$$

The theorem is proved.

Corollary 1. *If the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *is uniformly bounded,* $f(x)\in W_{1,m}^1(G)$, $f(0)=f(1)=0$ *and* $f'(x)\in H_{1,m}^{\alpha}(G)$, $0<\alpha<1$ *(*$H_{1,m}^{\alpha}(G)$ *is the Nikolsky class of* m *component vector-functions), then*

$$\sup_{x\in\overline{G}}|\sigma_\nu(x,f)-f(x)|\le const\ \nu^{-\alpha}\left\|f'\right\|_{1,m}^{\alpha},$$

where

$$\|g\|_{1,m}^{\alpha}=\|g\|_{1,m}+\sup_{\delta>0}\frac{\omega_{1,m}(g,\delta)}{\delta^{\alpha}}.$$

Corollary 2. *If the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *is uniformly bounded,* $f(x) \in W_{1,m}^1(G)$, $f(0) = f(1) = 0$ *and for some*$\beta > 0$ *it isfulflled the estimation*

$$\omega_{1,m}(f', \delta) = O\left(\ln^{-(1+\beta)} \frac{1}{\delta}\right), \quad \delta \to +0,$$

then

$$\sup_{x \in \overline{G}} |\sigma_\nu(x, f) - f(x)| = O\left(\ln^{-\beta} \nu\right), \quad \nu \to \infty.$$

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Veli M. Kurbanov
Institute of Mathematics and Mechanics of NAS of Azerbaijan
9, B.Vahabzade str., AZ 1141, Baku, Azerbaijan

Yasemen I. Huseynova
Baku State University,
23, B. Vahabzade str., AZ 1148, Baku, Azerbaijan
Tel.: (99412) 539 47 20 (off.).

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