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MATHEMATICS

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A–STATISTICAL APPROXIMATION OF ANALYTIC FUNCTIONS BY k-POSITIVE OPERATORS

Abstract

In the paper using the k-positive operators we obtain A -statistical analogies of Korovkin type theorems in the space of functions analytic in the closure of the domain D, in which the convergence is determined as a uniform convergence in some closed domain D_1, containing D strictly on compact subsets.

In theory of approximation of analytic functions, k-positive linear operators built-up by A.D. Gadjiev [1] play the same role that positive operators in theory of approximation of continuous real functions. In the papers [2-7], these operators are studied well and their application in theory of analytic functions are shown.

In the present paper, using k-positive operators, the A -statistical analogies of Korovkin type theorem in the space of functions analytic in the closure of domain are obtained.

Definition 1. *The matrix $A=(a_{i,j})_{i,j=1}^{\infty}$ is said to be regular if $a_{ij}\geq 0$ for any $i,j\in N$, and for any convergent sequence $\{x_j\}_{j=1}^{\infty}$ the sequence $y_i=\sum\limits_{j=1}^{\infty}a_{i,j}x_j$, $i=1,2,3,...$ converges, and $\lim\limits_{j\to\infty}x_j=\lim\limits_{i\to\infty}y_i$.*

It is known [8] that the matrix $A=(a_{i,j})_{i,j=1}^{\infty}$ is regular iff the following conditions are fulfilled:

i) $\exists C>0\ \ \forall i\in N\ \ \sum\limits_{j=1}^{\infty}|a_{i,j}|<C$;

ii) $\lim\limits_{i\to\infty}\sum\limits_{j=1}^{\infty}a_{i,j}=1$;

iii) $\forall j\in N\ \ \lim\limits_{i\to\infty}a_{i,j}=0$.

Note that for non-negative matrices the condition (i) follows from the condition (ii).

Definition 2 [9]. *Let $A=(a_{i,j})_{i,j=1}^{\infty}$ be a non-negative regular matrix, and $\{x_j\}_{j=1}^{\infty}$ be some sequence in the space X. If for any $\varepsilon>0$ the equality*

$$\lim_{n\to\infty}n^{-1}\left|\{i\leq n:|y_i-\alpha|>\varepsilon\}\right|=0,$$

where $y_i=\sum\limits_{j=1}^{\infty}a_{i,j}x_j$, $i=1,2,3,...$ is fulfilled, and $|B|$ is a cardinal number of the set B, then the sequence $\{x_j\}_{j=1}^{\infty}$ is called A-statistically convergent, and the element $\alpha\in X$ is said to be A-statistic limit of the sequence $\{x_j\}_{j=1}^{\infty}$ and is denoted by $\alpha=A_{st}-\lim\limits_{j\to\infty}x_j$.

In special case when $A = (\delta_{i,j})_{i,j=1}^{\infty}$, where $\delta_{i,j}$ is the Kronecker symbol, A-statistical convergence is called a statistical convergence.

Let D be a simply-connected bounded domain with a simply connected complement. Denote by $A(\overline{D})$ the space of functions analytic in the closure of domain D, where the convergence is determined as a uniform convergence in some closed domain D_1, contained strictly in compact subsets of $(\overline{D} \subset D_1)$.

Let $\varphi(z)$ be a function that one-to-one and conformally maps the exterior of domain D onto the exterior of a unit circle.

Assume

$$\varphi_n(z) = \frac{1}{2\pi i}\int_L [\varphi(t)]^n \frac{dt}{t-z}, \quad n \in Z_+,$$

where the contour L contains D interior to it. The system $\{\varphi_n(z)\}$ (Faber polynomials) form a basis in $A(\overline{D})$ [10]. This means that in $A(\overline{D})$ it is fulfilled the expansion

$$f(z) = \sum_{k=0}^{\infty} f_k \varphi_k(z), \tag{1}$$

where

$$f_k = \frac{1}{2\pi i}\int_L \frac{f(t)\varphi'(t)}{[\varphi(t)]^{k+1}}dt. \tag{2}$$

Theorem 1. *Let $A = (a_{i,j})_{i,j=1}^{\infty}$ be a non-negative regular matrix. For A - statistical convergence of the sequence $f_n(z)$ to zero in $A(\overline{D})$, it is necessary and sufficient that the coefficients of the expansion*

$$f_n(z) = \sum_{k=0}^{\infty} f_{n,k}\varphi_k(z), \quad n \in N \tag{3}$$

satisfy the condition

$$|f_{nk}| < \frac{\varepsilon_n}{(1+\delta)^k} \tag{4}$$

for any $n \in N$, $k \in Z_+$, where

$$A_{st} - \lim_{n\to\infty} \varepsilon_n = 0, \quad \delta > 0. \tag{5}$$

Proof. Sufficiency. If expansion (3) is fulfilled, and the coefficients of the expansion satisfy condition (4), then for any $z \in \overline{D_\delta}$, where $\overline{D_\delta} = C \backslash \left\{z : |\varphi(z)| > 1 + \frac{\delta}{2}\right\}$, by virtue of the estimation (see [10])

$$|\varphi_k(z)| \le \left(\sqrt{e} + \sqrt{k\ln(k+1)}\right)\left(1+\frac{\delta}{2}\right)^k \tag{6}$$

we get the inequality

$$|f_n(z)| \le \sum_{k=0}^{\infty} \frac{\varepsilon_n}{(1+\delta)^k}|\varphi_k(z)| < \varepsilon_n \sum_{k=0}^{\infty} \frac{\sqrt{e}+\sqrt{k\ln(k+1)}}{(1+\delta)^k}\left(1+\frac{\delta}{2}\right)^k. \tag{7}$$

Since the series $\sum\limits_{k=0}^{\infty}\frac{\sqrt{e}+\sqrt{k\ln(k+1)}}{(1+\delta)^k}\left(1+\frac{\delta}{2}\right)^k$ converges, then it follows from (7) that there exists a number $d_1>0$ such that for any $n\in N$ it is fulfilled the inequality

$$\|f_n\|_{C\left(\overline{D_\delta}\right)}=\max_{z\in\overline{D_\delta}}|f_n(z)|\le d_1\varepsilon_n.$$

From this inequality it follows that for any $\varepsilon>0$ it holds the inclusion

$$\left\{i\in N:\sum_{j=1}^{\infty}a_{i,j}\|f_j\|_{C\left(\overline{D_\delta}\right)}>\varepsilon\right\}\subset\left\{i\in N:\sum_{j=1}^{\infty}a_{i,j}\varepsilon_j>\frac{\varepsilon}{d_1}\right\}.$$

Hence and from the condition $A_{st}-\lim\limits_{n\to\infty}\varepsilon_n=0$ we get $A_{st}-\lim\limits_{n\to\infty}\|f_n\|_{C\left(\overline{D_\delta}\right)}=0$, i.e. the sequence $\{f_n(z)\}$ A -statistically converges to zero in the space $A\left(\overline{D}\right)$.

Necessity. If the sequence of functions $\{f_n(z)\}$ A -statistically converges to zero in $A\left(\overline{D}\right)$, then there exists a number $\delta>0$ such that the convergence $\varepsilon_n=\max\limits_{|\varphi(z)|=1+\delta}|f_n(z)|$ A -statistically converges to zero as $n\to\infty$. Then

$$|f_{nk}|=\left|\frac{1}{2\pi i}\int\limits_{|\varphi(z)|=1+\delta}\frac{f_n(z)\varphi'(z)}{[\varphi(z)]^{k+1}}dz\right|\le\frac{1}{2\pi}\int\limits_{|u|=1+\delta}|f_n(\psi(u))|\frac{|du|}{|u|^{k+1}}\le\frac{\varepsilon_n}{(1+\delta)^k},$$

where $\psi(u)$ is the inverse of the function $\varphi(z)$. The theorem is proved.

Definition 3 [7]. *The linear continuous operator $T:A\left(\overline{D}\right)\to A\left(\overline{D}\right)$ is called k-positive if the operator T leaves invariant the set of functions with non-negative coefficients in expansion in basis $\{\varphi_k(z)\}$.*

From expansion (1) it follows that each linear continuous operator $T:A\left(\overline{D}\right)\to A\left(\overline{D}\right)$ has the form

$$(Tf)(z)=\sum_{k=0}^{\infty}\left(\sum_{m=0}^{\infty}T_{k,m}f_m\right)\varphi_k(z),$$

where $f(z)=\sum\limits_{k=0}^{\infty}f_k\varphi_k(z)$. Hence it follows that k-positivity of the operator T is equivalent to non-negativity of the coefficients $T_{k,m}$.

Let $g=\{g_k\}_{k=0}^{\infty}$ be a strictly monotonic sequence of positive numbers, satisfying the condition

$$\lim_{k\to\infty}|g_k-g_{k-1}|^{\frac{1}{k}}=1. \tag{8}$$

Note that hence it follows the equality $\lim\limits_{k\to\infty}g_k^{\frac{1}{k}}=1$.

Denote by $A_g^{(\delta_0)}\left(\overline{D}\right)$ the set of analytic functions $f(z)=\sum\limits_{k=0}^{\infty}f_k\varphi_k(z)\in A\left(\overline{D}\right)$ satisfying the condition

$$|f_k|\le\frac{M_f^{(\delta_0)}g_k}{(1+\delta_0)^k}, \tag{9}$$

where $\delta_0 > 0$ and $M_f^{(\delta_0)}$ is a constant independent of k.

Denote

$$g_\nu(z) = \sum_{k=0}^{\infty} \frac{g_k^{\frac{\nu}{2}}}{(1+\delta_0)^k} \varphi_k(z), \quad \nu = 0,1,2, \quad \delta_0 > 0.$$

Let $T_n : A(\overline{D}) \to A(\overline{D})$ be a sequence of k-positive operators:

$$(T_n f)(z) = \sum_{k=0}^{\infty} \left(\sum_{m=0}^{\infty} T_{k,m}^{(n)} f_m \right) \varphi_k(z),$$

where $f(z) = \sum_{k=0}^{\infty} f_k \varphi_k(z)$.

Theorem 2. *In order the sequence of functions $T_n f - f$ A -statistically converge to zero in $A(\overline{D})$ for any function $f \in A_g^{(\delta_0)}(\overline{D})$, it is necessary and sufficient that A -statistically converge to zero of the sequence of functions $T_n g_\nu - g_\nu$, $\nu = 0,1,2$ in $A(\overline{D})$.*

Proof. From A-statistical convergence to zero in $A(\overline{D})$ of the sequence of functions $T_n g_\nu - g_\nu$, $\nu = 0,1,2$, by virtue of theorem 1, it follows the inequality

$$\sum_{m=0}^{\infty} \left(\sqrt{g_m} - \sqrt{g_k}\right)^2 \frac{T_{k,m}^{(n)}}{(1+\delta_0)^m} \le \frac{\varepsilon_n}{(1+\delta)^k}, \tag{10}$$

where $A_{st} - \lim_{n\to\infty} \varepsilon_n = 0$, $\delta > 0$.

If we denote

$$\Delta_k(g) = \min\left\{ \left|\sqrt{g_k} - \sqrt{g_{k-1}}\right|; \ \left|\sqrt{g_{k+1}} - \sqrt{g_k}\right| \right\},$$

then from inequality (10) and from strict monotonicity of the sequence $\{g_k\}_{k=0}^{\infty}$ it follows the equality

$$\sum_{\substack{m=0\\ m\neq k}}^{\infty} \frac{T_{k,m}^{(n)}}{(1+\delta_0)^m} \le \frac{\varepsilon_n}{(1+\delta)^k \Delta_k^2(g)}. \tag{11}$$

For any $f(z) = \sum_{k=0}^{\infty} f_k \varphi_k(z) \in A(\overline{D})$ we have

$$T_n f(z) - f(z) = \sum_{k=0}^{\infty} \left\{ \sum_{m=0}^{\infty} T_{k,m}^{(n)} f_m - f_k \right\} \varphi_k(z) =$$

$$= \sum_{k=0}^{\infty} \left\{ \sum_{m=0}^{\infty} \frac{T_{k,m}^{(n)}}{(1+\delta_0)^m} \left[f_m (1+\delta_0)^m - f_k (1+\delta_0)^k \right] \right\} \varphi_k(z) +$$

$$+ \sum_{k=0}^{\infty} \left[\sum_{m=0}^{\infty} \frac{T_{k,m}^{(n)}}{(1+\delta_0)^m} - \frac{1}{(1+\delta_0)^k} \right] f_k (1+\delta_0)^k \varphi_k(z) = J_n^{(1)}(z) + J_n^{(2)}(z). \tag{12}$$

If $f \in A_g^{\delta_0}\left(\overline{D}\right)$, then from inequalities (6), (9), (10) and (11) for any $z \in \overline{D_\delta}$, where $\overline{D_\delta} = C\backslash\left\{z : |\varphi(z)| > 1 + \frac{\delta}{2}\right\}$, the following estimations are fulfilled:

$$\left|J_n^{(1)}(z)\right| \le \sum_{k=0}^{\infty}\left\{\sum_{m=0}^{\infty}\frac{T_{k,m}^{(n)}}{(1+\delta_0)^m}\left|f_m(1+\delta_0)^m - f_k(1+\delta_0)^k\right|\right\}|\varphi_k(z)| \le$$

$$\le \sum_{k=0}^{\infty}\left\{\sum_{\substack{m=0\\ m\neq k}}^{\infty}\frac{T_{k,m}^{(n)}}{(1+\delta_0)^m}M_f^{(\delta_0)}[g_m+g_k]\right\}|\varphi_k(z)| \le$$

$$\le M_f^{(\delta_0)}\sum_{k=0}^{\infty}\left\{\sum_{\substack{m=0\\ m\neq k}}^{\infty}\frac{T_{k,m}^{(n)}}{(1+\delta_0)^m}\left[2\left(\sqrt{g_m}-\sqrt{g_k}\right)^2+3g_k\right]\right\}|\varphi_k(z)| \le$$

$$\le M_f^{(\delta_0)}\varepsilon_n\sum_{k=0}^{\infty}\left\{\frac{2}{(1+\delta)^k}+\frac{3g_k}{(1+\delta_0)^k\Delta_k^2(g)}\right\}\left(\sqrt{e}+\sqrt{k\ln(k+1)}\right)\left(1+\frac{\delta}{2}\right)^k;$$

$$\left|J_n^{(2)}(z)\right| \le \sum_{k=0}^{\infty}\left|\sum_{m=0}^{\infty}\frac{T_{k,m}^{(n)}}{(1+\delta_0)^m}-\frac{1}{(1+\delta_0)^k}\right|\left|f_k(1+\delta_0)^k\right||\varphi_k(z)| \le$$

$$\le \sum_{k=0}^{\infty}\frac{\varepsilon_n}{(1+\delta)^k}M_f^{(\delta_0)}g_k\left(\sqrt{e}+\sqrt{k\ln(k+1)}\right)\left(1+\frac{\delta}{2}\right)^k.$$

Hence, by virtue of (12) it follows that there exists a number $d_2 > 0$ such that for any $n \in N$ it is fulfilled the inequality

$$\|T_nf - f\|_{C\left(\overline{D_\delta}\right)} = \max_{z\in\overline{D_\delta}}|T_nf(z) - f(z)| \le d_2\varepsilon_n.$$

From this inequality, for any $\varepsilon > 0$ it holds the inclusion

$$\left\{i \in N : \sum_{j=1}^{\infty}a_{i,j}\|T_jf - f\|_{C\left(\overline{D_\delta}\right)} > \varepsilon\right\} \subset \left\{i \in N : \sum_{j=1}^{\infty}a_{i,j}\varepsilon_j > \frac{\varepsilon}{d_2}\right\}.$$

Hence and from the condition $A_{st}-\lim\limits_{n\to\infty}\varepsilon_n = 0$ we get that $A_{st}-\lim\limits_{n\to\infty}\|T_nf - f\|_{C\left(\overline{D_\delta}\right)} = 0$, i.e. the sequence $T_nf - f$ A statistically converges to zero in the space $A\left(\overline{D}\right)$. The theorem is proved.

Corollary 1. *Let* $g = \left\{(1+k)^2\right\}_{k=0}^{\infty}$. *In order the sequence of functions* $T_nf - f$ *A -statistically converge to zero in* $A\left(\overline{D}\right)$ *for any* $f \in A_g^{(\delta_0)}\left(\overline{D}\right)$, *it is necessary and sufficient the A-statistical convergence to zero in* $A\left(\overline{D}\right)$ *of the sequence of functions* $T_nh_\nu - h_\nu$, $\nu = 0,1,2,$ *where*

$$h_\nu(z) = \frac{1}{2\pi i}\int\limits_C\left(\frac{1}{1+\delta_0-\varphi(t)}\right)^\nu\frac{dt}{t-z}, \quad \nu = 0,1,2.$$

Indeed, in the case $g=\left\{(1+k)^2\right\}_{k=0}^{\infty}$

$$g_0(z)=\sum_{k=0}^{\infty}\frac{\varphi_k(z)}{(1+\delta_0)^k}=\sum_{k=0}^{\infty}\frac{1}{2\pi i}\int_C\left[\frac{\varphi(t)}{1+\delta_0}\right]^k\frac{dt}{t-z}=$$

$$=\frac{1}{2\pi i}\int_C\frac{1}{1-\frac{\varphi(t)}{1+\delta_0}}\frac{dt}{t-z}=(1+\delta_0)\,h_0(z);$$

$$g_1(z)=\sum_{k=0}^{\infty}\frac{1+k}{(1+\delta_0)^k}\varphi_k(z)=\sum_{k=0}^{\infty}\frac{1}{2\pi i}\int_C(1+k)\left[\frac{\varphi(t)}{1+\delta_0}\right]^k\frac{dt}{t-z}=(1+\delta_0)^2\,h_1(z);$$

$$g_2(z)=\sum_{k=0}^{\infty}\frac{(1+k)^2}{(1+\delta_0)^k}\varphi_k(z)=\sum_{k=0}^{\infty}\frac{1}{2\pi i}\int_C(1+k)^2\left[\frac{\varphi(t)}{1+\delta_0}\right]^k\frac{dt}{t-z}=$$

$$=2(1+\delta_0)^3\,h_2(z)-(1+\delta_0)\,h_1(z)$$

and therefore by virtue of theorem 2 the A -statistical convergence to zero of the sequence of functions $T_n g_\nu - g_\nu$, $\nu=0,1,2$ is equivalent to A -statistical convergence to zero of the functions $T_n h_\nu - h_\nu$, $\nu=0,1,2$.

Let the strict motone sequence of positive numbers $b=\{b_k\}_{k=0}^{\infty}$ and $g=\{g_k\}_{k=0}^{\infty}$ satisfy condition (8). Denote

$$b_\nu(z)=\sum_{k=0}^{\infty}b_k^{\frac{\nu}{2}}g_k\frac{\varphi_k(z)}{(1+\delta_0)^k},\quad \nu=0,1,2,\quad \delta_0>0.$$

Theorem 3. *Let $T_n : A\left(\overline{D}\right)\to A\left(\overline{D}\right)$ be a sequence of k-positive operators. In order the sequence of functions $T_n f - f$ A -statistical converge to zero in $A\left(\overline{D}\right)$ for any function $f\in A_g^{(\delta_0)}(D)$ it is necessary and sufficient that A statistically converge to zero in $A\left(\overline{D}\right)$ of the sequence of functions $T_n b_\nu - b_\nu$, $\nu=0,1,2$.*

Proof. Let $(T_n f)(z)=\sum_{k=0}^{\infty}\left(\sum_{m=0}^{\infty}T_{k,m}^{(n)}f_m\right)\varphi_k(z)$, where $f(z)=\sum_{k=0}^{\infty}f_k\varphi_k(z)$. From A -statistical convergence to zero in $A\left(\overline{D}\right)$ of the sequence of functions $T_n b_\nu - b_\nu$, $\nu=0,1,2$, by theorem 1 it follows the inequality

$$\sum_{m=0}^{\infty}\left(\sqrt{b_m}-\sqrt{b_k}\right)^2\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}\le\frac{\varepsilon_n}{(1+\delta)^k},\tag{13}$$

where $A_{st}-\lim\limits_{n\to\infty}\varepsilon_n=0$, $\delta>0$.

If we denote

$$\Delta_k(b)=\min\left\{\left|\sqrt{b_k}-\sqrt{b_{k-1}}\right|;\left|\sqrt{b_{k+1}}-\sqrt{b_k}\right|\right\},$$

then from inequality (13) it follows that

$$\sum_{\substack{m=0\\ m\neq k}}^{\infty}\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}\le\frac{\varepsilon_n}{(1+\delta)^k\Delta_k^2(b)}.\tag{14}$$

For any $f(z)=\sum\limits_{k=0}^{\infty}f_k\varphi_k(z)\in A_g^{(\delta_0)}\left(\overline{D}\right)$ we have

$$T_nf(z)-f(z)=\sum_{k=0}^{\infty}\left\{\sum_{m=0}^{\infty}\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}\left[\frac{f_m}{g_m}(1+\delta_0)^m-\frac{f_k}{g_k}(1+\delta_0)^k\right]\right\}\varphi_k(z)+$$

$$+\sum_{k=0}^{\infty}\left[\sum_{m=0}^{\infty}\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}-\frac{g_k}{(1+\delta_0)^k}\right]\frac{f_k}{g_k}(1+\delta_0)^k\varphi_k(z)=\widetilde{J}_n^{(1)}(z)+\widetilde{J}_n^{(2)}(z). \quad (15)$$

By virtue of inequalities (6), (9) and (14), for any $z\in\overline{D}_\delta$, where $\overline{D}_\delta=C\backslash\left\{z:|\varphi(z)|>1+\frac{\delta}{2}\right\}$ the following estimations are fulfilled

$$\left|\widetilde{J}_n^{(1)}(z)\right|\le\sum_{k=0}^{\infty}\left\{\sum_{\substack{m=0\\m\neq k}}^{\infty}\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}2M_f^{(\delta_0)}\right\}|\varphi_k(z)|\le$$

$$\le 2M_f^{(\delta_0)}\sum_{k=0}^{\infty}\frac{\varepsilon_n}{(1+\delta)^k}\frac{\left(\sqrt{e}+\sqrt{k\ln(k+1)}\right)}{\Delta_k^2(b)}\left(1+\frac{\delta}{2}\right)^k;$$

$$\left|\widetilde{J}_n^{(2)}(z)\right|\le\sum_{k=0}^{\infty}\left|\sum_{m=0}^{\infty}\frac{T_{k,m}^{(n)}g_m}{(1+\delta_0)^m}-\frac{g_k}{(1+\delta_0)^k}\right|\left|\frac{f_k}{g_k}(1+\delta_0)^k\right||\varphi_k(z)|\le$$

$$\le\sum_{k=0}^{\infty}\frac{\varepsilon_n}{(1+\delta)^k}M_f^{(\delta_0)}\left(\sqrt{e}+\sqrt{k\ln(k+1)}\right)\left(1+\frac{\delta}{2}\right)^k.$$

Hence, from equality (15) it follows that there exists a number $d_3>0$ such that for any $n\in N$ it is fulfilled the inequality

$$\|T_nf-f\|_{C\left(\overline{D}_\delta\right)}=\max_{z\in\overline{D}_\delta}|T_nf(z)-f(z)|\le d_3\varepsilon_n.$$

From this inequality it follows that for any $\varepsilon>0$ it holds the inclusion

$$\left\{i\in N:\sum_{j=1}^{\infty}a_{i,j}\|T_jf-f\|_{C\left(\overline{D}_\delta\right)}>\varepsilon\right\}\subset\left\{i\in N:\sum_{j=1}^{\infty}a_{i,j}\varepsilon_j>\frac{\varepsilon}{d_3}\right\}.$$

Hence and from the condition $A_{st}-\lim\limits_{n\to\infty}\varepsilon_n=0$ we get that $A_{st}-\lim\limits_{n\to\infty}\|T_nf-f\|_{C\left(\overline{D}_\delta\right)}=0$, i.e. the sequence T_nf-f A- statistically converges to zero in the space $A\left(\overline{D}\right)$.

The theorem is proved

Corollary 2. *Let $D=\{z:|z|<1\}$ be a unit circle, $g=\{g_k\}_{k=0}^{\infty}$ be a strictly monotone sequence of positive numbers, $c_k=\frac{g_k}{(1+k)^2}$, $k\in Z_+$ and $c(z)=\sum\limits_{k=0}^{\infty}c_k\left(\frac{z}{1+\delta_0}\right)^k$, $\delta_0>0$. Then in order the sequence of functions T_nf-f A-statistically converge to zero in $A\left(\overline{D}\right)$ for any function $f\in A_g^{(\delta_0)}(D)$ it is necessary and sufficient the A*

-statistical convergence to zero of the sequence f functions $T_n\left(z^\nu c^{(\nu)}(z)\right) - z^\nu c^{(\nu)}(z)$, $\nu = 0, 1, 2$ *in* $A\left(\overline{D}\right)$.

Indeed, if we take the sequence $b = \left\{\frac{1}{(1+k)^2}\right\}_{k=0}^{\infty}$, then from theorem 3 and the equalities

$$h_0(z) = \sum_{k=0}^{\infty} g_k \frac{z^k}{(1+\delta_0)^k} = z^2 c''(z) - 3zc'(z) + 2c(z);$$

$$h_1(z) = \sum_{k=0}^{\infty} \frac{g_k}{1+k} \frac{z^k}{(1+\delta_0)^k} = zc'(z) + c(z);$$

$$h_2(z) = \sum_{k=0}^{\infty} \frac{g_k}{(1+k)^2} \frac{z^k}{(1+\delta_0)^k} = c(z)$$

it follows Corollary 2.

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