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A PROBLEM OF OPTIMAL CONTROL OF COEFFICIENTS OF A PARABOLIC EQUATION WITH OPTIMIZATION ALONG THE BOUNDARY OF THE DOMAIN

Abstract

For a problem of optimal control of coefficients of a parabolic equation with optimization along the boundary of the domain, the issues of well-posedness of the problem statement were studied and a necessary optimality condition in the form of variational inequality was established.

1. Introduction

The optimal control problems for parabolic equations are met in optimization of different controlled processes of thermal physics, diffusion, filtration and others. In a number of practical optimal control problems the functions enter into the coefficients of parabolic equations. Such problems arise also by solving the coefficient inverse problems for parabolic equations considered at variational statements [1-3].

In [4-10] and others the optimal control problems for parabolic equations with controls in coefficients were studied. Such problems were not studied enough in the cases when the optimizations along the domain having the important practical value were studied.

In the present paper, we consider an optimal control problem for a linear parabolic equation with controls in its coefficients and with optimization along the boundary of the domain.

The well-posedness of the problem statement was studied, the Frechet differentiability of optimization was proved and necessary optimality condition in the form of variational inequalities was established.

2. Problem statement

Let $l, T > 0$ be the given numbers, $Q_T = \{(x,t) : 0 < x < l, 0 < t < T\}$. The denotation of functional spaces and their norms used in the paper correspond to ones accepted in [11, p.12]. Below, the positive constants independent of the evaluated values and admissible controls, are denoted by $M_j\,(j = 1, 2, ...)$.

Let the controlled process be described in Q_T by the following initial-boundary value problem for the linear parabolic equation:

$$u_t - (k(x,t)\,u_x)_x + q(x,t)\,u = f(x,t)\,, \quad (x,t) \in Q_T, \tag{1}$$

$$u|_{t=0} = \varphi(x), \quad 0 \le x \le l, \tag{2}$$

$$[-k(x,t)\, u_x + p_0(t)\, u]\,|_{x=0} = g_0(t),$$

$$[k(x,t)\, u_x + p_1(t)\, u]\,|_{x=l} = g_1(t), \quad 0 < t \le T, \tag{3}$$

where $f(x,t) \in L_2(Q_T)$, $\varphi(x) \in W_2^1(0,l)$, $g_0(t)$, $g_1(t) \in W_2^1(0,T)$, are the given functions, $k(x,t)$, $q(x,t)$, $p_0(t)$, $p_1(t)$ are the control functions, $\upsilon = \big(k(x,t), q(x,t), p_0(t), p_1(t)\big)$ is a control, $u = u(x,t) = u(x,t,\upsilon)$ is the solution to problem (1)-(3) the state of the process corresponding to the control υ.

Introduce the set of admissible controls

$$K = \{k(x,t) \in W_{00}^{1,1}(Q_T) : 0 < \vartheta \le k(x,t) \le \mu,$$

$$|k_x(x,t)| \le d_1, \ |k_t(x,t)| \le d_2 \quad a.e. \ on \ \ Q_T\},$$

$$Q = \{q(x,t) \in L_{00}(Q_T) : 0 \le q_0 \le q(x,t) \le q_1 \quad a.e. \ on \ \ Q_T\}, \tag{4}$$

$$P_i = \{p_i(t) \in W_{00}^1(Q_T) : 0 \le p_0^{(i)} \le p_i(t) \le p_1^{(i)},$$

$$|p_i'(t)| \le p_2^{(i)} \quad a.e. \ on \quad [0,T]\,(i=0,1),$$

where $\mu \ge \vartheta > 0$, $q_1 \ge q_0 \ge 0$, $p_1^{(i)} \ge p_0^{(i)} \ge 0$, d_1, d_2, $p_2^{(i)} > 0 \quad (i=0,1)$ are the given functions.

Set the following optimal control problem: on the solutions $u = u(x,t,\upsilon)$ of problem (1)-(3) corresponding to all admissible controls $\upsilon \in V$ minimize the functional

$$J(\upsilon) = \int_0^T \Big[\alpha_0\, |u(0,t,\upsilon) - z_0(t)|^2 + \alpha_1\, |u(l,t,\upsilon) - z_1(t)|^2\Big]\, dt +$$

$$+ \int_0^l |u(x,T,\upsilon) - z_2(x)|^2\, dx \tag{5}$$

where α_0, α_1, $\alpha_2 \ge 0$, $\alpha_0 + \alpha_1 + \alpha_2 > 0$ are the given numbers, $z_0(t)$, $z_1(t) \in W_2^1(0,T)$, $z_2(x) \in W_2^1(0,l)$ are the given functions. This problem below will be called problem (1)-(5).

Under the solution of boundary value problem (1)-(3) corresponding to the control $\upsilon \in V$, we'll understand the generalized solution from $V_2^{1,1/2}(Q_T)$ i.e. the function $u = u(x,t,\upsilon)$ from $V_2^{1,1/2}(Q_T)$ satisfying the identity

$$\iint_{Q_T} [-u\eta_t + k(x,t)\, u_x \eta_x + q(x,t)\, u\eta]\, dxdt +$$

$$+ \int_0^T p_0(t)\, u(0,t,\upsilon)\, \eta(0,t)\, dt + \int_0^T p_1(t)\, u(l,t,\upsilon)\, \eta(l,t)\, dt =$$

$$= \int_0^t \varphi(x)\,\eta(x,0)\,dx + \int_0^T g_0(t)\,\eta(0,t)\,dt + \int_0^T g_1(t)\,\eta(l,t)\,dt + \iint_{Q_T} f\eta dxdt \qquad (6)$$

for any function $\eta = \eta(x,t)$ from $W_2^{1,1}(Q_T)$ equal to zero for $t = T$.

We can show that under the made suppositions, boundary value problem (1)-(3) has a unique generalized solution from $V_2^{1,1/2}(Q_T)$ and this solution belongs to the space $W_2^{2,1}(Q_T)$, satisfies equation (2) for almost all $(x,t) \in Q_T$ and it is valid a priori estimation [11, pp. 197-213]

$$\max_{0\le t\le T} \|u(x,t,\upsilon)\|_{2,[0,l]} + \max_{0\le t\le T} \|u_x(x,t,\upsilon)\|_{2,[0,l]} + \|u_{xx}\|_{2,Q_T} + \|u_t\|_{2,Q_T} \le$$

$$\le M_1 \left[\|f\|_{2,Q_T} + \|\varphi\|_{2,[0,l]}^{(1)} + \|g_0\|_{2,[0,T]}^{(1)} + \|g_1\|_{2,[0,T]}^{(1)} \right]. \qquad (7)$$

3. Well-posedness of the problem statement

Introduce the space $E = W_r^{1,1}(Q_T) \times L_2(Q_T) \times W_2^1(0,T) \times W_2^1(0,T)$, where $r > 2$ is some number.

Theorem 1. *Let the conditions accepted for the statement of problem (1)-(5) be fulfilled. Then there exists at least one optimal control* $\upsilon_* = \big(k_*(x,t), q_*(x,t), p_{0*}(t), p_{1*}(t)\big) \in V$ *of problem (1)-(5). i.e.*

$$V_* = \{\upsilon_* \in V : J(\upsilon_*) = \inf\{J(\upsilon) : \upsilon \in V\}\} \ne \varnothing,$$

the set of optimal controls V_* *in problem (1)-(5) is weakly compact in* E, *and any minimizing sequence* $\{\upsilon^{(m)}\} = \Big\{\Big(k^{(m)}(x,t), q^{(m)}(x,t), p_0^{(m)}(t), p_1^{(m)}(t)\Big)\Big\} \subset V$ *of the functional* $J(\upsilon)$ *in* E *weakly converges to the set* V_*.

Proof. We can show that the set V is weakly compact in the space E. And in the proof we use the fact that the imbedding operator $W_r^{1,1}(Q_T)$ in $L_{00}(Q_T)$ is continuous for any finite $r > 2$ [11.p.78].

Show that the functional $J(\upsilon)$ in E is weakly continuous on the set V. Let $\upsilon = (k(x,t), q(x,t), p_0(t), p_1(t)) \in V$ be some element,

$$\Big\{\upsilon^{(n)} = \Big(k^{(n)}(x,t), q^{(n)}(x,t), p_0^{(n)}(t), p_1^{(n)}(t)\Big)\Big\} \subset V$$

be an arbitrary sequence weakly in E converging to the element υ, etc.

$$k^{(n)}(x,t) \to k(x,t) \quad weakly \quad in \quad W_r^{1,1}(Q_T), \qquad (8)$$

$$q^{(n)}(x,t) \to q(x,t) \quad weakly \quad in \quad L_2(Q_T), \qquad (9)$$

$$p_i^{(n)}(t) \to p_i(t) \quad (i = 0,1) \quad weakly \quad in \quad W_2^1(0,T). \qquad (10)$$

From the compactness of imbeddings $W_r^{1,1}(Q_T) \to L_\infty(Q_T)$, $W_2^1(0,T) \to L_\infty(0,T)$ [11,p.78] and from (8), (10) it follows that

$$k^{(n)}(x,t) \to k(x,t) \quad strongly \quad in \quad L_\infty(Q_T), \qquad (11)$$

$$p_i^{(n)}(t) \to p_i(t) \ \ (i=0,1) \quad strongly \quad in \quad L_\infty(Q_T). \tag{12}$$

From the unique solvability of problem (1)-(3) to each $\upsilon^{(n)} \in V$ we assign $u^{(n)}(x,t) = u\left(x,t,\upsilon^{(n)}\right)$, a unique solution of problem (1)-(4) for $\upsilon = \upsilon^{(n)}$, furthermore,

$$\left\|u^{(n)}\right\|_{2,Q_T}^{(2,1)} \leq M_2, \tag{13}$$

i.e. the sequence $\left\{u^{(n)}(x,t)\right\}$ is uniformly bounded in the norm of the space $W_2^{2,1}(Q_T)$. Then from the compactness of imbedding $W_2^{2,1}(Q_T) \to L_s(Q_T)$ for any finite $s \geq 1$ [12, p. 33] and compactness of mappings $u \to u|_{x=0}$, $u \to u|_{x=l}$, $u \to u|_{t=T}$ of the space $W_2^{2,1}(Q_T)$ in $L_2(0,T)$, $L_2(0,T)$, $L_2(0,l)$ respectively [11, p.78], [13,p.130] it follows that from the sequence $\left\{u^{(n)}(x,t)\right\}$ we can extract the subsequence $\left\{u^{(n_k)}(x,t)\right\}$ such that

$$\left\{u^{(n_k)}(x,t)\right\} \to u(x,t) \ \ weakly \ \ in \quad W_2^{2,1}(Q_T), \ strongly$$

$$in \ L_s(Q_T) \quad a.e. \ in \ Q_T \tag{14}$$

$$\left\{u^{(n_k)}(0,t)\right\} \to u(0,t), \ \left\{u^{(n_k)}(l,t)\right\} \to u(l,t) \quad strongly \ \ in \ \ L_2(0,T), \tag{15}$$

$$\left\{u^{(n_k)}(0,T)\right\} \to u(0,T) \quad strongly \ \ in \ \ L_2(0,T), \tag{16}$$

where $u(x,t)$ is some element from $W_2^{2,1}(Q_T)$.

Further, on the basis of relations (9), (14)-(16) we can show [8] that $u(x,t)$ is the solution of problem (1)-(3) corresponding to the control $\upsilon \in V$, i.e. $u(x,t) = u(x,t,\upsilon)$.

Thus, it is established that subject to relations (8)-(10), from the subsequence $\left\{u^{(n)}(x,t)\right\}$ we can select the subsequence $\left\{u^{(n_k)}(x,t)\right\}$, for which relations (14)-(16), where $u(x,t) = u(x,t,\upsilon)$ are valid. From the uniqueness of the solution of problem (1)-(3) for the fixed $\upsilon \in V$ it follows that relations (14)-(16) are valid also for all of the sequence $\left\{u^{(n)}(x,t)\right\}$, i.e.

$$u^{(n)}(0,t) \to u(0,t,\upsilon), \quad u^{(n)}(l,t) \to u(l,t,\upsilon) \quad strongly \ \ in \ \ L_2(0,T),$$

$$u^{(n)}(x,T) \to u(x,T,\upsilon) \quad strongly \ \ in \ \ L_2(0,l).$$

Then using these relations, we get $J\left(\upsilon^{(n)}\right) \to J(\upsilon)$ as $n \to \infty$. Thus, it is established that the functional $J(\upsilon)$ in E is weakly continuous on the set V. Then the validity of theorem 1 follows from [14, p. 49; p. 51]. Theorem 1 is proved.

4. Differentiability of the functional and necessary optimality condition.

For problem (1)-(5) introduce the conjugate state $\psi = \psi(x,t,\upsilon)$ as the solution of the problem

$$\psi_t + (k(x,t)\psi_x)_x - q(x,t)\psi = 0, \quad (x,t) \in Q_T, \tag{17}$$

$$\psi|_{t-T} = -2\alpha_2 [u(x,T,\upsilon) - z_2(x)], \quad 0 \le x \le l, \tag{18}$$

$$[-k(x,t)\psi_x + p_0(t)\psi]|_{x=0} = 2\alpha_0 [u(0,t,\upsilon) - z_0(t)],$$

$$[k(x,t)\psi_x + p_1(t)\psi]_{x=l} = 2\alpha_1 [u(l,t,\upsilon) - z_1(t)], \quad 0 \le t < T. \tag{19}$$

Under the generalized solution of boundary value problem (17)-(19) corresponding to the control $\upsilon \in V$ we'll understand the function $\psi = \psi(x,t,\upsilon)$ from $W_2^{1,1/2}(Q_T)$ satisfying the identity

$$\iint_{Q_T} [\psi\eta_t + k(x,t)\psi_x\eta_x + q(x,t)\psi\eta]\,dxdt+$$

$$+\int_0^T p_0(t)\psi(0,t,\upsilon)\eta(0,t)\,dt + \int_0^T p_1(t)u(l,t,\upsilon)\eta(l,t)\,dt =$$

$$= 2\alpha_0 \int_0^T [u(0,t,\upsilon) - z_0(t)]\eta(0,t)\,dt+$$

$$+2\alpha_1 \int_0^T [u(l,t,\upsilon) - z_1(t)]\eta(l,t)\,dt + 2\alpha_2 \int_0^t [u(x,T,\upsilon) - z_2(t)]\eta(x,T)\,dx \tag{20}$$

for any function $\eta = \eta(x,t)$ from $W_2^{1,1}(Q_T)$ equal to zero for $t = 0$.

We can show that under the made suppositions the boundary value problem (17)-(19) has a unique solution from $V_2^{1,1/2}(Q_T)$ for each fixed $\upsilon \in V$, and the following estimation is valid [11, p. 197-213]

$$|\psi|_{Q_T} = \max_{0\le t\le T} \|\psi(x,t,\upsilon)\|_{2,[0,T]} + \|\psi_x\|_{2,Q_T} \le$$

$$\le M_3 \Big[\alpha_0 \|u(0,t,\upsilon) - z_0(t)\|_{2,[0,T]} + \alpha_1 \|u(l,t,\upsilon) - z_1(t)\|_{2,[0,T]} +$$

$$+\alpha_2 \|u(x,T,\upsilon) - z_2(t)\|_{2,[0,l]}\Big].$$

Then taking into account estimations (7) and continuity of mappings $u \to u|_{x=0}$, $u \to u|_{x=l}$, $u \to u|_{l=T}$ of the space $W_2^{2,1}(Q_T)$ in $L_2(0,T)$, $L_2(0,T)$, $L_2(0,l)$ respectively, [11, p. 78], [13, p. 130] we get the estimations

$$|\psi|_{Q_T} \le M_4 \Big[\|f\|_{2,Q_T} + \|\varphi\|_{2,[0,l]}^{(1)} + \alpha_0 \|z_0\|_{2,[0,T]} + \alpha_1 \|z_1\|_{2,[0,T]} + \alpha_2 \|z_2\|_{2,[0,l]}\Big]. \tag{21}$$

Theorem 2. *Let the conditions of theorem be fulfilled. Then functional (1) is continuously differentiable by Frechet on V and its differential at the point $\upsilon \in V$ with the increment $\Delta\upsilon = (\Delta k, \Delta q, \Delta p_0, \Delta p_1) \in B$, $\upsilon + \Delta\upsilon \in V$ is determined by the expression*

$$\left\langle J'(\upsilon), \Delta\upsilon \right\rangle_B = \iint\limits_{Q_T} (u_x \psi_x \Delta x + u\psi \Delta q)\, dxdt + \int\limits_0^T u(0,t,\upsilon)\, \psi(0,t,\upsilon)\, \Delta p_0(t)\, dt+$$

$$+\int\limits_0^T u(l,t,\upsilon)\, \psi(l,t,\upsilon)\, \Delta p_1(t)\, dt. \qquad (22)$$

Proof. Let $\upsilon, \upsilon + \Delta\upsilon \in V$ be arbitrary controls and $\Delta u = \Delta u(x,t) =$ $= u(x,t,\upsilon+\Delta\upsilon) - u(x,t,\upsilon)$, $u = u(x,t) = u(x,t,\upsilon)$. From conditions (1)-(3) it follows that Δu is the solution from $W_2^{2,1}(Q_T)$ of the problem

$$\Delta u_t - ((k+\Delta k)\Delta u_x)_x + (q+\Delta q)\Delta u = (\Delta k u_x)_x - \Delta q u, \quad (x,t)\in Q_T, \qquad (23)$$

$$\Delta u|_{t=0} = 0, \quad 0 \le x \le l, \qquad (24)$$

$$[-(k+\Delta k)\Delta u_x + (p_0+\Delta p_0)\Delta u]\,|_{x=0} = [\Delta k u_x - \Delta p_0 u]\,|_{x=0},$$

$$[(k+\Delta k)\Delta u_x + (p_1+\Delta p_1)\Delta u]\,|_{x=l} =$$

$$= [-\Delta k u_x - \Delta p_1 u]\,|_{x=l}, \quad 0 < t \le T. \qquad (25)$$

For the solution of problem (23)-(25) the following estimation [11, p. 197-213] is valid

$$|\Delta u|_{Q_T} \le M_5 \Big[\|\Delta k u_x\|_{2,Q_T} + \|\Delta q u\|_{2,Q_T} + \|\Delta k u_x|_{x=0}\|_{2,[0,T]} +$$

$$+ \|\Delta k u_x|_{x=l}\|_{2,[0,T]} + \|\Delta p_0 u|_{x=0}\|_{2,[0,T]} + \|\Delta p_1 u|_{x=l}\|_{2,[0,T]} \Big]. \qquad (26)$$

Obviously, the following inequalities are valid

$$\|\Delta k u_x\|_{2,Q_T} \le \|\Delta k\|_{00,Q_T} \cdot \|u_x\|_{2,Q_T},$$

$$\|\Delta q u_x\|_{2,Q_T} \le \|\Delta q\|_{00,Q_T} \cdot \|u\|_{2,Q_T}. \qquad (27)$$

Furthermore, using the boundedness of the imbedding $W_2^{2,1}(Q_T) \to L_2(0,T)$ [11, p. 98], we get the inequalities

$$\|\Delta k u_x|_{x=0}\|_{2,[0,T]} \le \|\Delta k\|_{00,Q_T} \cdot \|u_x|_{x=0}\|_{2,[0,T]} \le M_6 \|\Delta k\|_{00,Q_T} \|u\|_{2,Q_T}^{(2,1)},$$

$$\|\Delta k u_x|_{x=l}\|_{2,[0,T]} \le \|\Delta k\|_{00,Q_T} \cdot \|u_x|_{x=l}\|_{2,[0,T]} \le M_7 \|\Delta k\|_{00,Q_T} \|u\|_{2,Q_T}^{(2,1)},$$

$$\|\Delta p_0 u_x|_{x=0}\|_{2,[0,T]} \le M_8 \|\Delta p_0\|_{00,[0,T]} \|u\|_{2,Q_T}^{(2,1)},$$

$$\|\Delta p_1 u_x|_{x=l}\|_{2,[0,T]} \le M_9 \|\Delta p_1\|_{00,[0,T]} \|u\|_{2,Q_T}^{(2,1)}. \qquad (28)$$

Taking into account in (26) the inequalities (27), (28) and (7), we get the estimation

$$|\Delta u|_{Q_T} \leq M_{10}\left[\|f\|_{2,Q_T} + \|\varphi\|^{(1)}_{2,[0,l]} + \|g_0\|^{(1)}_{2,[0,T]} + \|g_1\|^{(1)}_{2,[0,T]}\right] \cdot \|\Delta \upsilon\|_B . \qquad (29)$$

The increment of the functional (5) has the form

$$\Delta J(\upsilon) = J(\upsilon + \Delta\upsilon) - J(\upsilon) =$$

$$= \int_0^T \{2\alpha_0 [u(0,t,\upsilon) - z_0(t)]\Delta u(0,t) + 2\alpha_1 [u(l,t,\upsilon) - z_1(t)]\Delta u(l,t)\}\, dt +$$

$$+2\alpha_2 \int_0^t [u(x,T,\upsilon) - z_2(x)]\Delta u(x,T)\, dx + \alpha_0 \|\Delta u(0,t)\|^2_{2,[0,T]} +$$

$$+\alpha_1 \|\Delta u(l,t)\|^2_{2,[0,T]} + \alpha_2 \|\Delta u(x,T)\|^2_{2,[0,l]} . \qquad (30)$$

Using the integral identity for the generalized solution of boundary value problem (23)-(25) and identity (20), we can show that it is valid the equality

$$\int_0^T \{2\alpha_0 [u(0,t,\upsilon) - z_0(t)]\Delta u(0,t) + 2\alpha_1 [u(l,t,\upsilon) - z_1(t)]\Delta u(l,t)\}\, dt +$$

$$+2\alpha_2 \int_0^t [u(x,T,\upsilon) - z_2(x)]\Delta u(x,T)\, dx =$$

$$= \iint_{Q_T} [(u_x + \Delta u_x)\psi_x \Delta k + (u + \Delta u)\psi\Delta q]\, dxdt +$$

$$+ \int_0^T \{[u(0,t,\upsilon) + \Delta u(0,t)]\psi(0,t,\upsilon)\Delta p_0(t) +$$

$$+ [u(l,t,\upsilon) + \Delta u(l,t)]\psi(l,t,\upsilon)\Delta p_1(t)\}\, dt.$$

Taking this equality into account in (30), we have

$$\Delta J(\upsilon) = \iint_{Q_T} (u_x \psi_x \Delta k + u\psi\Delta q)\, dxdt +$$

$$+ \int_0^T [u(0,t,\upsilon)\psi(0,t,\upsilon)\Delta p_0(t) + u(l,t,\upsilon)\psi(l,t,\upsilon)\Delta p_1(t)]\, dt + R, \qquad (31)$$

where

$$R = \alpha_0 \|\Delta u(0,t)\|^2_{2,[0,T]} + \alpha_1 \|\Delta u(l,t)\|^2_{2,[0,T]} + \alpha_2 \|\Delta u(x,T)\|^2_{2,[0,l]} +$$

$$+\iint\limits_{Q_T}(\Delta u_x\psi_x\Delta k+\Delta u\psi\Delta q)\,dxdt+$$

$$+\int\limits_0^T[\Delta u(0,t)\,\psi(0,t,\upsilon)\,\Delta p_0(t)+\Delta u(l,t)\,\psi(l,t,\upsilon)\,\Delta p_1(t)]\,dt. \qquad (32)$$

Arguing as in deriving estimation (29), we have

$$\left|\iint\limits_{Q_T}(\Delta u_x\psi_x\Delta k+\Delta u\psi\Delta q)\,dxdt+\right.$$

$$\left.+\int\limits_0^T[\Delta u(0,t)\,\psi(0,t,\upsilon)\,\Delta p_0(t)+\Delta u(l,t)\,\psi(l,t,\upsilon)\,\Delta p_1(t)]\,dt\right|\leq$$

$$\leq M_{11}\,|\Delta u|_{Q_T}\,|\psi|_{Q_T}\,|\Delta\upsilon|_B\,.$$

Hence and from (21), (29) it follows that for the remainder term R defined by the equality (32), it is valid the estimation

$$|R|\leq M_{12}\,\|\Delta\upsilon\|_B^2\,.$$

Taking into account this estimation in (31) we conclude that functional (5) is differentiable by Frechet on V and its differential is determined by the expression (22). It is easy to show that the mapping $\upsilon\to J'(\upsilon)$ defined by equality (22) continuously acts from V to B^*, where B^* is a space conjugate to B [8]. Theorem 2 is proved.

Theorem 3. *Let the conditions of theorem 2 be fulfilled. Then for the optimality of the control* $\upsilon_*=(k_*(x,t),q_*(x,t),p_{0*}(t),p_{1*}(t))\in V$ *in problem (1)-(5) it is necessary that the inequality*

$$\iint\limits_{Q_T}\{u_{*x}\psi_{*x}[k(x,t)-k_*(x,t)]+u_*\psi_*[q(x,t)-q_*(x,t)]\}\,dxdt+$$

$$+\int\limits_0^T\{u_*(0,t)\,\psi_*(0,t)\,[p_0(t)-p_{0*}(t)]+$$

$$+u_*(l,t)\,\psi_*(l,t)\,[p_1(t)-p_{1*}(t)]\}\,dt\geq 0 \qquad (33)$$

be fulfilled for any $\upsilon=(k(x,t),q(x,t),p_0(t),p_1(t))\in V$*, where* $u_*(x,t)=u(x,t,\upsilon_*)$*,* $\psi_*(x,t)=\psi(x,t,\upsilon_*)$ *is the solution of problem (1)-(3) and (17)-(19) for* $\upsilon=\upsilon_*$*.*

Proof. The set V defined by relation (4) is convex in B. Furthermore, according to theorem 2, the functional $J(\upsilon)$ is continuously differentiable by Frechet on V. Then by virtue of theorem 5 from [14, p. 28] on the element $\upsilon_*\in V$ the inequality

$\langle J'(\upsilon_*), \upsilon - \upsilon_* \rangle_B \geq 0$ should be fulfilled for all $\upsilon \in V$. Hence and from (22) the validity of inequality (33) follows. Theorem 3 is proved.

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