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THE ACTION OF TEMPERATURE FIELD ON CRACK RETARDATION IN STRIP (BEAM) BENDING

Abstract

Some problems on the action of temperature field on crack retardation in a strip (beam) when a strip bends in its plane by the given system of external loads (constant bending moments, uniformly distributed by pressure and others), are considered.

Problem statement. The strips (beams) are widely used in engineering and construction. They are subjected to the action of power load. Investigation of strips (beam) failure issues is of great importance in practice.

Let's consider a homogeneous isotropic strip (beam). Denote by $2c$ and $2h$ the length and the width of the strip, respectively. The cartesian coordinates x,y in the median surface of the beam are a symmetry plane. Accept that in the plane xOy there is a crack arranged along the axis x for $a \leq x \leq b$, and a, b are the abscissas of the crack's ends.

Let the external loads (bending moments uniformly distributed along the strip's length, pressure or concentrated forces) arranged in the strip's median plane act on such a strip (beam). The strip's bounds parallel to the plane xOy are assumed to be free from external loads. The crack faces are assumed to be free from external loads. For retarding the crack, by means of heating of area S by thermal source to temperature T_0 the contractive stresses zone is built-up on its propagation way.

The problem under consideration determines the stress-strain state of the strip (beam), and also finds the ultimate external load at attaining of which the crack will grow across the strip's section.

Accept the following assumptions:

1) All thermoelastic features of the strip's (beam's) material are temperature-independent;

2) The strip's material is a homogeneous and isotropic body.

It is assumed that at moment $t = 0$, an arbitrary area S on the crack growth way in the strip (beam) instantly heats up to constant temperature $T = T_0$.

The boundary conditions on the crack faces are of the form

$$\sigma_y - i\tau_{xy} = 0 \quad for \quad y = 0, \quad a \leq x \leq b. \tag{1}$$

The stress state of a cracked beam is sought in the form

$$\sigma_x = \sigma_x^0 + \sigma_x^1, \quad \sigma_y = \sigma_y^0 + \sigma_y^1, \quad \tau_{xy} = \tau_{xy}^0 + \tau_{xy}^1, \tag{2}$$

where the first addends σ_x^0, σ_y^0, τ_{xy}^0 is the solution of an thermo-elasticity problem for a continuous strip.

We have earlier found the solution of the thermo-elasticity problem for a continuous strip in [1] (see formula (5)).

Allowing for (2), boundary condition (1) on the crack faces takes the following form

$$\sigma_y^1 - \tau_{xy}^1 = f_1(x) - if_2(x) \quad for \quad y = 0, \quad a \le x \le b, \tag{3}$$

where $f_1(x) = -\sigma_y^0(x, 0)$; $f_2(x) = -\tau_{xy}^0(x, 0)$.

We define the stress-strain state in the crack's vicinity approximately in the sense of that [2] that the boundary conditions of the problem on the crack's contour (condition (3)) will be satisfied, and require that at significant distance from the crack the stress state in the beam coincide with the stress state defined by the functions:

$$\lim_{|z|\to\infty} \Phi(z) = \Phi_0(z) = A_0 z^3 + A_1 z^2 + A_2 z + A_3; \tag{4}$$

$$\lim_{|z|\to\infty} \Omega(z) = \Omega_0(z) = B_0 z^3 + B_1 z^2 + B_2 z + B_3.$$

The functions (4) subject to the values of the coefficients A_j and B_j $(j = 0, 1, 2, 3)$ define the stress state in a crackless strip (beam).

Assuming in formulae (4)

$$A_0 = 0; \quad A_1 = 0; \quad A_2 = \frac{M}{4I}; \quad A_3 = 0; \tag{5}$$

$$B_0 = 0; \quad B_1 = 0; \quad B_2 = \frac{3M}{4I}; \quad B_3 = 0,$$

where I is the inertia moment of cross-section area, one can be convinced that in this case the functions $\Phi_0(z)$ and $\Omega_0(z)$ give the solution of a problem on pure bending of an infinite crackless strip (beam) by the moments M.

For

$$A_0 = \frac{q}{24I}; \quad A_1 = 0; \quad A_2 = \frac{q}{8I}\left(L^2 + \frac{2c^2}{5}\right); \quad A_3 = -\frac{qc^3}{12I}; \tag{6}$$

$$B_0 = \frac{7q}{24I}; \quad B_1 = 0; \quad B_2 = \frac{q}{8I}\left(3L^2 - \frac{11c^2}{5}\right); \quad B_3 = \frac{qc^3}{12I}.$$

The functions (4) give the solution of a problem on bending of a crackless beam of length $2L$, when the beam is loaded by a uniform pressure of intensity q. In this case, it is accepted that the beam is freely arranged in two supports, and the support reactions are determined as tangential forces applied to the crack's end faces.

If

$$A_0 = 0; \quad A_1 = -\frac{iQ}{8I}; \quad A_2 = -\frac{Q(2L - d)}{4I}; \quad A_3 = 0; \tag{7}$$

$$B_0 = 0; \quad B_1 = \frac{5iQ}{8I}; \quad B_2 = -\frac{3Q(2L - d)}{4I}; \quad B_3 = \frac{iQc^2}{2I},$$

then functions (4) give the solution of a problem on bending of rigidly built-in crackless cantilever beam under constant lateral force Q applied on its free end.

Following N.I. Muskheleshvili [2], we reduce boundary value problem (3) to the problem on linear conjugation of boundary values of complex potentials $\Phi(z)$ and $\Omega(z)$

$$[\Phi(t)+\Omega(t)]^{+}+[\Phi(t)+\Omega(t)]^{-}=2f(t) \tag{8}$$

$$[\Phi(t)-\Omega(t)]^{+}-[\Phi(t)-\Omega(t)]^{-}=0,$$

where $a\le t\le b$; t is the affix of the crack's contour points.

By solving this problem [2] and taking into account the behavior of functions $\Phi(z)$ and $\Omega(z)$ at infinity, we get

$$\Phi(z)=\frac{1}{2\pi i\sqrt{(z-a)(z-b)}}\int_a^b\frac{\sqrt{(t-a)(t-b)}}{t-z}f(t)\,dt+$$

$$+\frac{P_n(z)}{\sqrt{(z-a)(z-b)}}+\frac{1}{2}\left[\Phi_0(z)-\Omega_0(z)\right]; \tag{9}$$

$$\Omega(z)=\frac{1}{2\pi i\sqrt{(z-a)(z-b)}}\int_a^b\frac{\sqrt{(t-a)(t-b)}}{t-z}f(t)\,dt+$$

$$+\frac{P_n(z)}{\sqrt{(z-a)(z-b)}}-\frac{1}{2}\left[\Phi_0(z)-\Omega_0(z)\right],$$

where the functions $\Phi_0(z)$ and $\Omega_0(z)$ are determined by relations (4), and the polynomial $P_n(z)$ is of the form

$$P_n(z)=D_nz^n+D_{n-1}z^{n-1}+...+D_0; \tag{10}$$

Here, as $z\to\infty$ $\quad\sqrt{(z-a)(z-b)}\to z+O(1/z)$.

The root under the integral sign is the value of the branch of the appropriate analytic function separated by the condition on the upper face of the crack.

The degree of polynomial (10) and its coefficients $D_0,\ D_1,...,D_n$ are determined from the behavior conditions of functions $\Phi(z)$ and $\Omega(z)$ in the vicinity of the point $|z|=\infty$.

The complex potentials $\Phi(z)$ and $\Omega(z)$ are analytic in the domain exterior to the crack and for large values of $|z|$ have the form

$$\Phi(z)=\Phi_0(z)+O\left(\frac{1}{z}\right);\quad \Omega(z)=\Omega_0(z)+O\left(\frac{1}{z}\right). \tag{11}$$

Thus, for determining the coefficients $D_0,\ D_1,...,D_n$ it is necessary to expand the function $\Phi(z)$ represented in (9), in series in degree of z in the vicinity of the point $|z|=\infty$ and compare this expansion with expression (4).

Taking into account the previous relations and conducting necessary calculations, we can define the coefficient of polynomial (10) in the form:

1) at pure bending of a beam with crack

$$D_0 = -\frac{M}{16I}(b-a)^2;\ D_1 = -\frac{M}{4I}(b+a);$$

$$D_2 = \frac{M}{2I}; D_n = 0 \quad for \quad n \geq 3$$

2) at bending a strip with a crack under uniformly distributed load

$$D_0 = -\frac{q}{32I}(b-a)^2 L^2 - \frac{3}{2}h^2 + \frac{1}{24}\left(5a^2 + 6ab + 5b^2\right);$$

$$D_1 = -\frac{q}{96I}(a+b)\left[(b-a)^2 + 12\left(L^2 - \frac{2}{5}h^2\right)\right];$$

$$D_2 = -\frac{q}{48I}\left[(b-a)^2 - 12\left(L^2 - \frac{2}{5}h^2\right)\right];$$

$$D_3 = -\frac{q}{12I}(b+a);\ D_4 = \frac{q}{6I};\quad D_n = 0 \quad for \quad n \geq 5.$$

3) at bending of a cantilever beam with a crack

$$D_0 = -\frac{iQ}{64I}(b+a)\left[(b-a)^2 - 8h^2\right] + \frac{Q}{16I}(2L-2)(b-a)^2;$$

$$D_1 = -\frac{iQ}{32I}\left[(b-a)^2 + 8h^2 + 8i(b+a)(2L-d)\right];$$

$$D_2 = -\frac{Q}{8I}\left[4(2L-d) + (b+a)i\right];\ D_3 = \frac{iQ}{4I};$$

$$D_n = 0 \quad for \quad n \geq 4.$$

Let at each of the considered cases, the crack be entirely arranged in the tensile stresses area. In failure mechanics, definition of behavior of stresses near the cracks ends described by the stress coefficients, is of great interest.

1) at pure bending of a strip with a non-centrally arranged crack, the complex potentials have the form

$$\Phi(z) = \frac{M}{16I}\frac{8z^2 - 4(a+b) - (b-a)^2}{\sqrt{(z-a)(z-b)}} - \frac{M}{4I}z + F(z); \tag{12}$$

$$F(z) = \frac{1}{2\pi i}\frac{1}{\sqrt{(z-a)(z-b)}}\int_a^b \frac{\sqrt{(t-a)(t-b)}}{t-z}f(t)\,dt;$$

$$\Omega(z) = \Phi(z) + \frac{M}{2I}z;\quad I = \frac{4hc^3}{3}$$

2) at bending of a strip with a non-centrally arranged crack under the action of uniformly distributed load q, the complex potentials have the form

$$\Phi(z) = F(z) + f_0(z) - \frac{q}{8I}\left[z^3 + \left(L^2 - \frac{7}{5}c^2\right)z + \frac{2}{3}c^3\right]; \tag{13}$$

$$\Omega(z) = F(z) + f_0(z) + \frac{q}{8I}\left[z^3 + \left(L^2 - \frac{7}{5}c^2\right)z + \frac{2}{3}c^3\right];$$

$$f_0(z) = \frac{q}{96I\sqrt{(z-a)(z-b)}}\left\{16z^4 - 8(a+b)z^3 + \right.$$

$$+2\left[12\left(L^2 - \frac{2}{5}c^2\right) - (b-a)^2\right]z^2 - (a+b)\left[12\left(L^2 - \frac{2}{5}c^2\right) + (b-a)^2\right]z -$$

$$\left. - \frac{(b-a)^2}{8}\left[24\left(L^2 - \frac{2}{5}c^2\right) + 5a^2 + 6ab + 5b^2\right]\right\}$$

3) at bending of a contiliver beam with a non-centrally arranged crack under the action of lateral force Q, the complex potentials have the form

$$\Phi(z) = F(z) + f_0(z) + \frac{iQ}{64I\sqrt{(z-a)(z-b)}} \times$$

$$\times\left\{16z^3 + \left[-8(a+b) + 32i(2L-d)\right]z^2 + \right.$$

$$+\left[-16i(a+b)(2L-d) - 16c^2 - 2(b-a)^2\right]z - (a+b)\times$$

$$\left.\times\left[(b-a)^2 - 8c^2\right] - 4i(2L-d)(b-a)^2\right\} - \frac{iQ}{8I}\left[3z^2 + 2i(2L-d)z - 2c^2\right];$$

$$\Omega(z) = \Phi(z) + \frac{iQ}{8I}\left[3z^2 + 2i(2L-d)z - 2c^2\right].$$

Using formulas (12)-(14) and the relation

$$K_I - iK_{II} = 2\lim_{z\to l}\left[\sqrt{2\pi|z-l|}\Phi(z)\right],$$

one can find the stress intensity coefficients for the vicinity of the cracks tips

For example, for a pure bending of the strip we have:

at the point $x = a$

$$K_I = \frac{\sqrt{2}}{\sqrt{\pi(b-a)}}\int_a^b f(x)\sqrt{\frac{b-x}{x-a}}dx + \frac{3\sqrt{\pi}}{8\sqrt{2}}M\frac{3a^2 - b^2 - 2ab}{c^3\sqrt{b-a}} \quad (15)$$

$$K_{II} = 0$$

at the point $x = b$

$$K_I = \frac{\sqrt{2}}{\sqrt{\pi(b-a)}}\int_a^b f(x)\sqrt{\frac{x-a}{b-x}}dx + \frac{3\sqrt{\pi}}{8\sqrt{2}}M\frac{3b^2 - a^2 - 2ab}{c^3\sqrt{b-a}}$$

$$K_{II} = 0.$$

The stress intensity coefficients for other cases are found in the same way.

Knowing the stress intensity coefficients for any time, by means of the generalized criterion of brittle failure [3,4] we analyze the crack's growth in the strip.

For calculating the critical load and the deviation angle θ_* we have [4] the following equations

$$\theta_* = 2arctg\left[\frac{1-\sqrt{1+8\lambda^2}}{4\lambda}\right], \quad \left(\lambda = \frac{K_I}{K_{II}}\right) \tag{16}$$

$$4\sqrt{2}K_I\lambda^3\frac{1+3\sqrt{1+8\lambda^2}}{\left(12\lambda^2+1-\sqrt{1+8\lambda^2}\right)^{3/2}} = K_c\left(T\right). \tag{17}$$

Equations (16)-(17) allow to study the action of temperature induced by thermal source on crack propagation by numerical analysis.

In the relations of the brittle failure criterion, the characteristics of fracture toughness depends of temperature T. Knowing the dependence of fracture toughness of the material on temperature is necessary for complete analysis of the construction's strength. As the investigations results show [5], at practical range of temperature change the dependence of K_c on temperature outside of cold-brittleness interval may be with sufficient accuracy approximated by a second degree polynomial from the temperature

$$K_c = A_0 + A_1T + A_2T^2, \tag{18}$$

where A_0 is a constant of K_{c0} at temperature $T = 0$, A_1, A_2 are some empiric constants.

Substituting in condition (17) the found stress intensity coefficients and instead of the quantity $K_c\left(T\right)$ the relation (18), in the implicit form we get the dependence of the crack's length on the applied load, time, geometrical and physical parameters of the problem.

Analyzing numerically the dependence of the stress intensity coefficients, we can define the action of thermal source action, geometrical parameters of elevated temperatures area, time on the value of critical stress intensity coefficients. Based on the calculation results, we can make some conclusions. As is known a crack is stable if the tensile stress σ_0 necessary for holding it in moving-equilibrium state increases due to increase of the crack's length. The condition of stable growth of a crack has the form:

$$\frac{\partial}{\partial l}\left[-\frac{1-\nu}{2\mu}\left(K_I^2+K_{II}^2\right)+\frac{K_c^2}{E}\right] > 0.$$

The investigation shows that the crack's growth occurs stably (the quantity K_I, decreases as it approaches to the elevated temperatures area (i.e. to thermal source). As the crack is arranged at the compression area, the tensile stresses are partially compensated with thermoelastic stresses (fields), and to a certain extent this reduces to "strengthening" of the plate.

At great distance of the crack tip from the domain S (of thermal source), the decrease of stress intensity at the crack tip is small. As the crack's tip approaches to the thermal source, because of non-uniform heating of the growing crack faces, the break trajectory changes its direction. The crack propagation initiation originated at angle $14^0 - 20^0$ to the plane's symmetry axis subject to geometrical parameters of elevated temperatures zone (domain S).

As the crack's tip approaches to the domain S, the stress intensity decrease becames more weighty, and attains the great value when the crack directly is found in elevated temperatures area.

The greater the stress intensity decrease, the higher temperature of the domain S (heated area).

In the considered case, the contractive stresses of thermoelastic fields and change of viscosity of the material fracture is the cause of the crack retardation. As is known, the viscosity of a great majority of metals increase monotonically according to temperature increase.

As numerical analysis shows, the more efficient way of crack retardation by means of thermal source is the change of stress field symmetry near the crack tip. This way is based on the fact that intrusion of the crack into the area of action of thermo-elastic stresses causes stress redistribution at the crack's end and near the thermal source. And as a result, the tensile stresses direction changes and the crack is compelled to return to the side of thermal source. This reduces to decrease of failure rate, and finally to short-term or full shut-down of crack propagation. This result is of great practical value since it admits to struggle actively against dangerous through cracks of operational or technological origin, attains full retardation of such cracks by creating elevated temperatures area on the crack's tip by means of heat source.

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Received October 24, 2013; Revised December 28, 2013.