**MECHANICS**

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# ON A METHOD FOR WEAR REDUCTION OF A FRICTION PAIR HUB

**Abstract**

*Theoretical analysis on definition of the external contour points of a friction pair hub providing the absence of initial wear taking place to the end of the running-in period is conducted on the base of the model of a rough friction surface.*

One of the most responsible units of machines that determine reliability and durability of operation of transport systems are friction (kinematic) pairs that are included as components of many transport machines. It is known that it is impossible to prevent entirely the friction pair elements wear in the operation process. In this connection, for increasing service life of a friction pair, different measures for lowering the wear of friction pair components should be used. In the process of friction pair operation, at repeatedly reciprocating motion of a plunger the surface layer of the hub and plunger undergo maximum stress. In this connection, the failure (wear) usually begins from the surface. Thus, it is very important to impart higher operational properties to the surface layer of the hub and plunger. One of the effective measures for increasing wear-resistance of friction pair hub by decreasing initial wear is to determine such function of displacements of external contour points of the hub at which the contact pressure of a friction pair is as one that is shaped at conditions of friction node operation to the end of running-in period.

Thus, by changing the function of displacement of external contour points of the hub, one can lower the wear, at the expense of its degradation at running – in period. At present, there is no mathematical model for the hub-plunger pair that admits to calculate optimal displacements at given conditions of plunger's motion. The goal of the paper is to construct such relations.

**Problem Statement.** Let's consider stress-strain state of the hub of a friction pair at the action of loads normal and tangential to internal contour. Suppose that the internal contour of the hub and the external contour of the plunger are close to circular one. It is assumed that the hub of a friction pair on the external contour has some displacements. The displacement function of the external contour points are not known beforehand and should be determined in the process of problem solution from the additional condition.

It is accepted that the plane deformation conditions are fulfilled. Refer the hub to the polar system of coordinates $r\theta$ having chosen the origin at the center of concentric circles $L_0$ and $L$ with radii $R_0$ and $R$, respectively.

Represent the boundary of the internal contour $L_0'$ in the form

$$\rho = R_0 + \varepsilon H(\theta), \quad H(\theta) = \sum_{k=0}^{\infty} \left( a_k^0 \cos k\theta + b_k^0 \sin k\theta \right), \tag{1}$$

where $\varepsilon = R_{\max}/R_0$ is a small parameter; $R_{\max}$ is the greatest height of unevenness of the hub's interior surface.

In the operation of a "hub-plunger" friction pair there occurs force interaction between the contacting surfaces of a hub and plunger, there arise friction forces reducing to wear of conjunction materials. It is accepted that wear of friction pair components is of abrasive character. It is required to define the displacement function of the external contour points of the friction pair hub at which the contact pressure of the friction pair was as one that is shaped at conditions of operations to the end of running-in period.

**Solution method.** For the solution of the stated problem at first a wearressistance problem on pressing the plunger into the hub's surface. On this stage of the problem solution we formally assume that the displacement function of the external contour points of the hub is known and we can represent it in the form of expansion in Fourier series

$$g(\theta) = a_0 + \sum_{k=1}^{\infty} (a_k \cos k\theta + b_k \sin k\theta). \tag{2}$$

The condition connecting the displacements of the hub and plunger on the contact surface (the main contact equation) is written [1] in the following way

$$\upsilon_1 + \upsilon_2 = \delta(\theta), \quad (\theta_1 \leq \theta \leq \theta_2). \tag{3}$$

Here $\delta(\theta)$ is the sag of the hub and plunger's surface points defined by the form of the internal surface of the hub and plunger, and also by the quantity of compressing force $P$, $\theta_2 - \theta_1$ is the quantity of the contact angle (area).

In the contact zone, in addition to normal pressure, there is tangential stress connected with contact pressure $p(\theta, t)$ by Amontone-Coulomb law.

The tangential forces (friction forces) $\tau_{r\theta}(\theta, t)$ promote heat release in the contact area at which the amount of heat per a unit of time is proportional to friction power, and the amount of heat released at the point of contact zone with the coordinate $\theta$ will be determined by the formula

$$Q(\theta, t) = V\tau_{r\theta}(\theta, t) = Vfp(\theta, t),$$

where $V$ is the mean velocity on the period of plungers motion with respect to the hub.

For displacements of the hub and plunger we have

$$\upsilon_1 = \upsilon_{1y} + \upsilon_{1u} + \upsilon_1; \quad \upsilon_2 = \upsilon_{2e} + \upsilon_{2w} + \upsilon_{2m}.$$

Here $\upsilon_{1y}$ are thermo-elastic displacements of the contact surface points of the hub; $\upsilon_{1u}$ are displacements caused by the wear of hub's surface a $\upsilon_{1m}$ are displacements caused by removal of micro bulges of the hub surface; $\upsilon_{2e}+\upsilon_{2w}+\upsilon_{2m}$ the same for the plunger.

For determining $\upsilon_{1e}$, $\upsilon_{1w}$ and $\upsilon_{2e}$, $\upsilon_{2w}$ the thermo-elasticity problems are solved for the hub and plunger, respectively.

For the hub it holds

$$\Delta T = 0$$

$$for \quad r=\rho(\theta) \;\; A_{T1}\lambda\frac{\partial T}{\partial r}-A_{T2}\alpha_1(T-T_c)=-Q_*$$

$$Q_*=\begin{cases} Q_b & on\ the\ \ contac\ area \\ 0 & outside\ of \quad the\ contact\ \ area\end{cases}$$

$$for \quad r=R \quad \lambda\frac{\partial T}{\partial r}+\alpha_2(T-T_c)=0;$$

$$for \quad r=\rho(\theta) \quad \sigma_n=-p(\theta,t); \quad \tau_{nt}=-fp(\theta,t) \quad on\ the\ contact\ \ area$$

$$\sigma_n=0; \quad \tau_{nt}=0 \quad outside\ \ of \quad the\ contact\ \ area$$

$$for \quad r=R \quad u+iv=g(\theta).$$

Here $\lambda$ is the hub's heatconcluctivity coefficient; $\Delta$ is a Laplace operator; $\alpha_1$ is the heatconcluctivity coefficient from the internal surface of the hub; $\alpha_2$ is the heat release coefficient of the external surface of the hub with the external medium of temperature $T_c$; $n,t$ is the normal and tangential to the internal contour of the hub; $A_{T_1}$ is the heat-absorbing surface; $A_{T_2}$ is the cooling surface; $Q_*$ is a part of the heat amount released at friction per hub's heating; $g(\theta)$ is the desired displacement function.

A thermoelasticity problem for determining the displacements $\upsilon_{2e}$ and $\upsilon_{2w}$ of the plunger's contact surface is stated in the same way.

For determining $\upsilon_{1e}$ and $\upsilon_{2w}$ the kinetic wear equation [2] for the hub and plunger are used.

The found quantities $\upsilon_1$ and $\upsilon_2$ are substituted into the main contact equation (3). For algebraization of the main contact equation, the desired contact pressure functions [3] are found in the form of expansions

$$p(\theta,t)=p_0(\theta)+tp_1(\theta)+...; \tag{4}$$

$$p_0(\theta)=\alpha_0^0+\sum_{k=1}^{\infty}\left(\alpha_k^0\cos k\theta+\beta_k^0\sin k\theta\right)$$

$$p_1(\theta)=\alpha_0^1+\sum_{k=1}^{\infty}\left(\alpha_k^1\cos k\theta+\beta_k^1\sin k\theta\right).$$

In order to construct algebraic system for finding the desired coefficients $\alpha_k^0$, $\beta_k^0$ and $\alpha_k^1$, $\beta_k^1$, we equate the coefficients at the identical trigonometric functions.

As a result, we get an infinite algebraic system with respect to $\alpha_k^0\,(k=0,1,2,...)$, $\beta_k^0\,(k=0,1,2,...)$ and $\alpha_k^1$, $\beta_k^1$ etc.

Because of unknown quantities $\theta_1$ and $\theta_2$ the system of equations becomes nonlinear. The quantities $\theta_1$ and $\theta_2$ being the ends of the area of the hub and plunger contact, are defined from the equalities [4]:

$$p(\theta_1)=0;\quad p(\theta_2)=0. \tag{5}$$

The right sides of infinite algebraic systems contain the coefficients of displacement functions expansion. At the known displacements function $g(\theta)$ the obtained systems allow to find contact pressure by numerical calculations.

The obtained formula for contact pressure shows that the pressure depends on the coefficients $a_k$ and $b_k$ of the Fourier series of displacements functions $g(\theta)$.

Symbolically, for the contact pressure of friction node we can write

$$p(\theta,t)=f(\theta,t,a_0,a_k,b_k)\qquad(k=1,2,...,m). \tag{6}$$

Because of wear, to the end of the friction node running-in, the contact pressure decreases to the quantity $\tau^0/f$ and in fact its wear stops; Here $\tau^0$ is the ultimate friction force below of which galling of the friction pair doesn't occur; $f$ is the friction coefficient of the friction pair.

Thus, the displacements function $g(\theta)$ should be chosen so that the contact pressure was equal to $\tau^0/f$.

For constructing missing equations for determination of the coefficients of the expansion of displacements functions $g(\theta)$, we use the principle of least squares. For finding unknown parameters $a_k$ and $b_k$ we perform some calculations.

On the segment $[\theta_1,\theta_2]$ we choose $M$ points, where $M>2m+1$

$$\theta_i=\theta_1+i\Delta\theta;\quad \Delta\theta=\frac{\theta_2-\theta_1}{M};\quad F(\theta_i,t,a_0,a_k,b_k)\quad(i=1,2,...,M).$$

Time is assumed as a free parameter.

It is required to find such parameters of unknown parameters that will provide a constant value to the values (7) of the contact pressure function in the best way

$$F(\theta_i,t,a_0,a_k,b_k)=\tau^0/f\quad(i=1,2,...,M). \tag{7}$$

In other words, it is required to find the most probability values of unknown parameters. The deviations from the values of the contact value from the constant one will be

$$\varepsilon_i=F(\theta_i,t,a_0,a_k,b_k)-\tau^0/f\quad(i=1,2,...,M).$$

The principle of least squares states that the most probable values of the parameters are the ones at which the sum of the squares of deviations $\varepsilon_i$, will be least, i.e.

$$U=\sum_{i=1}^{M}\left[F(\theta_i,t,a_0,a_k,b_k)-\tau^0/f\right]\to\min. \tag{8}$$

For any moment we consider $a_0$, $a_k$, $b_k$ $(k = 1, 2, ..., m)$ as independent variables, and equating to zero the partial derivatives of the left side of (8) with respect to these variables, we get $2m + 1$ equations with $2m + 1$ unknowns

$$\frac{\partial U}{\partial a_0} = 0; \quad \frac{\partial U}{\partial a_k} = 0; \quad \frac{\partial U}{\partial b_k} = 0 \quad (k = 1, 2, ..., m). \tag{9}$$

The system of equations (9) closes the infinite algebraic system of the contact problem. Linear system (9) jointly with the infinite algebraic system of the contact problem should be solved for fixed values of time.

The greatest interest for a designer is the solution at initial time, i.e. when $t = 0$. The joint solution of the obtained system of equations admits to find approximate values of the coefficients $a_k$, $b_k$ and $\alpha_k$, $\beta_k$. The combined system of equations became nonlinear because of the unknowns $\theta_1$ and $\theta_2$. For its solution it is suggested to use the method of successive approximations [5] the essence of which is the following.

We solve the combined algebraic system at some definite values of $\theta_{1*}$ and $\theta_{2*}$ with respect to remaining unknowns $a_k$, $b_k$ and $\alpha_k$, $\beta_k$. At this it is necessary to solve the nonlinear algebraic system. The values of $\theta_{1*}$ and $\theta_{2*}$ and the found values of the remaining unknowns are substituted to the unused equation (5). The taken values of $\theta_{1*}$ and $\theta_{2*}$ and the appropriate values of the remaining unknowns, will not, generally speaking, satisfy equations (5). Therefore, choosing the values of the parameters $\theta_1$ and $\theta_2$, we'll repeat calculations untill the last equations of system (5) will be satisfied with the given accuracy.

The obtained system of equations, admits to choose optimal geometrical parameters of the displacement function of the external contour points of the hub of the contact pair providing the wear decrease by the numerical calculations on the design stage.

## References

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