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GLOBAL SOLVABILITY AND BEHAVIOR OF SOLUTIONS THE CAUCHY PROBLEM FOR SYSTMES OF SEMILINEAR HYPERBOLIC EQUATIONS WITH DISSIPATION

Abstract

In this paper we study the global solvability and the behavior of the Cauchy problem for systems of semilinear dissipative equations of the nonlinear part consists of sums of functions of two variables. We find conditions when the nonlinear part of which ensure the existence of global solutions

1.Introduction. Global solvability of the Cauchy problem for semilinear hyperbolic equations with dissipation is studied in the papers [1-6]. In these studies, sufficient conditions for the growth of the nonlinear part ensuring the existence of global solutions are obtained. The existence of global solutions for general systems of semilinear dissipative equations is investigate in the work [7]. The existence of global solutions for the special case of the systems of two semilinear hyperbolic equations was studied in [8-10]. In this paper we study the global solvability and the behavior of the Cauchy problem for systems of semilinear dissipative equations of the nonlinear part consists of sums of functions of two variables. We find conditions when the nonlinear part of which ensure the existence of global solutions.

2. Formulation of the problem and main results. In the domain $R_+ \times R_N$ consider the Cauchy problem for systems of semilinear hyperbolic equations with dissipations:

$$u_{ktt} + u_{kt} + (-1)^{l_k} u_k = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{kij}(u_i, u_j), \quad k = 1, ..., n \tag{1}$$

with initial conditions

$$u_k(0, x) = \varphi_k(x), \quad u_{kt}(0, x) = \psi_k(x), \quad x \in R^N, k = 1, ..., n. \tag{2}$$

Assume that the following conditions hold:

1. $l_1 \geq ... \geq l_n \geq \dfrac{N}{2}$;
2. $f_{kij}(\cdot) \in C^1(R^2), k, i, j = 1, ..., n$;
3. $|f_{kij}(u_i, u_j)| \leq c\, |u_i|^{\alpha_{kij}} \cdot |u_j|^{\beta_{kij}}$.

where

$$\alpha_{kij} + \beta_{kij} > \frac{2}{m_k} \tag{3}$$

$$\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j} > \frac{2}{N} + \frac{r_{kij}}{m_k}, \ i, j, k = 1, ..., n. \tag{4}$$

Here

$$r_{kij} = r_k(l_i, l_j, \alpha_{kij}, \beta_{kij}, m_k) =$$
$$= \begin{cases} \dfrac{1}{l_i} \quad \alpha_{kij} \geq \dfrac{2}{m_k}, \beta_{kij} \geq 0, \\ \dfrac{m_k\alpha_{kij}}{2l_i} + \dfrac{2 - m_k\alpha_{kij}}{2l_j}, \quad 0 \leq \alpha_{kij} < \dfrac{2}{m_k}, \beta_{kij} > 0. \end{cases}$$

We introduce the following notation:

$$U^l_{\delta,m} = \{(u,v) \cdot u \in W^1_2(R^N) \cap L_m(R^N), \quad v \in L_2(R^N) \cap L_m(R^N)\ ,$$
$$\|u\|_{W^l_2(R^N)} + \|u\|_{L_m(R^N)} + \|v\|_{L_2(R^N)} + \|v\|_{L_m(R^N)} < \delta.$$

We prove the following main theorem.

Theorem 1. *Suppose that conditions 1-3 are satisfied. Then there exists a real number* $\delta_0 > 0$, *such that for any* $(\varphi_k, \psi_k) \in U^{l_k}_{\delta_0, m_k}$ $k = 1, ..., n$ *problem (1), (2) has a unique solution*

$$u = (u_1, ..., u_n) \in C\left([0,\infty); \prod_{k=1}^{n} W_2^{l_k}(R^N)\right) \cap C^1\left([0,\infty); \left[L_2(R^N)\right]^n\right)$$

for $u_1, ..., u_n$ *the following estimates are valid:*

$$\sum_{|\alpha|=r} \|D^\alpha u_k(t,\cdot)\|_{L_2(R^N)} \leq c(1+t)^{\frac{N\ \frac{1}{m_k} - \frac{1}{2}\ + r}{2l_k}}, \ r = 0, 1, ..., l_k, \tag{5}$$

$$\|D_t u_k(t,\cdot)\|_{L_2(R^N)} \leq c(1)t)^{-\min(1 + \frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}\ , \Lambda_k)}, \tag{6}$$

where

$$\Lambda_k = \min_{i,j=1,...,n} \left\{ \frac{N}{2}\left(\frac{\alpha_{kij}}{l_j m_j} - \frac{\beta_{kij}}{l_j m_j}\right) - \frac{r_{kij}}{m_i} \right\}, \quad k = 1, ..., n.$$

3. Local solvability. In the Hilbert space $H = \left[L_2(R^N)\right]^n$ we write problem (1), (2), as the Cauchy problem

$$\left.\begin{matrix} y'' + y' + Ay = F(y) \\ y(0) = y_0, \quad y'(0) = y \end{matrix}\right\}, \tag{7}$$

where

$$y = \begin{pmatrix} u_1 \\ ... \\ u_n \end{pmatrix} \quad y_0 = \begin{pmatrix} \varphi_1 \\ ... \\ \varphi_3 \end{pmatrix}, \quad y_1 = \begin{pmatrix} \psi_1 \\ ... \\ \psi_3 \end{pmatrix},$$

A is linear operator in H defined by the equalities

$$D(A) = \prod_{k=1}^{n} W_2^{2l_k}(R^N),$$

$$A = \begin{pmatrix} (-1)^{l_1+1}\Delta^{l_1} + 1 & ... & 0 \\ ... & ... & ... \\ 0 & ... & (-1)^{l_n+1}\Delta^{l_n} + 1 \end{pmatrix};$$

$$F(y)=\begin{pmatrix}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} f_{1ij}(u_i,u_j)+u_1\\ \dots\dots\dots\dots\dots\dots\\ \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} f_{nij}(u_i,u_j)+u_n\end{pmatrix}$$

is anon- linear operator acting from $D\left(A^{1/2}\right)=\prod\limits_{k=1}^{n}W_2^{l_k}(R^N)$ to H.

A is a self-adjoint positive definite operator, and conditions 1 and 2 imply that the nonlinear operator $F(\cdot)$ satisfies the local Lipschitz condition,i.e.

$$\|F(y_1)-F(y_2)\|_H\le c(r)\left\|A^{1/2}(y_1-y_2)\right\|_H,$$

where $c(\cdot)\in C(R_+),\ \ c(r)\ge 0, r=\sum\limits_{i=1}^{2}\left\|A^{\frac{1}{2}}y_i\right\|_H$

By the theorem of solvability of the Cauchy problem for the operator differential equation we have the following local solvability theorem (see [12]) for problem (7).

Theorem 2. *Let the conditions of 1.2 be satisfied. Then for any*

$$y_0\in D\left(A^{1/2}\right)=\prod_{k=1}^{n}W_2^{l_k}(R^N)$$

there exists $T'\in(0,\infty)$ such that problem (7) has a unique solution
$y\in C\left([0,T'],\ D\left(A^{1/2}\right)\right)\cap C^1([0,T'],H)$..

If T_0 is the length of the maximum interval of the existence of solutions $y\in C\left([0,T_0],\ D\left(A^{1/2}\right)\right)\cap C^1([0,T_0],H)$. then one of the following statements is true:

1) $T_0=+\infty$

2) If $T_0<+\infty$, then $\lim\limits_{t\to T_0-0}\left(\left\|A^{1/2}y(t)\right\|+\|y'(t)\|\right)=+\infty$.

It follows that , if the a priori estimate

$$\left\|A^{1/2}y(t)\right\|+\|y'(t)\|\le[0,\infty). \tag{8}$$

holds then problem (7) has a global solution.

4. Proof of Theorem 1. By Theorem 2 and (8), to prove Theorem 1 we need to get a priori estimate:

$$\sum_{i=1}^{2}\left\|\nabla^{l_k}u_k(t,\cdot)\right\|_{L_2(R^N)}\le c,\ t\in[0,\infty)\ \ k=1,2,...,m. \tag{9}$$

Using Fourier transform, we obtain the following inequality (see [11])

$$\|u_k(t,\cdot)\|_{L_2(R^N)}\le c(1|t)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}}\ E_k(\varphi_k,\psi_k)+$$

$$+c\int_0^t(1+T-\tau)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}}\ \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\|f_{kij}\left(u_i(\tau,\cdot)\right)\|_{L_{m_k}(R^N)}+\right.$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|f_{kij}\left(u_i(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_2(R^N)}\Bigg]d\tau, \tag{10}$$

$$\sum_{|\alpha|=l_k}\left\|D^{\alpha}u_k(t,\cdot)\right\|_{L_2(R^N)}\leq c(1+t)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}\ -\frac{1}{2}}E_k(\varphi_k,\psi_k)+$$

$$+c\int_0^t(1+t-\tau)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}\ -\frac{1}{2}}\Bigg[\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|f_{kij}\left(u_i(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_{m_k}(R^N)}+$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|f_{kij}\left(u_i(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_2(R^N)}\Bigg]d\tau, \tag{11}$$

and

$$\sum_{|\alpha|=l_k}\left\|D_t u_k(t,\cdot)\right\|_{L_2(R^N)}\leq c(1+t)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}\ -1}E_k(\varphi_k,\psi_k)+$$

$$+c\int_0^t(1+t-\tau)^{-\frac{N}{2l_k}\ \frac{1}{m_k}-\frac{1}{2}\ -1}\Bigg[\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|f_{kij}\left(u_i(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_{m_k}(R^N)}+$$

$$++\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|f_{kij}\left(u_i(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_2(R^N)}\Bigg]d\tau, \tag{12}$$

where $1\leq m_k\leq 2,\quad k=1,...,3,\ t\in[0,T_0)$.

Further using the Holder inequality and conditions 1-3 , we obtain that

$$\left\|f_{kij}(u_i,u_j)\right\|_{L_2(R^N)}\leq c\left\|u_i\right\|^{\alpha_{kij}}_{L_{p_{lij}\alpha_{kij}}(R^N)}\cdot\left\|u_j\right\|^{\beta_{kij}}_{L_{q_{kij}\beta_{kij}}(R^N)}, \tag{13}$$

where $p_{kij}>1,\ q_{kij}>1,\ \dfrac{1}{p_{kij}}+\dfrac{1}{q_{kij}}=1$ (if $p_{kij}=0$, then $q_{kij}=\infty$, if $q_{kij}=0$, then $p_{kij}=\infty$).

By applying the multiplicative inequality (see [13]) from (13) we obtain that

$$\left\|f_{kij}(u_i,u_j)\right\|_{L_{m_k}(R^N)}\leq$$

$$\leq c\left\|u_i\right\|^{(1-\theta_{1kij})\alpha_{kij}}_{L_2(R^N)}\cdot\left\|\nabla^{l_i}u_i\right\|^{\theta_{1kij}\alpha_{kij}}_{L_2(R^N)}\cdot\left\|u_j\right\|^{(1-\theta_{2kij})\beta_{kij}}_{L_2(R^N)}\cdot\left\|\nabla^{l_j}u_j\right\|^{\theta_{2kij}\beta_{kij}}_{L_2(R^N)}, \tag{14}$$

where

$$\theta_{1kij}=\frac{N}{l_i}\left(\frac{1}{2}-\frac{1}{p_{kij}\alpha_{kij}m_k}\right),\quad \theta_{2kij}=\frac{N}{l_j}\left(\frac{1}{2}-\frac{1}{q_{kij}\beta_{kij}m_k}\right), \tag{15}$$

$$p_{kij}>\frac{2}{\alpha_{k,i,j}m_k},\quad q_{kij}>\frac{2}{\beta_{k,i,j}m_k},\quad \frac{1}{p_{kij}}+\frac{1}{q_{kij}}=1 \tag{16}$$

In a similar way we find thatwhere

$$\left\|f_{kij}(u_i,u_j)\right\|_{L_2(R^N)}\leq c\left\|u_i\right\|^{(1-\theta'_{1kij})\alpha_{kij_1}}_{L_2(R^N)}\cdot\left\|\nabla^{l_i}u_i\right\|^{(1-\theta'_{1kij})\alpha_{kij_1}}_{L_2(R^N)}.$$

$$\|u_j\|_{L_2(R^N)}^{(1-\theta'_{2kij})\beta_{kij}} \cdot \left\|\nabla^{l_j} u_j\right\|_{L_2(R^N)}^{\theta'_{2kij}\beta_{kij}}, \tag{17}$$

where

$$\theta'_{1kij} = \frac{N}{2l_j}\left(1 - \frac{1}{p'_{kij}\alpha_{kij}m_k}\right), \quad \theta'_{2kij} = \frac{N}{2l_k}\left(1 - \frac{1}{q'_{kij}\beta_{kij}m_k}\right)' \tag{18}$$

$$p'_{kij} > \frac{2}{\alpha_{kij}m_k}1, \quad q'_{kij} > \frac{2}{\beta_{kij}m_k}, \quad \frac{1}{p'_{kij}} + \frac{1}{q'_{kij}} = 1. \tag{19}$$

Proposition 1: The exponents $p_{kij} > 1$, $p'_{kij} > 1$ $q_{kij} > 1$ and $q'_{kij} > 1$, $k, i, j = 1, ..., n$ can be chosen so that the inequalities (16) and (19) are satisfied.

We introduce the following notations

$$X_k(t) = (1+t)^{\frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}}\ \|u_k(t,\cdot)\|_{L_2(R^N)}$$

$$Y_k(t) = (1+t)^{\frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}\ + \frac{1}{2}} \left\|\nabla^{l_k} u_k(t,\cdot)\right\|_{L_2(R^N)}.$$

From (10)-(17) we obtain that

$$X_k(t) \le cE_k(\varphi_k, \psi_k) + (1+t)^{\frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}} \int_0^t (1+t-\tau)^{-\frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}} \times$$
$$\times \sum_{i=1}^{n}\sum_{j=1}^{n} \left[(1+t)^{\gamma_{ijk}} F_{1kij}(\tau) + (1+\tau)^{\gamma_{ijk}} F_{2kij}(\tau)\right] d\tau \tag{20}$$

$$Y_k(t) \le cE_k(\varphi_k, \psi_k) + (1+t)^{\frac{N}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}\ + \frac{1}{2}} \int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\ \frac{1}{m_k} - \frac{1}{2}\ - \frac{1}{2}} \times$$

$$\times \sum_{i=1}^{n}\sum_{j=1}^{n} \left[(1+t)^{\gamma_{ijk}} F_{1kij}(\tau) + (1+\tau)^{\gamma_{ijk}} F_{2kij}(\tau)\right] d\tau, \tag{21}$$

where

$$F_{1kij}(\tau) = X_i^{(1-\theta_{1ki})\alpha_{kij}}(\tau) \cdot X_j^{(1-\theta_{2kj})\beta_{kij}} \cdot Y_j^{\theta_{2ki}\beta_{kij}}(\tau),$$

$$F_{2kij}(\tau) = X_i^{(1-\theta'_{1ki})\alpha_{kij}}(\tau) \cdot X_j^{(1-\theta'_{2kj})\beta_{kij}} \cdot Y_j^{\theta'_{2ki}\beta_{kij}}(\tau),$$

$$\gamma_{ijk} = \frac{N}{2l_i}\left(\frac{1}{m_i} - \frac{1}{2}\right)(1-\theta_{1kij})\,\alpha_{kij} + \frac{N}{2l_j}\left(\frac{1}{m_j} - \frac{1}{2}\right)(1-\theta_{2kij})\,\beta_{kij} +$$
$$+ \left(\frac{N}{2l_i}\left(\frac{1}{m_j} - \frac{1}{2}\right) + \frac{1}{2}\right)\theta_{1kij}\alpha_{kij} + \left(\frac{N}{2l_j}\left(\frac{1}{m_j} - \frac{1}{2}\right) + \frac{1}{2}\right)\theta_{2kij}\beta_{kij}, \tag{22}$$

$$\gamma'_{kij} = \frac{N}{2l_i}\left(\frac{1}{m_i} - \frac{1}{2}\right)(1-\theta'_{1kij})\,\alpha_{kij} + \frac{N}{2l_j}\left(\frac{1}{m_j} - \frac{1}{2}\right)(1-\theta'_{2kij})\,\beta_{kij} +$$
$$+ \left(\frac{N}{2l_i}\left(\frac{1}{m_{i1}} - \frac{1}{2}\right) + \frac{1}{2}\right)\theta'_{1kij}\alpha_{kij} + \left(\frac{N}{2l_j}\left(\frac{1}{m_j} - \frac{1}{2}\right) + \frac{1}{2}\right)\theta'_{2kij}\beta_{kij}, \tag{23}$$

From (16), (19), (23) and (24) we have

$$\gamma_{kij}=\frac{N}{2}\left[\left(\frac{\alpha_{kij}}{l_i m_i}+\frac{\beta_{kij}}{l_j m_j}\right)-\frac{1}{m_k}\left(\frac{1}{l_i p_{kij}}+\frac{1}{l_j q_{kij}}\right)\right]$$

$$\gamma'_{kij}=\frac{N}{2}\left[\left(\frac{\alpha_{kij}}{l_i m_i}+\frac{\beta_{kij}}{l_j m_j}\right)-\frac{1}{2m_k}\left(\frac{1}{l_i p_{kij}}+\frac{1}{l_j q_{kij}}\right)\right].$$

It is obvious that $\gamma'_{kij}>\gamma_{kij},\ k,\ i,\ j=1,...,n.$

Proposition 2: From (3), (4) follows that

$$\gamma_{kij}>\gamma'_{kij},\ k,\ i,\ j=1,...,n.$$

Therefore, by Proposition 2 and Segal's Lemma (see [12]) we obtain the following inequalities

$$(1+t)^{\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\int\limits_0^t(1+t-\tau)^{-\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}(1+\tau)^{-\gamma_{kij}}d\tau\leq c,\quad t\in[0,T_0); \tag{24}$$

$$(1+t)^{\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\int\limits_0^t(1+t-\tau)^{-\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}(1+\tau)^{-\gamma'_{kij}}d\tau\leq c,\quad t\in[0,T_0); \tag{25}$$

$$(1+t)^{\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\int\limits_0^t(1+t-\tau)^{-\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\times$$

$$\times(1+\tau)^{-\gamma_{kij}}d\tau\leq c,\quad t\in[0,T_0); \tag{26}$$

$$(1+t)^{\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\int\limits_0^t(1+t-\tau)^{-\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\times$$

$$\times(1+\tau)^{-\gamma'_{kij}}di\leq c,\quad t\in[0,T_0). \tag{27}$$

By taking into account (24)-(27) , from (20)-(21) we obtain the inequality

$$Z(t)\leq c_1\eta+c_2Z^q(t),\quad t\in[0,T_0), \tag{28}$$

where

$$Z(t)\leq\sum_{k=1}^{n}\underset{0\leq s\leq t}{Sup}\left[X_k(s)+Y_k(s)\right],$$

$$q=\max_{k,i,j=1,2,3}\left(\alpha_{kij}+\beta_{kij}\right),\quad\eta=\sum_{k=1}^{n}E_k(\varphi_k,\psi_k).$$

From (28) implies that for sufficiently small $\eta>0$

$$Z(t)\leq M,\quad t\in[0,T_0). \tag{29}$$

Therefore

$$\sum_{\|\alpha\|=l_k}\|D^\alpha u_k(t,\cdot)\|_{L_2(R^N)}\leq c(1+t)^{-\frac{N}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{1}{2}},$$

$$k = 1, ..., n, t \in [0, T_0), \tag{30}$$

$$\|u_k(t, \cdot)\|_{L_2(R^N)} \leq c(1+t)^{\frac{N}{2l_k} \ \frac{1}{m_k} - \frac{1}{2}}, \quad k = 1, ..., n. \tag{31}$$

By using the theorem on intermediate derivatives, we obtain

$$\sum_{\|\alpha\|=r} \|D^\alpha u_k(t, \cdot)\|_{L_2(R^N)} \leq c(1+t)^{-\frac{N}{2l_k} \ \frac{1}{m_k} - \frac{1}{2} \ - \frac{r}{2l_k}},$$

$$r = 0, 1, ..., l_k, \ k = 1, ..., n; \tag{32}$$

Then, using (29) and (31), from (12) we obtain that

$$\|D_t u_k(t, \cdot)\|_{L_2(R^n)} \leq c(1+t)^{-\min \ \frac{n}{2l_k} \ \frac{1}{m_i} - \frac{1}{2} \ , \Lambda_k},$$

where

$$\Lambda_k = \min_{i,j=1,...,n} \frac{n}{2} \left[\left(\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j} \right) - \frac{r_{kij}}{m_k} \right].$$

Therefore, for sufficiently small d the Cauchy problem (1)-(2) has a global solution, i.e. $T_0 = +\infty$

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