APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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EXISTENCE AND CALCULATION FORMULA OF THE DERIVATIVE OF DOUBLE LAYER ACOUSTIC POTENTIAL

Abstract

In the paper, the existence conditions are found and a formula for calculation of the derivative of a double layer acoustic potential is obtained.

Consider an acoustic potential of double layer

$$W_{k,p}(x) = \int_S \frac{\partial \Phi_k(x,y)}{\partial \overrightarrow{n}(y)} \rho(y) dS_y, \quad x \in S,$$

where $S \subset R^3$ is Lyapunov's surface with the exponent α, $\overrightarrow{n}(y)$ is an external unit normal at the point $y \in S$, $\Phi_k(x,y) = \dfrac{\exp(ik\,|x-y|)}{4\pi\,|x-y|}$ is the fundamental solution of the Helmholts equation, k is a wave number, $\operatorname{Im} k \geq 0$, and $\rho(y)$ is a continuous function on S.

It is known that some problems of physics and mechanics are reduced to singular integral equations dependent on a normal derivative of a double layer acoustic potential (for example, Dirichlet and Neumann external boundary value problems and others (see [1]). However, the counterexamples constructed by Hunter (see [2]) show that for a double layer potential with continuous density the derivatives, generally speaking, don't exist. But in [1] it is shown that if S is a twice differentiable surface, and $\rho \in C^{1,\alpha}$ ($C^{1,\alpha}$ is a class of continuously differentiable functions with Holder's uniform continuous derivative) then the double layer acoustic potential has a derivative, and by means of surface gradient the calculation formula of the derivative of the double layer acoustic potential is given. But this formula is not efficient , i.e. by means of this formula, generally speaking, it is impossible to construct a cubic formula for a normal derivative of a double layer potential. Therefore, there arises interest to development of a more practical formula for calculating the derivative of a double layer acoustic potential.

For the vector function S continues on $\varphi(x)$ introduce the continuity modulus of the form

$$\omega(\varphi, \delta) = \delta \sup_{\tau \geq \delta} \frac{\overline{\omega}(\varphi, \tau)}{\tau}, \quad \delta > 0,$$

where

$$\overline{\omega}(\varphi, \tau) = \max_{\substack{|x-y| \leq \tau \\ x,y \in S}} |\varphi(x) - \varphi(y)| \,.$$

Theorem. *Let S be the Lyapunov surface with the exponent $0<\alpha<1$, $\rho(x)$ be a continuously differentiable function on S, and* $\int\limits_0^{diamS}\frac{\omega(\operatorname{grad}\rho,t)}{t}dt<+\infty$. *Then the double layer acoustic potential $W_{k,\rho}(x)$ has on S a derivative, and*

$$\operatorname{grad}W_{k,p}(x)=\int\limits_S \operatorname{grad}_x\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)\rho(y)dS_y-$$

$$-\frac{3}{4\pi}\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\overrightarrow{xy}}{|x-y|^5}(\rho(y)-\rho(x))dS_y+\frac{1}{4\pi}\int\limits_S\frac{\rho(y)-\rho(x)}{|x-y|^3}\overrightarrow{n}(y)dS_y,\quad x\in S,$$

where the last integral exists in the sense of the Cauchy principle value.

Proof. Obviously,

$$W_{k,\rho}(x)=(W_{k,\rho}(x)-W_{0,\rho}(x))+W_{0,\rho}(x),$$

where

$$W_{k,\rho}(x)-W_{0,\rho}(x)=\int\limits_S\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)\rho(y)dS_y,\quad x\in S,$$

$$W_{0,\rho}(x)=W_{k,\rho}(x)\left.\right|_{k=0},\quad x\in S \text{ and } \Phi_0(x,y)=\Phi_k(x,y)\left.\right|_{k=0},\quad x,y\in S.$$

Denote by $\alpha(\overrightarrow{xy},\overrightarrow{n}(y))$ an angle between the vectors $\overrightarrow{xy}$ and $\overrightarrow{n}(y)$.

It is easy to calculate

$$\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)=-\frac{\cos\alpha(\overrightarrow{xy},\overrightarrow{n}(y))}{4\pi|x-y|^2}\times$$

$$\times((1-ik|x-y|)\exp(ik|x-y|)-1),\quad x,y\in S.$$

Since

$$|\cos\alpha(\overrightarrow{xy},\overrightarrow{n}(y))|\le M\cdot|x-y|^\alpha,\quad x,y\in S \tag{1}$$

(here and in sequel, by M we'll denote positive constants dependent only on S and k)

and

$$((1-ik|x-y|)\exp(ik|x-y|)-1)\le M\cdot|x-y|^2,\quad x,y\in S, \tag{2}$$

then

$$\left|\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right|\le M\cdot|x-y|^\alpha,\quad x,y\in S.$$

Then the expression $W_{k,\rho}(x)-W_{0,\rho}(x)$ has on S a derivative, and

$$\operatorname{grad}(W_{k,\rho}(x)-W_{0,\rho}(x))=\int\limits_S\operatorname{grad}_x\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)\cdot\rho(y)dS_y=$$

$$=\int\limits_S\left[\overrightarrow{yx}\cdot\frac{(\overrightarrow{n}(y),\overrightarrow{xy})}{4\pi|x-y|^5}\cdot\left(\left(3-3ik\cdot|x-y|-k^2\cdot|x-y|^2\right)\cdot e^{ik|x-y|}-3\right)+\right.$$

$$+\frac{\overrightarrow{n}(y)}{4\pi\cdot|x-y|^3}\cdot\left((1-ik\cdot|x-y|)\cdot e^{ik|x-y|}-1\right)\Bigg]\cdot\rho(y)dS_y.$$

Obviously,

$$\left|\operatorname{grad}_x\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)\right|\leq\frac{M}{|x-y|}.$$

Therefore, the integral

$$\int\limits_S\operatorname{grad}_x\left(\frac{\partial(\Phi_k(x,y)-\Phi_0(x,y))}{\partial\overrightarrow{n}(y)}\right)\cdot\rho(y)dS_y$$

converges as singular.

From the Gauss theorem (see [3])

$$W_{0,1}(x)=\int\limits_S\frac{\partial\Phi_0(x,y)}{\partial\overrightarrow{n}(y)}dS_y=-\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{4\pi\,|x-y|^3}dS_y=$$

$$=-\frac{1}{4\pi}\int\limits_S\frac{\cos\alpha(\overrightarrow{xy},\overrightarrow{n}(y))}{|x-y|^2}dS_y=-\frac{1}{4\pi}\cdot 2\pi=-\frac{1}{2},\quad x\in S.$$

Then we can represent $W_{0,\rho}(x)$ in the form

$$W_{0,\rho}(x)=-\frac{1}{4\pi}\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{4\pi\,|x-y|^3}\cdot(\rho(y)-\rho(x))\,dS_y-\frac{1}{2}\rho(x).$$

The function $\rho(x)$ is continuously differentiable, therefore there exists a point $y^*=x+\theta\cdot(y-x)$ such that (here $\theta=(\theta_1,\theta_2,\theta_3)$ and $\theta_i\in(0,1),\ i=\overline{1,3}$)

$$\rho(y)-\rho(x)=(\operatorname{grad}\rho(y^*),\overrightarrow{xy}),\ \ x,y\in S. \tag{3}$$

Then

$$\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{|x-y|^3}\cdot(\rho(y)-\rho(x))=\frac{\cos(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\cos\alpha(\operatorname{grad}\rho(y^*),\overrightarrow{xy})}{|x-y|}\cdot|\operatorname{grad}\rho(y^*)|\,.$$

Taking into account (1), we get that the expression $W_{0,\rho}(x)$ has on S a derivative (see [4]), and

$$\operatorname{grad}W_{0,\rho}(x)=-\frac{1}{4\pi}\int\limits_S\operatorname{grad}_x\left[\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{|x-y|^3}\cdot(\rho(y)-\rho(x))\right]dS_y-\frac{1}{2}\operatorname{grad}\rho(x)=$$

$$=\frac{1}{4\pi}\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{|x-y|^3}\operatorname{grad}\rho(x)dS_y-\frac{3}{4\pi}\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\overrightarrow{xy}}{|x-y|^5}(\rho(y)-\rho(x))dS_y+$$

$$+\frac{1}{4\pi}\int\limits_S\frac{\overrightarrow{n}(y)}{|x-y|^3}\cdot(\rho(y)-\rho(x))dS_y-\frac{1}{2}\operatorname{grad}\rho(x)=$$

$$=\operatorname{grad}\rho(x)\cdot\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))}{4\pi\,|x-y|^3}dS_y-\frac{3}{4\pi}\int\limits_S\frac{(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\overrightarrow{xy}}{|x-y|^5}\cdot(\rho(y)-\rho(x))dS_y+$$

$$+\frac{1}{4\pi}\int\limits_S \frac{\rho(y)-\rho(x)}{|x-y|^3}\cdot \overrightarrow{n}(y)dS_y-\frac{1}{2}\,\mathrm{grad}\,\rho(x)=$$

$$=-\frac{3}{4\pi}\int\limits_S \frac{(\overrightarrow{xy},\overrightarrow{n}(y))\overrightarrow{xy}}{|x-y|^5}\left(\rho(y)-\rho(x)\right)dS_y+\frac{1}{4\pi}\int\limits_S \frac{\rho(y)-\rho(x)}{|x-y|^3}\cdot \overrightarrow{n}(y)dS_y,\quad x\in S.$$

Taking into account (1) and (3), we get

$$\left|\frac{(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\overrightarrow{xy}}{|x-y|^5}\cdot(\rho(y)-\rho(x))\right|\le\frac{M\max\limits_{x\in S}|\mathrm{grad}\,\rho(x)|}{|x-y|^{2-\alpha}},$$

this means that the integral

$$\int\limits_S \frac{(\overrightarrow{xy},\overrightarrow{n}(y))\cdot\overrightarrow{xy}}{|x-y|^5}\cdot(\rho(y)-\rho(x))\,dS_y$$

converges.

Obviously,

$$\int\limits_S \frac{\rho(y)-\rho(x)}{|x-y|^3}\cdot \overrightarrow{n}(y)dS_y=$$

$$=\int\limits_S \frac{\overrightarrow{n}(y)-\overrightarrow{n}(x)}{|x-y|^3}\cdot(\rho(y)-\rho(x))\,dS_y+\overrightarrow{n}(x)\cdot\int\limits_S \frac{\rho(y)-\rho(x)}{|x-y|^3}dS_y.$$

Taking into account (3) and inequality $|\overrightarrow{n}(y)-\overrightarrow{n}(y)|\le M\cdot|x-y|^\alpha$, we get

$$\left|\frac{\overrightarrow{n}(y)-\overrightarrow{n}(x)}{|x-y|^3}\cdot(\rho(y)-\rho(x))\right|\le M\cdot\frac{\max\limits_{x\in S}|\mathrm{grad}\,\rho(x)|}{|x-y|^{2-\alpha}},$$

and therefore the integral

$$\int\limits_S \frac{\overrightarrow{n}(y)-\overrightarrow{n}(x)}{|x-y|^3}\cdot(\rho(y)-\rho(x))\,dS_y$$

converges.

It remains to prove that the integral

$$\int\limits_S \frac{\rho(y)-\rho(x)}{|x-y|^3}dS_y \tag{4}$$

exists in the sense of Cauchy principal value.

Denote by $d>0$ a radius of standard sphere for S (see [3]). Then for any point $x\in S$ the vicinity $S_d(x)=\left\{y\in S\middle||y-x|<d\right\}$ intersects with a straight line parallel to the normal $\overrightarrow{n}(x)$ at the unique point, or doesn't intersect at all, i.e. the set $S_d(x)$ is uniquely projected on the set $\Omega_d(x)$ lying in the circle of radius d centered at the point x in tangential plane $\Gamma(x)$ to S at the point x. On the segment $S_d(x)$ choose a local rectangular system of coordinates (u,v,w) with origin at the point x, where the axis w is directed along the normal $\overrightarrow{n}(x)$, the axes u and v lie

in the tangential plane $\Gamma(x)$. Then at these coordinates we can define the vicinity $S_d(x)$ by the equation

$$w = f(u,v), \quad (u,v) \in \Omega_d(x),$$

and

$$f \in C^{1,\alpha}(\Omega_d(x)) \quad \text{and} \quad f(0,0) = 0, \quad \frac{\partial f(0,0)}{\partial u} = 0, \quad \frac{\partial f(0,0)}{\partial v} = 0.$$

Furthermore, if $\widetilde{y} \in \Gamma(x)$ is the projection of the point $y \in S$, then (see [5])

$$|x - \widetilde{y}| \le |x - y| \le C_1 \cdot |x - \widetilde{y}|,$$

where C_1 is a positive constant dependent only on S (for the sphere $C_1 = \sqrt{2}$)

Taking into account (3), we get:

$$\int_S \frac{\rho(y) - \rho(x)}{|x-y|^3} dS_y = \int_{S \backslash S_d(x)} \frac{\rho(y) - \rho(x)}{|x-y|^3} dS_y +$$

$$+ \int_{S_d(x)} \frac{\rho(y) - \rho(x)}{|x-y|^3} dS_y = \int_{S \backslash S_d(x)} \frac{\rho(y) - \rho(x)}{|x-y|^3} dS_y +$$

$$+ \int_{S_d(x)} \frac{(\operatorname{grad}\rho(y^*) - \operatorname{grad}\rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y + \int_{S_d(x)} \frac{(\operatorname{grad}\rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y.$$

The integral $\int_{S \backslash S_d(x)} \frac{\rho(y) - \rho(x)}{|x-y|^3} dS_y$ exists as non-singular.

Passing to double integral, we get

$$\left| \int_{S_d(x)} \frac{(\operatorname{grad}\rho(y^*) - \operatorname{grad}\rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y \right| \le \int_{S_d(x)} \frac{|\operatorname{grad}\rho(y^*) - \operatorname{grad}\rho(x)|}{|x-y|^2} dS_y \le$$

$$\le M \int_0^d \frac{\omega(\operatorname{grad}\rho, t)}{t} dt < +\infty$$

It remains to prove that the integral $\int_{S_d(x)} \frac{(\operatorname{grad}\rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y$ exists in the sense of the Cauchy principal value.

Obviously,

$$\int_{S_d(x)} \frac{(\operatorname{grad}\rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y =$$

$$= \int_{S_d(x)} \frac{(y_1 - x_1) \cdot \dfrac{\partial \rho(x)}{\partial x_1} + (y_2 - x_2) \dfrac{\partial \rho(x)}{\partial x_2} + (y_3 - x_3) \dfrac{\partial \rho(x)}{\partial x_3}}{\left((y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2 \right)^{\frac{3}{2}}} dS_y.$$

Let $d_0 = \dfrac{d}{C_1}$ and $O_{d_0} = \left\{ u^2 + v^2 < d_0 \right\} \subset \Gamma(x)$ (obviously, $O_{d_0} \subset \Omega_d(x)$).

Then by the formula of reduction of a surface integral to repeated one, we get

$$\int\limits_{S_d(x)} \frac{(\operatorname{grad} \rho(x), \overrightarrow{xy})}{|x-y|^3} dS_y =$$

$$= \int\limits_{\Omega_d(x)} \frac{a \cdot u + b \cdot v + c \cdot f(u,v)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} dudv =$$

$$= \int\limits_{\Omega_d(x) \backslash O_{d_0}} \frac{a \cdot u + b \cdot v + c \cdot f(u,v)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} dudv +$$

$$+ \int\limits_{O_{d_0}} \frac{a \cdot u + b \cdot v + c \cdot f(u,v)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} dudv =$$

$$= \int\limits_{\Omega_d(x) \backslash O_{d_0}} \frac{a \cdot u + b \cdot v + c \cdot f(u,v)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} dudv +$$

$$+ \int\limits_{O_{d_0}} \frac{au+bv}{\left(\sqrt{u^2+v^2}\right)^3} dudv + \int\limits_{O_{d_0}} \frac{cf(u,v)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} dudv +$$

$$+ \int\limits_{O_{d_0}} \frac{au+bv}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} \cdot \left(\sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2} - 1 \right) dudv +$$

$$+ \int\limits_{O_{d_0}} (au+bv) \cdot \left(\frac{1}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3} - \frac{1}{\left(\sqrt{u^2+v^2}\right)^3} \right) dudv,$$

where a, b and c are constants dependent on the surface S, on the point $x \in S$ and on the function $\rho(x)$. Denote the additive integral in the right side of this equality by A_1, A_2, A_3, A_4 and A_5, respectively.

The integral A_1, exists as non-singular.

The integral A_2 exists in the sense of the Cauchy principal value and equals zero. Indeed, let $O_\varepsilon = \left\{ u^2 + v^2 < \varepsilon,\ w = 0 \right\}$, where $\varepsilon > 0$. Then

$$\int\limits_{O_{d_0}} \frac{au+bv}{\left(\sqrt{u^2+v^2}\right)^3} dudv = \lim_{\varepsilon \to 0} \int\limits_{O_{d_0} \backslash O_\varepsilon} \frac{au+bv}{\left(\sqrt{u^2+v^2}\right)^3} dudv =$$

$$= \lim_{\varepsilon \to 0} \int\limits_{\varepsilon}^{d_0} \int\limits_{0}^{2\pi} \frac{ar\cos\varphi + br\sin\varphi}{r^3} \cdot r \cdot d\varphi \cdot dr = \lim_{\varepsilon \to 0} \int\limits_{\varepsilon}^{d_0} \int\limits_{0}^{2\pi} \left(\frac{a}{r}\cos\varphi + \frac{b}{r}\sin\varphi \right) d\varphi dr = 0$$

Since $|f(u,v)| \le M \cdot \left(\sqrt{u^2+v^2}\right)^{1+\alpha}$ (see [3]), then for the integral A_3 we have:

$$|A_3| \le |M \cdot c| \int\limits_{O_{d_0}} \frac{1}{\left(\sqrt{u^2+v^2}\right)^{2-\alpha}} dudv = |M \cdot c| \int\limits_0^{d_0} \int\limits_0^{2\pi} \frac{1}{r^{1-\alpha}} d\varphi dr = 2\pi \cdot M\,|c|\, \frac{d_0^\alpha}{\alpha}$$

Furthermore, taking into account (see [3]),

$$\left|\frac{\partial f}{\partial u}\right| \le M \cdot \left(\sqrt{u^2+v^2}\right)^{\alpha} \quad \text{and} \quad \left|\frac{\partial f}{\partial v}\right| \le M \cdot \left(\sqrt{u^2+v^2}\right)^{\alpha},$$

we get

$$|A_4| = \left| \int\limits_{O_{d_0}} \frac{(au+bv)\left(\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2\right)}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3 \cdot \left(1+\sqrt{1+\left(\frac{\partial f}{\partial u}\right)^2+\left(\frac{\partial f}{\partial v}\right)^2}\right)} dudv \right| \le$$

$$\le M \cdot \int\limits_{O_{d_0}} \left| \frac{(au+bv)\left(\sqrt{u^2+v^2}\right)^{2\alpha}}{\left(\sqrt{u^2+v^2}\right)^3} \right| dudv = M \int\limits_0^{d_0} \int\limits_0^{2\pi} \frac{|a\cos\varphi + b\sin\varphi|}{r^{1-2\alpha}} d\varphi dr \le$$

$$\le \frac{\pi \cdot M \cdot (|a|+|b|)}{\alpha} \cdot d_0^{2\alpha}.$$

For the integral A_5 we have:

$$A_5 = \left| \int\limits_{O_{d_0}} \frac{(au+bv)f^2(u,v)(2(u^2+v^2)+f^2(u,v)+\sqrt{(u^2+v^2)\cdot(u^2+v^2+f^2(u,v))}dudv}{\left(\sqrt{u^2+v^2+f^2(u,v)}\right)^3 \cdot \left(\sqrt{u^2+v^2}\right)^3 \cdot \left(\sqrt{u^2+v^2+f^2(u,v)}+\sqrt{u^2+v^2}\right)} \right| \le$$

$$\le M \cdot \int\limits_{O_{d_0}} \frac{|au+bu| \cdot (u^2+v^2)^{1+\alpha}(u^2+v^2)\left(2+(u^2+v^2)^\alpha+\sqrt{1+(u^2+v^2)^\alpha}\right)}{2 \cdot \left(\sqrt{u^2+v^2}\right)^7} dudv =$$

$$= M \cdot \int\limits_0^{d_0} \int\limits_0^{2\pi} \frac{|a\cos\varphi + b\sin\varphi| \cdot \left(2+r^{2\alpha}+\sqrt{1+r^{2\alpha}}\right)}{2r^{2-2\alpha}} rd\ \varphi\ dr \le$$

$$\le M \cdot \frac{(|a|+|b|)\cdot\left(2+d_0^{2\alpha}+\sqrt{1+d_0^{2\alpha}}\right)}{2} \cdot 2\pi \int\limits_0^{d_0} \frac{1}{r^{1-\alpha}} dr =$$

$$= M\pi\,(|a|+|b|) \cdot \left(2+d_0^{2\alpha}+\sqrt{1+d_0^{2\alpha}}\right) \cdot \frac{d_0^{2\alpha}}{2\alpha}.$$

So, integral (4) exists in the sense of the Cauchy principal value. The theorem is proved.

Corollary. *Let S be the Lyapunov surface, $\rho(x)$ be a continuously differentiable function on S, and* $\int\limits_0^{diamS} \frac{\omega(\operatorname{grad}\rho, t)}{t} dt < +\infty$. *Then the double layer acoustic potential $W_{k,\rho}(x)$ has on S a normal derivative, and*

$$\frac{\partial}{\partial \overrightarrow{n}(x)} W_{k,\rho}(x) = \int\limits_S \frac{\partial}{\partial \overrightarrow{n}(x)} \left(\frac{\partial(\Phi_k(x,y) - \Phi_0(x,y))}{\partial \overrightarrow{n}(x)} \right) dS_y -$$

$$-\frac{3}{4\pi} \int\limits_S \frac{(\overrightarrow{xy}, \overrightarrow{n}(y)) \cdot (\overrightarrow{xy}, \overrightarrow{n}(x))}{|x-y|^5} \cdot (\rho(y) - \rho(x))\, dS_y +$$

$$+\frac{1}{4\pi} \int\limits_S \frac{(\overrightarrow{n}(y), \overrightarrow{n}(x))}{|x-y|^3} \cdot (\rho(y) - \rho(x))\, dS_y, \quad x \in S,$$

where the last integral exists in the sense of the Cauchy principal value.

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