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INVESTIGATION OF CURVILINEAR CRACK STIMULATION OF THERMOELASTIC STRESS FIELD

Abstract

The curvilinear crack stimulation of thermoelastic stress field in the extendable plane is investigated. The crack faces are free of external loads. The stress intensity factors depending on physical and geometrical parameters of the plate and thermal source are found.

Let's consider an unbounded elastic plane weakened with one linear crack of length $2l$ at the origin of coordinates (fig. 1). In actual materials, the crack surfaces have unevenness and curvatures because of structural and technological factors. Let's consider a problem of mechanics on a crack in the plane, assuming that the crack's contour has roughnesses (small deviations from the linear form). The crack faces are free from external loads. At infinity the plate is subjected to homogeneous extension along the axis oy $\sigma_y^\infty = \sigma_0$.

In order to retard the curvilinear crack propagation, the contractive stresses areas are created on the way of its propagation from the both ends by heating the areas S_1 and S_2 by means of thermal source to temperature T_0. The choice of the system of cartesian coordinates and denotation are explained on fig.1.

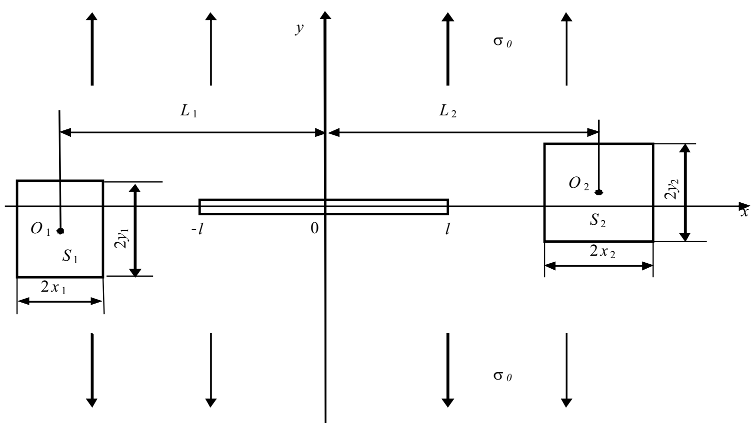

Fig.1.

The problem is to define the stress-strain state outside of the crack and to define the size of the ultimate external load by attaining of which the crack will propagate along the section of the plate.

We accept the following assumptions:

1) all thermoelastic characteristics of the plate's material are temperature independent;

2) The plate's material is a homogeneous and isotropic body.

For many metallic materials (steels, aluminium alloy and etc.) in the range of temperature to $300^0 - 400^0 C$ the dependence of thermoelastic characteristics weakly changes with temperature.

This is an experimentally established fact [1,2]. Thus for all the structural materials there exists the temperature range where the assumption on constancy of thermoelastic characteristics of the material is correct and is established on the base of temperature dependence of the modulus of elasticity. The experiments [3] show that by heating the crack's route the crack retardation and inhibition is observed by $70^0 - 100^0$.

It is assumed that at moment $t = 0$, an arbitrary domain $S = S_1 + S_2$ on the way of crack propagation in the plate instantly heats up to the constant temperature $T = T_0$. At the initial moment, the remaining part of the plate has the temperature $T = 0$. The crack is assumed to be close to the linear form with only small deviations of the crack line from the straight line $y = 0$. The crack's equation is accepted in the form $y = f(x)$. $-l \leq x \leq l$.

For solving the thermoelasticity boundary value problem, at first we determine the temperature field with a crack. We'll use the superposition method. The distribution temperature of $T(x, y, t)$ may represented as follows:

$$T(x,y,t) = T_1(x,y,t) + T_2(x,y,t), \tag{1}$$

where $T_1(x, y, t)$ is temperature distribution in the entire plane when at moment $t = 0$ the arbitrary domain S instantly heats up to the constant temperature $T = T_0$, and $T_2(x, y, t)$ is the perturbed temperature field caused by the availability of the crack,

The temperature field for the entire plane is determined from the solution of the boundary value problem of heat conuction theory:

$$a\Delta T = \frac{\partial T}{\partial t}, \qquad T = \begin{cases} T_0 & (x, y \in S) \\ 0 & (x, y \notin S) \end{cases} \qquad for \;\; t = 0, \tag{2}$$

where Δ is the Laplace operator; a is the thermal diffusivity of the plane's material.

For the generalized plane stress state it is assumed that the plate was heatisolated on lateral surfaces. For definiteness, without loss of generality of the problem, let each of the domains S_1 and S_2 represent a rectangle (fig. 1).

The solution of the boundary value problem is achieved by the superposition method. Therefore, not going into details, we give the solution of the boundary value problem.

For temperature distribution in the entire plate we have

$$T(x,y,t) = T_1(x,y,t) + T_2(x,y,t), \tag{3}$$

$$T_1(x,y,t) = \frac{T_0}{4}\left[Erf\left(\frac{x-L_1+x_1}{2\sqrt{at}}\right) + Erf\left(\frac{-x+L_1+x_1}{2\sqrt{at}}\right)\right]\times$$
$$\times\left[Erf\left(\frac{y-b_1+y_1}{2\sqrt{at}}\right) + Erf\left(\frac{y_1+b_1-y}{2\sqrt{at}}\right)\right];$$
$$T_2(x,y,t) = \frac{T_0}{4}\left[Erf\left(\frac{x-L_2+x_2}{2\sqrt{at}}\right) + Erf\left(\frac{x_2+L_2-x}{2\sqrt{at}}\right)\right]\times$$
$$\times\left[Erf\left(\frac{y-b_2+y_2}{2\sqrt{at}}\right) + Erf\left(\frac{y_2+b_2-y}{2\sqrt{at}}\right)\right];$$

In this case for distribution of the stresses σ_{y_0} and τ_{xy_0} we find

$$\sigma_{y_0} = \sigma_{y_1} + \sigma_{y_2}; \quad \tau_{xy_0} = \tau_{xy_1} + \tau_{xy_2}; \tag{4}$$

$$\sigma_{y_1} = -\frac{\mu(1+\nu)\alpha T_0}{4\sqrt{\pi}}\left\{4\sqrt{\pi}A(x,y) + \frac{4}{\sqrt{\pi}}\left[arctg\left(\frac{y-b_1+y_1}{x-L_1+x_1}\right)+\right.\right.$$
$$\left.+arctg\left(\frac{y_1+b_1-y}{x_1+L_1-x}\right) + arctg\left(\frac{y_1+b_1-y}{x-L_1+x_1}\right) + arctg\left(\frac{y-b_1+y_1}{x_1+L_1-x}\right)\right]-$$
$$-\int_0^1 \frac{1}{\tau\sqrt{a\tau}}\left[(x-L_1+x_1)\exp\left(-\frac{(x-L_1+x_1)^2}{4a\tau}\right)+\right.$$
$$\left.+(x_1+L_1-x)\exp\left(-\frac{(x_1+L_1-x)^2}{4a\tau}\right)\right]\times$$
$$\left.\times\left[Erf\left(\frac{(y-b_1+y_1)}{2\sqrt{a\tau}}\right) + Erf\left(\frac{(y_1+b_1-y)}{2\sqrt{a\tau}}\right)\right]d\tau\right\};$$
$$\sigma_{y_2} = -\frac{\mu(1+\nu)\alpha T_0}{4\sqrt{\pi}}\left\{4\sqrt{\pi}A(x,y) + \frac{4}{\sqrt{\pi}}\left[arctg\left(\frac{y-b_2+y_2}{x-L_2+x_2}\right)+\right.\right.$$
$$\left.+arctg\left(\frac{y_2+b_2-y}{x_2+L_2-x}\right) + arctg\left(\frac{y_2+b_2-y}{x-L_2+x_2}\right) + arctg\left(\frac{y-b_2+y_2}{x_2+L_2-x}\right)\right]-$$
$$-\int_0^1 \frac{1}{\tau\sqrt{a\tau}}\left[(x-L_2+x_2)\exp\left(-\frac{(x-L_2+x_2)^2}{4a\tau}\right)+\right.$$
$$\left.+(x_2+L_2-x)\exp\left(-\frac{(x_2-L_2+x)^2}{4a\tau}\right)\right]\times$$
$$\left.\times\left[Erf\left(\frac{(y-b_2+y_2)}{2\sqrt{a\tau}}\right) + Erf\left(\frac{(y_2+b_2-y)}{2\sqrt{a\tau}}\right)\right]d\tau\right\};$$
$$\tau_{xy_1} = -\frac{\mu(1+\nu)\alpha T_0}{2\pi}\left\{\ln\frac{(x-x_1-L_1)^2+(y-b_1+y_1)^2}{(x-x_1-L_1)^2+(x-b_1-y_1)^2}+\right.$$

$$+\ln\frac{(x-L_1+x_1)^2+(y-y_1-b_1)^2}{(x-L_1+x_1)^2+(y-b_1+y_1)^2}-$$

$$-\int\limits_0^t\frac{1}{\tau}\left[\exp\left(-\frac{(x-L_1+x_1)^2}{4a\tau}\right)-\exp\left(-\frac{(x_1+L_1-x)^2}{4a\tau}\right)\right]\times$$

$$\times\left[\exp\left(-\frac{(y-b_1-y_1)^2}{4a\tau}\right)-\exp\left(-\frac{(y_1+b_1-y)^2}{4a\tau}\right)\right]d\tau;$$

$$\tau_{xy_2}=-\frac{\mu(1+\nu)\alpha T_0}{2\pi}\left\{\ln\frac{(x-x_2-L_2)^2+(y-b_2+y_2)^2}{(x-x_2-L_2)^2+(x-b_2-y_2)^2}+\right.$$

$$+\ln\frac{(x-L_2+x_2)^2+(y-y_2-b_2)^2}{(x-L_2+x_2)^2+(y-b_2+y_2)^2}-$$

$$-\int\limits_0^t\frac{1}{\tau}\left[\exp\left(-\frac{(x-L_2+x_2)^2}{4a\tau}\right)-\exp\left(-\frac{(x_2+L_2-x)^2}{4a\tau}\right)\right]\times$$

$$\times\left[\exp\left(-\frac{(y-b_1+y_1)^2}{4a\tau}\right)-\exp\left(-\frac{(y_1+b_1-y)^2}{4a\tau}\right)\right]d\tau;$$

$$A(x,y)=\left(\begin{array}{ll}1 & (x,y)\in S\\ 0 & (x,y)\notin S\end{array}\right.$$

The boundary conditions on the crack faces take the form

$$\sigma_n^i-i\tau_m^1=-(\sigma_{n_0}-i\tau_{m_0}) \qquad (5)$$

Consider some arbitrary realization of the rough (with small deviations from the linear form) surfaces of the crack faces. Since the functions $f(x)$ and $f'(x)$ are small quantities, we can represent the function $f(x)$ in the form

$$f(x)=\varepsilon H(x) \qquad -l\le x\le l \qquad (6)$$

where ε is a small parameter.

Expand the stress tensor components $\sigma_x^1,\sigma_y^1,\tau_{xy}^1$, in small parameter ε

$$\sigma_x^1=\sigma_x^{(0)}+\varepsilon\sigma_x^{(1)}+...,\quad \sigma_y^1=\sigma_y^{(0)}+\varepsilon\sigma_y^{(1)}+...,\quad \tau_{xy}^1=\tau_{xy}^{(0)}+\varepsilon\tau_{xy}^{(1)}+... \qquad (7)$$

By expanding in series the expression for $y=f(x)$ the stresses in the vicinity $y=0$ we find the values of stresses for $y=f(x)$. Using the perturbations method with regard to preceding formulas, we find the boundary conditions for $y=0$, $-l\le x\le l$:

in a zero approximation

$$\sigma_y^{(0)}=-f_1(x),\quad \tau_{xy}^{(0)}=-f_2(x),\quad f_1(x)=\sigma_{y10}(x,0),\quad f_2(x)=\tau_{xy_0}(x,0) \qquad (8)$$

in a first approximation

$$\sigma_y^{(1)} = N, \quad \tau_{xy}^{(1)} = T \tag{9}$$

Here

$$N = 2\tau_{xy}^{(0)} \frac{dH}{dx} - H \frac{\partial \sigma_y^{(0)}}{\partial y}, \quad T = \left(\sigma_x^{(0)} - \sigma_y^{(0)}\right) \frac{dH}{dx} - H \frac{\partial \tau_{xy}^{(0)}}{\partial y} \tag{10}$$

For the solution of boundary value problems at each approximation we use the Kolosov-Muskheleshvili complex potentials [4].

In brittle failure mechanics the stress intensity factors are of greatest interest.

For the stress intensity factors $K_I^{(1)}$ and $K_{II}^{(1)}$ for the right vertex of the crack we find

$$K_I^{(1)} = K_{I_1}^{(1)} + K_{I_2}^{(1)} + K_{I_3}^{(1)}; \quad K_{II}^{(1)} = K_{II_1}^{(1)} + K_{II_2}^{(1)} + K_{II_3}^{(1)} \tag{11}$$

$$K_{I_1}^{(1)} = -\frac{\mu (1+\nu)\alpha T_0}{\sqrt{\pi l}} \left\{ \pi l + \frac{1}{\pi} \int_{-l}^{l} \left[arctg \left(\frac{y_1 - b_1}{x - L_1 + x_1} \right) + \right.\right.$$

$$+ arctg \left(\frac{y_1 + b_1}{x_1 + L_1 - x} \right) + arctg \left(\frac{y_1 + b_1}{x - L_1 + x_1} \right) + arctg \left(\frac{y_1 - b_1}{x_1 + L_1 - x} \right) \Big] \times$$

$$\times \sqrt{\frac{l+x}{l-x}} dx - \frac{1}{4\sqrt{\pi}} \int_{-l}^{l} \int_{0}^{t} \frac{1}{\tau \sqrt{a\tau}} \left[(x - L_1 + x_1) \exp \left(- \frac{(x - L_1 + x_1)^2}{4a\tau} \right) + \right.$$

$$\left. + (x_1 + L_1 - x) \exp \left(- \frac{(x_1 + L_1 - x)^2}{4a\tau} \right) \right] \times$$

$$\left. \times \left[Erf \left(\frac{(y_1 - b_1)}{2\sqrt{a\tau}} \right) + Erf \left(\frac{(y_1 + b_1)}{2\sqrt{a\tau}} \right) \right] \sqrt{\frac{l+x}{l-x}} d\tau dx \right\};$$

$$K_{I_2}^{(1)} = -\frac{\mu (1+\nu)\alpha T_0}{\sqrt{\pi l}} \left\{ \pi l + \frac{1}{\pi} \int_{-l}^{l} \left[arctg \left(\frac{y_2 - b_2}{x - L_2 + x_2} \right) + \right.\right.$$

$$+ arctg \left(\frac{y_2 + b_2}{x_2 + L_2 - x} \right) + arctg \left(\frac{y_2 + b_2}{x - L_2 + x_2} \right) + arctg \left(\frac{y_2 - b_2}{x_2 + L_2 - x} \right) \Big] \times$$

$$\times \sqrt{\frac{l+x}{l-x}} dx - \frac{1}{4\sqrt{\pi}} \int_{-l}^{l} \int_{0}^{t} \frac{1}{\tau \sqrt{a\tau}} \left[(x - L_2 + x_2) \exp \left(- \frac{(x - L_2 + x_2)^2}{4a\tau} \right) + \right.$$

$$\left. + (x_2 + L_2 - x) \exp \left(- \frac{(x_2 + L_2 - x)^2}{4a\tau} \right) \right] \times$$

$$\left. \times \left[Erf \left(\frac{(y_2 - b_2)}{2\sqrt{a\tau}} \right) + Erf \left(\frac{(y_2 + b_2)}{2\sqrt{a\tau}} \right) \right] \sqrt{\frac{l+x}{l-x}} d\tau dx \right\};$$

$$K_{II_1}^{(1)}=-\frac{\mu\left(1+\nu\right)\alpha T_0}{\sqrt{\pi l}}\left\langle\frac{1}{2\pi}\int_{-l}^{l}\left[\ln\frac{\left(x-x_1-L_1\right)^2+\left(y_1-b_1\right)^2}{\left(x-x_1-L_1\right)^2+\left(y_1+b_1\right)^2}+\right.\right.$$

$$\left.+\ln\frac{\left(x-L_1+x_1\right)^2+\left(y_1+b_1\right)^2}{\left(x-L_1+x_1\right)^2+\left(y_1-b_1\right)^2}\right]\sqrt{\frac{l+x}{l-x}}dx-$$

$$-\frac{1}{2\pi}\int_{-l}^{l}\int_{0}^{t}\frac{1}{\tau}\left[\exp\left(-\frac{\left(x-L_1+x_1\right)^2}{4a\tau}\right)-\exp\left(-\frac{\left(x_1+L_1-x\right)^2}{4a\tau}\right)\right]\times$$

$$\left.\times\left[\exp\left(-\frac{\left(y_1-b_1\right)^2}{4a\tau}\right)-\exp\left(-\frac{\left(y_1+b_1\right)^2}{4a\tau}\right)\right]\sqrt{\frac{l+x}{l-x}}d\tau dx\right\rangle;$$

$$K_{II_2}^{(1)}=-\frac{\mu\left(1+\nu\right)\alpha T_0}{\sqrt{\pi l}}\left\langle\frac{1}{2\pi}\int_{-l}^{l}\left[\ln\frac{\left(x-x_2-L_2\right)^2+\left(y_2-b_2\right)^2}{\left(x-x_2-L_2\right)^2+\left(y_2+b_2\right)^2}+\right.\right.$$

$$\left.+\ln\frac{\left(x-L_2+x_2\right)^2+\left(y_2+b_2\right)^2}{\left(x-L_2+x_2\right)^2+\left(y_2-b_2\right)^2}\right]\sqrt{\frac{l+x}{l-x}}dx-$$

$$-\frac{1}{2\pi}\int_{-l}^{l}\int_{0}^{t}\frac{1}{\tau}\left[\exp\left(-\frac{\left(x-L_2+x_2\right)^2}{4a\tau}\right)-\exp\left(-\frac{\left(x_2+L_2-x\right)^2}{4a\tau}\right)\right]\times$$

$$\left.\times\left[\exp\left(-\frac{\left(y_2-b_2\right)^2}{4a\tau}\right)-\exp\left(-\frac{\left(y_2+b_2\right)^2}{4a\tau}\right)\right]\sqrt{\frac{l+x}{l-x}}d\tau dx\right\rangle;$$

$$K_{I_3}^{(1)}=-\frac{\varepsilon}{\sqrt{\pi l}}\int_{-l}^{l}N\sqrt{\frac{l+t}{l-t}}dt,\quad K_{II_3}^{(1)}=-\frac{\varepsilon}{\sqrt{\pi l}}\int_{-l}^{l}T\sqrt{\frac{l+t}{l-t}}dt \qquad (12)$$

The stress intensity factors $K_I^{(1)}$ and $K_{II}^{(1)}$ for the vicinity of the left vertex of the crack are found in the same way.

We obtain the stress intensity factors for the perturbed temperature field in the following form: for the right end of the crack

$$K_I^{(2)}=0;\quad K_{II}^{(2)}=-\frac{2\delta}{\sqrt{\pi l}}\int_{-l}^{l}q_0\left(\tau\right)\sqrt{l^2-\tau^2}d\tau \qquad (13)$$

for the left end of the crack

$$K_I^{(2)}=0;\quad K_{II}^{(2)}=\frac{2\delta}{\sqrt{\pi l}}\int_{-l}^{l}q_0\left(\tau\right)\sqrt{l^2-\tau^2}d\tau, \qquad (14)$$

where $\delta=\frac{\beta}{1+k_0}$, $k_0=\left(3-\nu\right)/\left(1+\nu\right)$, $\beta=\dfrac{\alpha E}{1+\nu}$, ν is the Poisson ratio.

Now we give an expression for the function $q_0(x)$. In this case

$$q_0(x) = q_{0_1}(x) + q_{0_2}(x) \tag{15}$$

$$q_{0_1}(x) = -\frac{T_0 e^{\omega t}}{4\sqrt{\pi}\sqrt{at}}\left[\exp\left(-\frac{(y_1+b_1)^2}{4a\tau}\right) - \exp\left(-\frac{(y_1+b_1)^2}{4a\tau}\right)\right]\times$$

$$\times\left[Erf\left(\frac{(x-L_1+x_1)}{2\sqrt{a\tau}}\right) + Erf\left(\frac{(x_1+L_1+x)}{2\sqrt{a\tau}}\right)\right];$$

$$q_{0_2}(x) = -\frac{T_0 e^{\omega t}}{4\sqrt{\pi}\sqrt{at}}\left[\exp\left(-\frac{(y_2-b_2)^2}{4a\tau}\right) - \exp\left(-\frac{(y_2+b_2)^2}{4a\tau}\right)\right]\times$$

$$\times\left[Erf\left(\frac{(x-L_2+x_2)}{2\sqrt{a\tau}}\right) + Erf\left(\frac{(x_2+L_2-x)}{2\sqrt{a\tau}}\right)\right];$$

For the right end of the crack, the ctress intensity coefficient for the perturbed temperature field will take the following form:

$$K_{II}^{(2)} = K_{II_1}^{(2)} + K_{II_2}^{(2)}; \tag{16}$$

$$K_{II_1}^{(2)} = \frac{\delta T_0 e^{\omega t}}{4\sqrt{\pi l}}\cdot\frac{1}{\sqrt{\pi at}}\left[\exp\left(-\frac{(y_1-b_1)^2}{4a\tau}\right) - \exp\left(-\frac{(y_1+b_1)^2}{4a\tau}\right)\right]\times$$

$$\times\int_{-l}^{l}\left[Erf\left(\frac{(x-L_1+x_1)}{2\sqrt{a\tau}}\right) + Erf\left(\frac{(x_1+L_1-x)}{2\sqrt{a\tau}}\right)\right]\sqrt{l^2-x^2}dx-$$

$$-\frac{2\delta e^{\omega t}}{\sqrt{\pi l}}\int_{-l}^{l}\left[\varepsilon q_1(x) + \varepsilon^2 q_2(x)\right]\times\sqrt{l^2-x^2}dx;$$

$$K_{II_2}^{(2)} = \frac{\delta T_0}{2\sqrt{\pi l}}\cdot\frac{e^{\omega t}}{\sqrt{\pi at}}\left[\exp\left(-\frac{(y_2-b_2)^2}{4at}\right) - \exp\left(-\frac{(y_2+b_2)^2}{4at}\right)\right]\times$$

$$\times\int_{-l}^{l}\left[Erf\left(\frac{(x-L_2+x_2)}{2\sqrt{at}}\right) + Erf\left(\frac{(x_2+L_2-x)}{2\sqrt{at}}\right)\right]\sqrt{l^2-x^2}dx-$$

$$-\frac{2\delta e^{\omega t}}{\sqrt{\pi l}}\int_{-l}^{l}\left[\varepsilon q_1(x) + \varepsilon^2 q_2(x)\right]\times\sqrt{l^2-x^2}dx$$

The stress intensity coefficient $K_{II}^{(2)}$ for the left end of the crack is found in the same way. Finally, for the stress intensity coefficients we have the relations defind by the formulae

$$K_1 = \sigma_0\sqrt{\pi l} + K_I^{(1)} + K_I^{(2)} \tag{17}$$

$$K_{II} = \frac{\sigma_0}{\sqrt{\pi l}}\int_{-l}^{l}\frac{df}{dt}\sqrt{\frac{l+t}{l-t}}dt + K_{II}^{(1)} + K_{II}^{(2)}$$

Similarly we can consider the case when the domain S in the vicinity of each vertex is a totality of right domains ($S = S_1 + S_2 + ... + S_n$).

Knowing the stress intensity coefficients for any time by maens of the generalized criteria of brittle failure called also a maximal hoop stress σ_θ [5, 6] we analyze the crack propagation. According to this criterion the initial propagation of the crack at its spreading occurs in the plane for which the singular part of normal tensile stresses σ_θ has maximum intensity. We find the quantity of the angle $\theta = \theta_*$ characterizing the direction of this plane from the relation

$$\lim_{r\to 0}\left[\sqrt{2\pi r}\frac{\partial\sigma_\theta}{\partial\theta}\right] = 0 \quad for \quad \theta = \theta_*$$

To calculate the critical load $\sigma_0 = \sigma_0^*$ and deviation angle θ_*, we have [5] the following equations

$$\theta_* = 2arctg\left[\left(1 - \sqrt{1 + 8\lambda^2}\right)/(4\lambda)\right], \quad \left(\lambda = \frac{K_I}{K_{II}}\right) \tag{18}$$

$$4\sqrt{2}K_I\lambda^3\frac{1 + 3\sqrt{1 + 8\lambda^2}}{\left(12\lambda^2 + 1 - \sqrt{1 + 8\lambda^2}\right)^{3/2}} = K_C\,(T) \tag{19}$$

Equations (18)-(19) allow to investigate the thermal source stimulation of thermoelastic field on crack development by numerical method.

In the relations of brittle failure criterion, the characteristics of crack-resistance (failure toughness) depends on tempreature T. For total analysis of the construction's strength it is necessary to know the temperature dependence of crack resistance of the material. As the results [7] of the investigation shown, in practical range of temperature change, the temperature dependence of K_C outside of the cold brittleness may be with sufficient accuracy approximated by the polynomial of second degree on temperature

$$K_C = A_0 + A_1 T + A_2 T \tag{20}$$

where A_0 is a constant of K_{C_0} at temperature $T = 0$, A_1, A_2 are some empiric constants.

Substituting the found stress intensity coefficients in condition (19), and instead of the quantity $K_C\,(T)$ relation (20), we get the dependence of the crack's length on the applied load, time, geometrical, physical parameters of the problem in the implicit form.

Analyzing numerically the stress intensity coefficient dependence we can determine the influence of thermal source intensity, geometrical parameters of the zone of evaluated temperatures and of the plate, time on the value of critical stress intensity coefficients. On the base of the calculation results, we can make some conclusions.

As is known, the crack is stable if the tensile stress σ_θ necessary for keeping it in mobile-equilibrium state increases due to increase of the crack length. The condition of stable growth of the crack is of the form [5]:

$$\frac{\partial}{\partial l}\left[-\frac{1-\nu}{2\mu}\left(K_I^2+K_{II}^2\right)+\frac{K_C^2}{E}\right]>0$$

The investigation shows that the normal break crack growth occurs steadily (the quantity K_I decreases as the crack approaches to the area of evaluated temperatures, i.e. to thermal source). In availability of the crack in the zone of compression, the tensile stresses partially are compensated by thermoelastic stresses, and in certain degree this reduces to "hardening" of the plate.

For great removal of the crack vertex from the domain S (thermal source), the decrease of stress intensity at the crack vertex is negligible. As the crack vertex approaches to the thermal source, because of non-uniform heating of the faces of the growing crack, the breaking path changes its direction. The crack propagation arises under the angle $14^0 - 20^0$ to the symmetry axis of the plate depending on geometrical parameters of the evaluated temperature zones (domain S).

As the crack vertex approaches to the domain S, the stress intensity decrease becomes more weighty and attains the greatest value when the track directly gets into the zone of evaluated temperatures. The stress intensity decreases according to temperature increase of the domain S (heated zone).

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Received: September 21, 2012; Revised: November 28, 2012.