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ABOUT THE STABILUTY OF NONHOMOGENEOUS THREE-LAYER RODS UNDER UNEVEN TEMPERATURE IN AN NONLINEAR ELASTIC FOUNDATION

Abstract

We study the problem of three-layered nonhomogeneous rectilinear rods on an nonlinear elastic foundation under the pressure of compressive loads in this article.

It is assumed that the rod is in the uneven temperature field and the elasticity modules of the material layers depend on temperature.

For the elastic foundation of nonlinear model is accepted and it is assumed that the hypothesis of plane sections is valid for the entire thickness of the element of the rod. In general, we achieve the steadiness equation of the considered rod, and a formula is found for determining the critical load in the certain case.

Introduction

Single and three-layer rods are often used as bearing elements in many complex structures, operating in different loading modes. These structures are often placed on an nonlinear elastic foundation and uneven temperature field.

The surveys [1-3] investigated the steadiness of single and three-layer rods in normal temperature and under the impact of high temperature.

The current survey investigates the steadiness task of three-layer nonhomogeneous rods which are on an nonlinear elastic foundation and uneven temperature field.

The problem statement

We consider the steadiness of three-layer nonhomogeneous rods on an nonlinear elastic foundation, which has two symmetry axes of the cross section compressed at the ends with P forces in uneven heating.

The coordinate system has been chosen in the following way: OY and OZ axes are at the cross section of rod. OX axe is directed along the rod's axe

It is assumed that the temperature in each layer is a function of the thickness coordinate $(T_i = T_i(z))$.

The layers of the rod are made from a variety of nonhomogeneous materials, and the elasticity modules depend on the coordinates and temperature in the following way:

$$E_i = E_i\left(X, Z, T(z)\right) = E_i \cdot f_i(x)\,\varphi_i(z) \cdot \frac{T(z)}{T_0}, \quad (i = 0, 1, 2) \tag{1}$$

The connection between the icrements of tension and deformation will have the following state in a strained state of the rod:

$$\Delta\sigma' = E_{10} f_1(x)\,\varphi_1(z)\,T'(z)\,\Delta\varepsilon, \quad \left(-h_1 - \frac{h}{2} \le z \le -\frac{h}{2}\right)$$

$$\Delta\sigma = E_0 f(x)\,\varphi(z)\,T'(z)\,\Delta\varepsilon, \quad \left(-\frac{h}{2} \le z \le \frac{h}{2}\right) \tag{2}$$

$$\Delta\sigma^2 = E_{20} f_2(x)\,\varphi_2(z)\,T'(z)\,\Delta\varepsilon, \quad \left(\frac{h}{2} \le z \le \frac{h}{2} + h_2\right), \quad \left(T' = T/T_0\right).$$

Here $h_1, h \;\; h_2$ –define the appropriate layer thickness. Let's assume that the hypothesis of plain sections is valid for the entire thickness of the rod i.e.

$$\Delta\varepsilon = e_0 + z\text{æ} \tag{3}$$

Here e_0 – defines the additional deformation of the rod's axe, æ – curvature of the central line.

In this case, the force and time increment are identified by formulas:

$$\Delta P = \int_{-\frac{h}{2}-h_1}^{-h/2} \Delta\sigma^1 b(z)\,dz + \int_{-\frac{h}{2}}^{h/2} \Delta\sigma b(z)\,dz + \int_{h/2}^{\frac{h}{2}+h_2} \Delta\sigma^2 b(z)\,dz, \tag{4}$$

$$\Delta M = \int_{-\frac{h}{2}-h_1}^{-h/2} \Delta\sigma^1 z b(z)\,dz + \int_{-\frac{h}{2}}^{h/2} \Delta\sigma z b(z)\,dz + \int_{h/2}^{\frac{h}{2}+h_2} \Delta\sigma^2 z b(z)\,dz,$$

Where $b(z)$- the cross-section width of the rod.

By taking (1)-(3) into account from (4) we can get:

$$\Delta P = E_0 f(x)\left[e\left(e_{10}\gamma_1 S_1^0 + S^0 + e_{20}\gamma_2 S_2^0\right) + \text{æ}\left(e_{10}\gamma_1 S_1^1 + S^1 + e_{20}\gamma_2 S_2^1\right)\right], \tag{5}$$

$$\Delta M = E_0 f(x)\left[e\left(e_{10}\gamma_1 S_1^1 + S^1 + e_{20}\gamma_2 S_2^1\right) + \text{æ}\left(e_{10}\gamma_1 S_1^2 + S^2 + e_{20}\gamma_2 S_2^2\right)\right], \tag{6}$$

The following definitions have been introduced in these formulas:

$$S_1^i = \int_{-\frac{h}{2}-h_1}^{-h/2} \varphi_1(z)\,T'(z)\,b(z)\,z^i dz,$$

$$S^i = \int_{-h/2}^{h/2} \varphi(z)\,T'(z)\,b(z)\,z^i dz, \tag{7}$$

$$S_2^i = \int_{h/2}^{h/2+h_2} \varphi_2(z)\,T'(z)\,b(z)\,z^i dz$$

$$e_{10} = \frac{E_{10}}{E_0}, \quad e_{20} = \frac{E_{20}}{E_0}, \quad \gamma_1 = \frac{f_1(x)}{f(x)}, \quad \gamma_2 = \frac{f_2(x)}{f(x)}.$$

The generation of the steadiness equation

The equilibrium equation of the considered rod is as follows:

$$\Delta P = 0$$

$$\frac{d^2}{dx^2}(\Delta M) + P\frac{d^2 w}{dx^2} + C_0 W + C_1 W^3 = 0 \tag{8}$$

Here $C0, C1$ - coefficient of nonlinear elastic foundation, w – deflection the rod.

Considering (5) from the first equation of the system (8) we'll get:

$$e = -\frac{e_{10}\gamma_1 S_1^1 + S^1 + e_{20}\gamma_2 S_2^1}{e_{10}\gamma_1 S_1^0 + S^0 + e_{20}\gamma_2 S_2^0}æ. \tag{9}$$

Considering (9) from (6) for incrementing the moment we get the following expression:

$$\Delta M = KIæ \cdot E_0 f(x) \tag{10}$$

where it is defined that:

$$KI = \left(e_{10}\gamma_1 S_1^2 + S^2 + e_{20}\gamma_2 S_2^2\right) - \frac{\left(e_{10}\gamma_1 S_1^1 + S^1 + e_{20}\gamma_2 S_2^1\right)^2}{\left(e_{10}\gamma_1 S_1^0 + S^0 + e_{20}\gamma_2 S_2^0\right)}. \tag{11}$$

Taking into consideration that $æ = \frac{d^2w}{dx^2}$, from the second equation of the system (8) including (10) we get:

$$\frac{d^4w}{dx^4} + r^2\frac{d^2w}{dx^2} + S_0^2 W + S_1^2 W^3 = 0 \tag{12}$$

$$r^2 = \frac{P}{KI}, \quad S_0^2 = \frac{C_0}{KI}, \quad S_1^2 = \frac{C_1}{KI} \tag{13}$$

Thus, The steadiness equation of the considered rod has become as (12).

The problem solution

In order to solve certain tasks it is necessary to set the dependence of the elasticity modules of the coordinates.

Let's consider the case when the elasticity modules do not depend on the length coordinates (i.e.$f_1(x) = f(x) = f_2(x) = 1$)

We consider the pinning of the rod ends. In this case, we can take the following expression for the deflection:

$$w = w_0 \sin\frac{\pi x}{e} \tag{14}$$

By substituting (14) into (13) to determine the critical load we obtain:

$$P_{cr} = \left(\frac{\pi}{l}\right)^2 KI + \left(\frac{l}{\pi}\right)^2 \left(C_0 + \frac{3}{4}C_1 W_0^2\right). \tag{15}$$

Here KI is determined based on the formula (11).

In order to obtain high quality results for the heterogeneity and temperature functions, we adopt the following expressions:

$$\varphi_1(z) = 1 + \mu_1\frac{z}{h_1}, \quad \varphi(z) = 1 + \mu\frac{z}{h},$$

$$\varphi_2(z) = 1 + \mu_2\frac{z}{h_2}, \quad T(z) = T_0 + T_1\frac{z}{h}. \tag{16}$$

By substituting (16) into (7) after several conversions we'll get:

$$S_1^0 = bh_1\left[1 - \frac{\beta_1}{2}h(1+\delta_1) + \frac{\beta_1^1 h_1}{3}h\left(\frac{3}{4}\frac{1}{\delta_1} + \frac{3}{2} + \delta_1\right)\right],$$

$$S_1^1 = bh_1^2 \left[\frac{\beta_1^1}{4} \cdot \frac{h^2}{16\delta_1^2} - \frac{1}{2\delta_1} - \frac{1}{2} + \right.$$

$$\left. + \frac{B_1 h}{3} \left(\frac{3}{4\delta_1} - \frac{3}{2} + \delta_1 \right) - \frac{\beta_1^1 h_1^2}{4} \left(1 + \frac{1}{2\delta_1} \right)^4 \right],$$

$$S_1^2 = bh_1^3 \left[-\frac{1}{8\delta_1^3} + \frac{B_1}{4} \cdot \frac{h}{16\delta_1^3} - \frac{B_1^1}{5} \cdot \frac{h_2}{32\delta_1^3} + \right.$$

$$\left. + \frac{1}{3} \left(1 + \frac{1}{2\delta_1} \right)^3 - \frac{B_1 h_1}{4} \left(1 + \frac{1}{2\delta_1} \right)^4 + \frac{B_1^1 h_1^2}{5} \left(1 + \frac{1}{2\delta_1} \right)^5 \right],$$

$$S^0 = bh \left[1 + \frac{\beta^1 h^2}{12} \right], \quad S^1 = bh^2 \left[\frac{\beta h}{3} \right], \tag{17}$$

$$S^2 = bh^3 \left[\frac{1}{12} + \frac{\beta^1}{5} \cdot \frac{h^2}{16} \right],$$

$$S_2^0 = bh_2 \left[1 + \frac{\beta_2}{2} h + \frac{\beta_2}{2} h_2 + \frac{B_2^1 h_2^2}{3} \left(1 + \frac{1}{2\delta_2} \right)^2 - \frac{B_2^1 h^2}{3} \cdot \frac{1}{8\delta_2} \right],$$

$$S_2^1 = bh_2^2 \left[\frac{1}{2} \left(\frac{1}{2\delta_2} + 1 \right)^2 + \frac{B_2 h_2}{3} \left(\frac{1}{2\delta_2} + 1 \right)^3 + \right.$$

$$\left. + \frac{\beta_2^1 h_2^2}{4} \left(\frac{1}{2\delta_2} + 1 \right)^4 - \frac{1}{8\delta_2^2} - \frac{\beta_2}{3} h \cdot \frac{1}{8\delta_2^2} - \frac{\beta_2^1 h^2}{4} \cdot \frac{1}{16\delta_2^2} \right],$$

$$S_2^2 = bh_2^3 \left[\frac{1}{3} \left(\frac{1}{2\delta_2} + 1 \right)^3 + \frac{B_2 h_2}{4} \left(\frac{1}{2\delta_2} + 1 \right)^4 + \right.$$

$$\left. + \frac{\beta_2^1 h_2^2}{5} \left(\frac{1}{2\delta_2} + 1 \right)^5 - \frac{1}{3} \frac{1}{8\delta_2^3} - \frac{\beta_2}{4} h \cdot \frac{1}{16\delta_2^3} - \frac{\beta_2^1 h^2}{5} \cdot \frac{1}{32\delta_2^3} \right].$$

$$\beta_1 = \frac{\mu_1}{h_1} + \frac{T_1^1}{h}, \quad \beta = \frac{\mu}{h} + \frac{T_1^1}{h}, \quad \beta_2 = \frac{\mu_2}{h_2} + \frac{T_1^1}{h},$$

$$\beta_1^1 = \frac{\mu_1 T_1}{h_1 h}, \quad \beta^1 = \frac{\mu T_1^1}{h^2}, \quad \beta_2^1 = \frac{\mu_2 T_1^1}{h_1 h},$$

$$\delta_1 = \frac{h_1}{h}, \quad \delta_2 = \frac{h_2}{h}.$$

The generalized stiffness of the rod and the critical load based on (15) are determined by substituting the expression (17) into (11).

The numerical calculations were conducted for various parameter values and the results are presented in Fig. 1. Here, the dotted line indicates the solution of a similar task at a constant temperature.

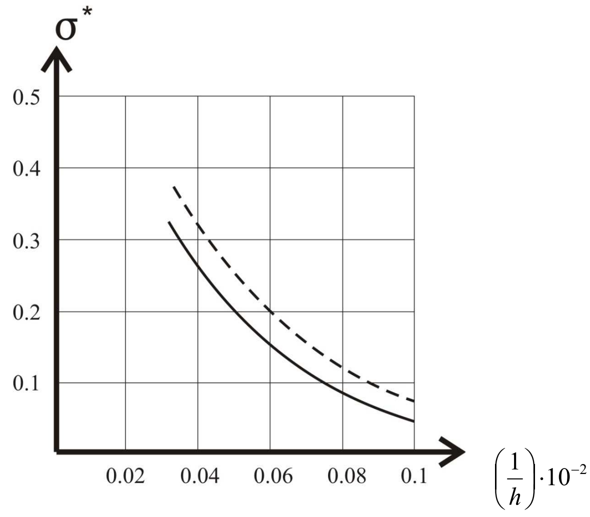

Fig.1.

$$E_{10} = E_{20} = 2 \cdot 10^6 \frac{kg}{sm^2}; \quad E_0 = 1.8 \cdot 10^6 \frac{kg}{sm^2}; \quad \delta_1 = \delta_2 = 0.2;$$

$$h/b = 0.5; \quad \mu\_1 = \mu\_2 = 1.5; \quad \mu = 1;$$

$$T_0 = 300K; \quad T_1' = 400K; \quad T_1^2 = 400K; \quad T_2^2 = 700K; \quad T_3' = 700K$$

$$\sigma^* = \frac{P}{E_0 b h_1}; \quad \frac{c_0}{\pi^2 E_0} = 0,001; \quad \frac{c_1}{\pi^2 E_0 h^2} = 0,0005; \quad \frac{w_0}{h} = 0,1.$$

References

[1]. Volmir A. S. *Steadiness of deformable systems.* M. Nauka, 1967, 984 p.

[2]. Lomakin V.A. *The elasticity theory of heterogeneous bodies.* M. Izd-vo MSU Press, 1976, 320 p.

[3]. Shapovalov L.A. *Effect of uneven heating on the steadiness of the compressed core.* PMM, 1957, vol. XXII, B.1., pp.119-123.

[4]. Yang Y.B., Lin T.J., Len L.I. *Thermal effect on the Postbuckling Behavior of an elastic or elasto-plastic truss.* Journal of Mechanics, 2008, vol.134, No 4, pp. 330-338.

[5]. Amin Heydarpour and Mark Andrew Bradford. *Nonlinear Analysis of Composite Beams with Partial Interaction in steel Frame Structures at Elevated Temperature.* Journal of Structural Engineering. 2010, vol.136, pp. 968-978.

[6]. Hosseini M., Fazelzadeh S.A. *Thermomechanical stability analysis of functionally graded thin-walled cantilever pipe with fluid subjected to axial load.* International Journal of Structural Stability and Dynamics.2011, vol. 11, No 3, pp.513-534.

[7]. İsayev F.Q., Mamedli R.E. *Stability of three-layer non-homegeneous rods subject to thermo-mechanical loads.* 2011, No 32, pp. 35-39.

[8]. Voshoughi A.R.Malekzadeh P.Banan Mo.R.*Thermal postbuckling of laminated composite skev plates with temperature-dependent properties.* J.Thin Walled structuresv.2011, vol. 47, No7, pp.804-811.

[9]. Thuz P.Vo, Huu-Tai Thai. *Vibration and buckling of composite beams using refined shear deformation theory.* International Journal of Mechanical Sciences. 2012, vol. 62, No1, pp. 67-76.

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