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MECHANICS

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ON NON-HOMOGENEOUS UNDERGROUND PIPELINES STABILITY

Abstract

The paper studies the non-homogeneous pipeline stability with regard to resistance of Pasternak type foundation. The problem is solved by the approximate analytic method for the specific cases.

As is known, the main gas-oil pipelines are complex engineering constructions used in different environmental conditions. The underground gaskets of pipelines, peculiarities of civil-engineering practice and constructive decisions create probability wide spectrum of strength and durability parameters of pipelines sections.

One of the important factors is the account of non-homogeneity of the pipe material, welding stresses and other possible deficiencies. Account of environment resistance that essentially depends on nature environmental conditions [1,3] is also very important.

Note that the stability and strength issues with regard to above-mentioned specific peculiarities essentially complicate mathematical realization of the stated problems, and their ignorance leads to great errors.

In the present paper we investigate one of the variants of stability loss analysis along the length of the pipe with regard to Pasternak's two-constant elastic resistance [2].

Note that the stability loss of a pipeline under the action of longitudinal forces is accompanied by its wavy distortion (fig. 1 a,b).

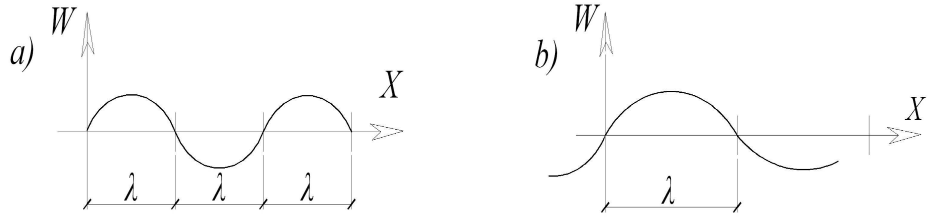

Fig. 1.

Within each wave or along the waves, the pipe's section moves in the lateral direction (fig. 1a,b), has force influence upon the soil and is subjected to reactive impedance that defines the quantity of critical force. The soil reaction essentially depends on environmental conditions and this is mathematically modelled by different relations: hydrostatic (Winkler's), two-constant (Pasternak), inhomogeneous (Fuss-Winkler) and inelastic.

At present, by constructing pipelines of different purpose, the pipes with non-homogeneous properties (for instance, fiber-glass plastic, reinforced composites and etc.) are widely used. This not least of the factors should be taken into account at stability and strength analysis.

Non-homogeneity in a pipe may happen as a result of technological process of making, non-homogeneity of compositions, at welding and etc [2].

The goal of the paper is to solve the stability problem with regard to continuous non-homogeneity along the pipe's length and soil resistance in the lateral direction on the shear (Pasternak).

Let the soil's resistance to the lateral displacement of pipes be characterized by two compression bed coefficients k_0 and k, to longitudinal by the shear bed coefficient k_1 [2]

$$q = k_0 w - k_1 \frac{d^2 w}{dx^2}, \tag{1}$$

where w is flexure.

The modulus of elasticity of the pipe's material is a length coordinate function

$$E = E_0 f(x), \tag{2}$$

where E_0 corresponds to the homogeneous case, the function $f(x)$ with its derivatives to the second order is a continuous function.

We can show that the stability equation of a pipe in an elastic medium, allowing for (1) and (2) may be written in the form

$$E_0 J_0 \frac{d^2 w}{dx^2}(f(x)\frac{d^2 w}{dx^2}) + (P - 2A)\frac{d^2 w}{dx^2} k_0 w = 0, \tag{3}$$

where $E_0 J_0$ is a homogeneous pipe rigidity, P is a longitudinal compressible force, d is pipeline's diameter.

Equation (3) may be represented in the form:

$$E_0 J_0 (f(x)\frac{d^4 w}{dx^4}) + 2\frac{df}{dx}\frac{d^2 w}{dx^2} + \frac{d^2 f}{dx^2}\frac{d^2 w}{dx^2} + (P - 2A)\frac{d^2 w}{dx^2} + Bw = 0. \tag{4}$$

Here we accept the following denotation:

$$A = k_1 d\left(1 + \frac{1}{\beta d}\right); B = k_0 d\left(1 + \frac{2}{\beta d}\right); \beta = \sqrt{\frac{k_0}{2k_1}}.$$

As it is seen, equation (4) is complex and for arbitrary value of the function $f(x)$ it is difficult or impossible to find the exact solution.

Therefore, by solving (3) we'll use the Bubnov-Galerkin approved and effective orthogonalization method, and look for the deflection function in the following form:

$$W(x) = \sum_{i=1}^{n} a_i v_i(x),$$

where a_i are unknown constants, and each $v_i(x)$ should satisfy the following boundary conditions.

Allowing for (3) and (4) the error function has the following form:

$$\eta_i(x) = \sum_{i=1}^{n} a_i \left[J_0 E_0 \left(f(x) \frac{d^4 v_i}{dx^4} + 2 \frac{df}{dx} \frac{d^3 v_i}{dx^3} + \frac{d^2 f}{dx^2} \frac{d^2 v_i}{dx^2} \right) \right] +$$

$$+ (P - 2A) \frac{d^2 v_i}{dx^2} + B v_i \neq 0. \qquad (6)$$

Based around the orthogonalization method we get:

$$\int_0^{\lambda} \eta_i(x) v_q(x) dx = 0, (q = 1, 2, ...) . \qquad (7)$$

Removing necessary integrals from (7), we get a system of linear homogeneous algebraic equations with respect to the constants $a_1, a_2, ...$. For the existence of non-trivial solutions, the principal determinant of the given system should become zero:

$$\| P \| = 0.$$

However, for engineering analysis, it suffices to be restricted with the first approximation, though for an arbitrary approximation this doesn't give rise to special difficulty

$$\int_0^{\lambda} \eta_1(x) v_1(x) dx = 0. \qquad (8)$$

For calculation we should give the function $f(x)$ and the form $v_1(x)$. Let

$$v_1 = f \sin \frac{\pi x}{\lambda}; f(x) = 1 + \varepsilon \bar{x}; \bar{x} = x \cdot l^{-1}; \quad \overline{\lambda} = \lambda \cdot l^{-1}; \quad \varepsilon \in [0, 1]. \qquad (9)$$

Substituting (9) in (8) and making elementary transformations, we get

$$\int_0^{2} [(1 + \varepsilon \bar{x}) \left(\frac{\pi}{\lambda} \right)^4 \sin^2 \pi \bar{x} - 2\lambda^{-}1 \left(\frac{\pi}{\lambda} \right)^3 \sin 2\pi \bar{x}]$$

$$-[(P - 2A) \sin^2 \pi \bar{x} + B \sin^2 \pi \bar{x}] dx = 0. \qquad (10)$$

Hence we find

$$J_0 E_0 (1 + 0.5\varepsilon) \left(\frac{\pi}{\lambda} \right)^4 - \left(\frac{\pi}{\lambda} \right)^2 (P - 2A) + B = 0.$$

Differentiating (10) with respect to λ and accepting $\frac{dP}{d\lambda} = 0$, we find the value of λ that corresponds to the minimal value of P, under which the stability loss of a non-homogeneous pipeline is possible with regard to two-constant resistance of Pasternak type.

$$\lambda = \sqrt[4]{\frac{B}{E_0 J_0 (1 + 0.5\varepsilon)}}. \qquad (11)$$

Hence for $\varepsilon = 0$ we get the solution of the homogeneous problem for a homogeneous pipeline

$$\lambda_0 = \sqrt[4]{\frac{B}{E_0 J_0}}. \tag{12}$$

Denoting $\bar{\lambda} = \frac{\lambda_0}{\lambda}$, we get

$$\bar{\lambda} = \sqrt[4]{\frac{1}{(1+0.5\varepsilon)}}. \tag{13}$$

Note that for other:

$$f(x) = e^{\mu(\bar{\bar{x}}-1)};\, f(x) = 1 + c_1\, \bar{\bar{x}} + c_2 \bar{\bar{x}}^2 \tag{14}$$

and etc. it is not difficult to get formula (13). In the present case, as it was mentioned above, the numerical analysis was conducted for the case $f(x) = 1 + \varepsilon \bar{x}$.

The results of calculations are given in fig. 2 and in the table.

E	$\bar{\lambda}^2$
0	1
0.2	0,969
0.4	0,94
0.6	0,903
0.8	0,813
1	0,81

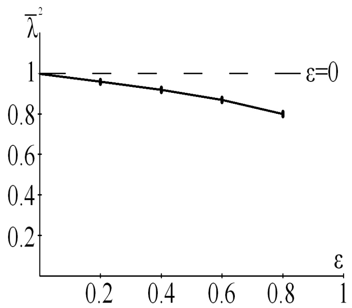

Fig.2.

As it is seen from the figure, the account of non-homogeneity essentially influences on the quantity of typical parameters.

References

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