MECHANICS

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NONLINEAR PARAMETRIC VIBRATIONS OF A RIDGE CYLINDRICAL SHELL DYNAMICALLY CONTACTING WITH MEDIUM

Abstract

In the paper, a problem on parametric vibration of a laterally stiffened cylindrical shell dynamically contacting with external viscoelastic medium and situated under the action of internal pressure is solved in a geometric nonlinear statement by means of the variation principle. Lateral shift of the shell is taken into account. Influences of environment have been taken into account by means of the Pasternak model. Dependencies of dynamical stability area on the construction parameters are given on the plane "load-frequency".

Introduction. A great number of works including [1-4] have been devoted to the solution in linear statement of the problem on parametric vibrations of a cylindrical shell under the action of axial harmonic load. Recently there is a great interest to nonlinear problem on parametric vibrations of thin-walled constructions, since the vibration process may be accompanied by their click. A few number of papers have been devoted to investigation of nonlinear parametric vibrations of shells [5-10].

The monograph [11] deals with nonlinear deformation of cylindrical shells under the action of different kind dynamical loads.

The results of investigations on parametric excitable vibrations of a compressed liquid or gas-filled cylindrical shells are cited in [12].

The mongraph [13] deals with the results on investigation of nonstationary interaction of weak shock waves with structiral elements in compressible liquid and elastic waves with bodies in an elastic medium.

The monographs [14-16] have been devoted to the investigation of stability vibration and optimization of ridge cylindrical shells.

The results of experimental investigation of the influence of strengthening ribs and adjoined solids on the frequency and form of free vibrations of thin elastic structurally infomogeneous shells are cited in [17].

Nonlinear vibrations of a strengthened visco-elastic medium-contacting cylindrical shell are investigated in [18-24] in geometrical nonlinear statement by using variational principle.

In the paper, a problem on parametric vibration of a laterally stiffened cylindrical shell dynamically contacting with external viscoelastic medium and situated under the action of internal pressure is solved in a geometric nonlinear statement by means of the variation principle. Lateral shift of the shell is taken into account. Influences of environment have been taken into account by means of the Pasternak model.

Dependencies of dynamical stability area on the construction parameters are given on the plane "load-frequency".

Note that similar problem regardless of lateral shift of the shell was investigated in [21].

Problem statement. On the base of Ostrogradsky-Hamilton variational principle we get differential motion equations and natural boundary conditions for laterally stiffened medium-contacting cylindrical shell with regard to lateral shift.

For applying the mentioned principle, we beforehand write the potential and kinetic energy of the system.

The potential energy of elastic deformation of a cylindrical shell is of the form [25]:

$$\Pi_0=\frac{1}{2}\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\left\{N_x\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)+N_y\left(\frac{\partial\vartheta}{\partial y}-\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)+\right.$$

$$+N_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial\vartheta}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)+M_x\frac{\partial\psi_x}{\partial x}+M_y\frac{\partial\psi_y}{\partial y}+M_{xy}\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right)+$$

$$\left.+Q_x\left(\frac{\partial w}{\partial x}+\psi_x\right)+Q_y\left(\frac{\partial w}{\partial y}+\psi_y\right)\right\}dxdy. \tag{1}$$

The expressions for potential energy of elastic deformation of the j-th lateral rib are the followings [14]:

$$\Pi_j=\frac{1}{2}\int\limits_{y_1}^{y_2}\left[E_jF_j\left(\frac{\partial\vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+E_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{w_j}{R^2}\right)^2+\right.$$

$$\left.+E_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y^2}-\frac{\varphi_{kpj}}{R}\right)^2+G_jJ_{kpj}\left(\frac{\partial\varphi_{kpj}}{\partial y}+\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]dy. \tag{2}$$

In expressions (1) and (2), x_1,x_2,y_1,y_2 are the coordinates of curvilinear and linear edges of the shell; $F_j,J_{zj},J_{xj},J_{kpj}$ are square and inertia moments of the cross-section of the j-th lateral bar with respect to the axis Oz, and the axes parallel to the axis Ox and passing through the gravity center of cross section, and also its torsional moment of inertia; E_j,G_j are elasticity and shift modules of the material of the j-th lateral bar.

The potential energy of the shell under the action of external surface and boundary loads applied to the shell is determined as a work done by these loads while shifting the system from deformed state to initial undeformed one, and is represented in the form

$$A_0=-\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\left(q_xu+q_y\vartheta+q_zw\right)dxdy-\int\limits_{y_1}^{y_2}\left(T_1u+S_1\vartheta+Q_1w+M_1\varphi_1\right)\Big|_{x=x_1}^{x=x_2}dy-$$

$$-\int\limits_{x_1}^{x_2}\left(S_2u+T_2\vartheta+Q_2w+M_2\varphi_2\right)\Bigg|_{y=y_1}^{y=y_2}dx. \tag{3}$$

The potential energies of external boundary loads applied to the end-faces of the j-th lateral bar, are determined similarlu by the following expressions (it is accepted that only boundary loads are applied to the ribs):

$$A_j=-\left(S_ju_j+T_j\vartheta_j+Q_jw_j+M_j\varphi_j+M_{1j}\varphi_{zj}+M_{kpj}\varphi_{kpj}\right)\Big|_{y=y_1}^{y=y_2} \tag{4}$$

The total potential energy of the system is equal to the sum of potential energy of elastic deformations of the shell and ribs, and also potential energies of all external loads:

$$\Pi=\Pi_0+\sum_{j=1}^{k_2}\Pi_j+A_0+\sum_{j=1}^{k_2}A_j. \tag{5}$$

Kinetic energies of the shell and ribs are written in the form:

$$K_0=\frac{Eh}{2(1-\nu^2)}\int\limits_0^{\zeta_1}\int\limits_0^{2\pi}\left[\left(\frac{\partial u}{\partial t_1}\right)^2+\left(\frac{\partial\vartheta}{\partial t_1}\right)^2+\right.$$

$$\left.+\left(\frac{\partial w}{\partial t_1}\right)^2+\frac{h^2}{12R^2}\left(\frac{\partial\psi_x}{\partial t_1}\right)^2+\frac{h^2}{12R^2}\left(\frac{\partial\psi_y}{\partial t_1}\right)^2\right]d\zeta d\theta, \tag{6}$$

$$K_j=\rho_jF_j\int\limits_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial\vartheta_j}{\partial t}\right)^2+\left(\frac{\partial w_1}{\partial t}\right)^2+\frac{J_{kpj}}{F_j}\left(\frac{\partial\varphi_{kpj}}{\partial t}\right)^2\right]dy, \tag{7}$$

where t is a temporary coordinate, $t_1=\omega_0t$, $\omega_0=\sqrt{\dfrac{E}{(1-\nu^2)\,\rho_0R^2}}$, ρ_0,ρ_j are densities of materials from which the shell and the j-th lateral bar were made.

The kinetic energy of the ridge shell is determined as follows:

$$K=K_0+\sum_{j=1}^{k_2}K_j \tag{8}$$

Intensity of the load acting on the shell as viewed from the visco-elastic filler, may be written in the following form:

$$q_z=k_cw-\int\limits_{-\infty}^{t}\Gamma\left(t-\tau\right)w\left(\tau\right)d\tau \tag{9}$$

where Γ is a relaxation kernel, the coefficient k_c is determined by the dependence $k_c=q_1+q_0\nabla^2$ (Pasternak's model) where ∇^2 is Laplace's two-dimensional operator on the contact surface, w is the shell's deflection, $q_0,q_1.$ are the constants.

The motion equations of a medium-contactiong ridge stiffened shell are obtained on the base of Ostrogradsky-Hamilton action stationarity principle

$$\delta W = 0 \tag{10}$$

where $W = \int\limits_{t'}^{t''} L dt$ is Hamilton's action $L = K - \Pi$ is a Lagrange function, t' and t'' are the given arbitrary times.

Taking into account that steadiness of radial deflections on the cross-sectional heights and also equality of corresponding twist angles following form the condition of rigid connection of ribs with a shell hold, we write the following relations:

$$u_j(y) = u(x_j, y) + h_j \varphi_1(x_j, y);$$

$$\vartheta_j(y) = \vartheta(x_j, y) + h_j \varphi_2(x_j, y);$$

$$w_j(y) = w(x_j, y);$$

$$\varphi_j(y) = \varphi_2(x_j, y);$$

$$\varphi_{kpj}(y) = \varphi_1(x_j, y), \tag{11}$$

where $h_j = 0,5h + H_j^1$, h is the shell's thickness; H_j^1 is the distance from axes of the j-th lateral bar to the shell's surface; (x_j, y) are the coordinates of conjugation lines of ribs and shell; φ_j, φ_{kpj} are the turning and twisting angles of lateral cross sections of annular ribs.

Allowing for relations (11), we express the displacement of bars by the displacements of the shell. From the stationarity condition (10) we get a system of nonlinear algebraic equations for the desired unknowns.

Problem solution. Consider nonlinear parametric vibrations of a laterally stiffened annular cyindrical shell with regard to lateral shift and under the action of radial load $q = q_0 + q_1 \sin \omega_1 t$, where q_0 is the average or main load. q_1 is the modification amplitude of the load, ω_1 is the frequency pressure modification of a visco-elastic medium-filled shell. Assuming that the edges of the shell are hingely-supported, i.e. for $x = 0;\ l$

$$N_x = 0;\ M_x = 0;\ w = 0;\ \vartheta = 0;\ \psi_y. \tag{12}$$

We approximate the unknown quantities as follows:

$$u = \cos\frac{\pi x}{l} \sin(m\varphi)\left(u_0 \cos\omega t + u_1 \sin\omega t\right),$$

$$\vartheta = \sin\frac{\pi x}{l} \cos(m\varphi)\left(\vartheta_0 \cos\omega t + \vartheta_1 \sin\omega t\right),$$

$$w=\sin\frac{\pi x}{l}\sin(m\varphi)\left(w_0\cos\omega t+w_1\sin\omega t\right),$$

$$N_x=\sin\frac{\pi x}{l}\sin(m\varphi)\left(N_{x0}\cos\omega t+N_{x1}\sin\omega t\right),$$

$$N_y=\cos\frac{\pi x}{l}\cos(m\varphi)\left(N_{y0}\cos\omega t+N_{y1}\sin\omega t\right), \tag{13}$$

$$N_{xy}=-qR+\sin\frac{\pi x}{l}\sin(m\varphi)\left(N_{xy0}\cos\omega t+N_{xy1}\sin\omega t\right),$$

$$M_x=\sin\frac{\pi x}{l}\sin(m\varphi)\left(M_{x0}\cos\omega t+M_{x1}\sin\omega t\right),$$

$$M_y=\sin\frac{\pi x}{l}\sin(m\varphi)\left(M_{y0}\cos\omega t+M_{y1}\sin\omega t\right),$$

$$M_{xy}=\sin\frac{\pi x}{l}\cos(m\varphi)\left(M_{xy0}\cos\omega t+M_{xy1}\sin\omega t\right),$$

$$\psi_x=\sin\frac{\pi x}{l}\sin(m\varphi)\left(\psi_{x0}\cos\omega t+\psi_{x1}\sin\omega t\right),$$

$$\psi_y=\sin\frac{\pi x}{l}\sin(m\varphi)\left(\psi_{y0}\cos\omega t+\psi_{y1}\sin\omega t\right),$$

$$Q_x=\sin\frac{\pi x}{l}\sin(m\varphi)\left(Q_{x0}\cos\omega t+Q_{x1}\sin\omega t\right),$$

$$Q_y=\sin\frac{\pi x}{l}\sin(m\varphi)\left(\psi_{y0}\cos\omega t+\psi_{y1}\sin\omega t\right),$$

where m is the number of waves in peripheral direction, ω is the frequency of vibrations of desired quantities: $u,\vartheta,w,N_x,N_y,N_{xy},M_x,M_y,M_{xy},\psi_x,\psi_y,Q_x,Q_y$. Substitute approximation (13) in functional L and taking into account $x_1=0,\ x_2=l,\ y_1=0,\ y_2=2\pi,\ t'=0,\ t''=\frac{\pi}{\omega}$ integrate with respect to x,y and t. Then, instead of function (5) we get a function of desired quantities $u_m,\vartheta_m,w_m,N_{xm},\ N_{ym},$ $N_{xym},M_{xm},M_{ym},M_{xym},\psi_{xm},\psi_{ym},Q_{xm},Q_{ym}$. The stationary value of the obtained function is determined by the following system:

$$1)\frac{\partial J}{\partial u_0}=0;\ 2)\frac{\partial J}{\partial u_1}=0;\ 3)\frac{\partial J}{\partial \vartheta_0}=0;\ 4)\frac{\partial J}{\partial \vartheta_1}=0;\ 5)\frac{\partial J}{\partial w_0}=0;$$

$$6)\frac{\partial J}{\partial w_1}=0;\ 7)\frac{\partial J}{\partial N_{x0}}=0;\ 8)\frac{\partial J}{\partial N_{x1}}=0;\ 9)\frac{\partial J}{\partial N_{y0}}=0;\ 10)\frac{\partial J}{\partial N_{y1}}=0;$$

$$11)\frac{\partial J}{\partial N_{xy0}}=0;\ 12)\frac{\partial J}{\partial N_{xy1}}=0;\ 13)\frac{\partial J}{\partial M_{x0}}=0;\ 14)\frac{\partial J}{\partial M_{x1}}=0;$$

$$15)\frac{\partial J}{\partial M_{y0}}=0;\ 16)\frac{\partial J}{\partial M_{y1}}=0;\ 17)\frac{\partial J}{\partial M_{xy0}}=0;\ 18)\frac{\partial J}{\partial M_{xy1}}=0; \tag{14}$$

$$19)\frac{\partial J}{\partial \psi_{x0}}=0;\ 20)\frac{\partial J}{\partial \psi_{x1}}=0;\ 21)\frac{\partial J}{\partial \psi_{y0}}=0;\ 22)\frac{\partial J}{\partial \psi_{y1}}=0;$$

$$23)\frac{\partial J}{\partial Q_{x0}}=0;\ 24)\frac{\partial J}{\partial Q_{x1}}=0;\ 25)\frac{\partial J}{\partial Q_{y0}}=0;\ 22)\frac{\partial J}{\partial Q_{y1}}=0.$$

Numerical analysis of vibrations. Nonlinear system of equations (14) was solved by the Newton method under the following input data:

$$E = E_j = 6,67 \cdot 10^9 \frac{n}{m^2}; \quad \nu = 0,3; \quad h = 0,45\ mm; \ R = 160\ mm; \ l = 800\ mm;$$

$$\rho_0 = 7,8\ g/sm^3; \quad \frac{q}{q_0} = 3; \quad \frac{q}{E} = 0,002; \quad \Gamma(t) = Ae^{-\Psi t} \left(\Psi = 0,05; \ A = 0,1615\right);$$

$$k_2 = 4; \ m = 8; \ h_j = 1,39\,mm; \ F_j = 5,75\,mm^2; \ J_{xj} = 19,9\,mm^4; \ \frac{J_{zj}}{2\pi R^3 h} = 0,23 \cdot 10^{-6};$$

$$J_{kpj} = 0,48\ mm^4; \ \tau_0 = \frac{q_0}{Eh^3} = 0,08; \ \tau_1 = \frac{q_1}{Eh^3}$$

The dependence of the dynamical stability zone on the construction's parameters on the plane "load-frequency" represented in the form of a curve, are given in the fugure.

The curve divides the plane into two domains: for the points of one domain the vibrations are restricted, for another one they are resticted in time. The shaded lines correspond to vibrations of laterally stiffened cylindrical shell in visco-elastic medium, the solid one in an elastic medium. It is seen form she picture that for visco-elastic medium the change point of the typical curve rises over the frequency axis. According to the calculation results, the discount of lateral shift in the shell reduces to contraction of stable areas of the shell.

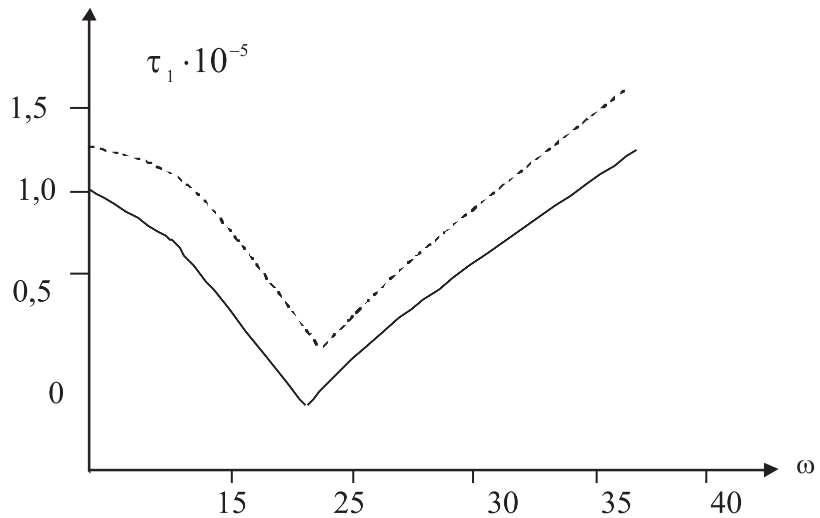

Fig. Domains of stable and unstable regions of parametric vibrations of a laterally stiffened shell.

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