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INFLUENCE OF KARLEMAN CONDITION BY INVESTIGATING BOUNDARY VALUE PROBLEMS FOR LAPLACE EQUATION

Abstract

The paper is devoled to the investigation of Fredholm property of boundary value problems for Laplace equation with nonlacal boundary conditions. Proceeding from non-locality of boundary conditions, at least two points move along the boundaries. If these two points move away from one point of the boundary, or they approach one point of the boundary, then the Karleman conditions are satisfied. If these two boundary points follow one another while moving, then the Karleman conditions don't hold.

Thus, in the first problem we obtain the Fredholm property of the stated boundary value problem since the Karleman conditions hold. In the second problem, in spite of the fact that the Karleman conditions don't hold, we again get the Fredholm property of the stated boundary value problems. For the third boundary value problem we show the existence of such a boundary condition for which the stated problem is not Fredholm if the Karleman conditions are not satisfied.

Introduction. When the Cauchy problem or a boundary value problem are considered, "k" conditions (initial or boundary) [2] are given for a k-th order ordinary differential equation.

If partial differential equations are considered, the number of initial conditions coincides with the highest order time derivative introduced into the equation under consideration, and the number of boundary conditions coincides with the half of the highest order spatial variable derivative (their number is at least two) contained in the equation of the problem under consideration [3]. Thus, with the help of nonlocal boundary conditions we eliminate the above-cited misunderstandings. In our case, the number of boundary conditions both for partial equations and ordinary differential equation will coincide with the highest order spatial variable derivative contained in the equation under consideration [4], [5].

The investigations are carried out as in [4], [5], i.e. proceeding from the fundamental solution we obtain Green's second formula and analogy of this formula from [4]-[6] that give us necessary conditions. These conditions contain singular integrals. After peculiar regularization of these singularities we get sufficient condtions on Fredholm property of the stated boundary value problems.

Problem statement. Let $D \subset R^2$ be converx in the direction of x_2, the boundary $\Gamma = \overline{D} \setminus D$ be a Lyapunov line. Consider the Laplace two-dimensional equation

$$\Delta u(x) \equiv \sum_{j=1}^{2} \frac{\partial^2 u(x)}{\partial x_j^2} = 0, \quad x \in D \qquad (1)$$

with boundary conditions

$$\left[\sum_{j=1}^{2}\alpha_{ij}\left(x_1\right)\frac{\partial u\left(x\right)}{\partial x_j}+\alpha_i\left(x_1\right)u\left(x\right)\right]\Bigg|_{x_2=\gamma_1(x_1)}+$$

$$+\left[\sum_{j=1}^{2}\beta_{ij}\left(x_1\right)\tfrac{\partial u(x)}{\partial x_j}+\beta_i\left(x_1\right)u\left(x\right)\right]\Bigg|_{x_2=\gamma_2(x_1)}=\varphi_i\left(x_1\right), \tag{2_1}$$

$$i=1,2,;\quad x_1\in\left[a_1,b_1\right]$$

or

$$\left[\sum_{j=1}^{2}\alpha_{ij}\left(x_1\right)\frac{\partial u\left(x\right)}{\partial x_j}+\alpha_i\left(x_1\right)u\left(x\right)\right]\Bigg|_{x_2=\gamma_1(x_1)}+$$

$$+\left[\sum_{j=1}^{2}\beta_{ij}\left(x_1\right)\tfrac{\partial u(\eta)}{\partial \eta_j}+\beta_i\left(x_1\right)u\left(\eta\right)\right]\Bigg|_{\substack{\eta_2=\gamma_2(\eta_1)\\ \eta_1=a_1+b_1-x_1}}=\varphi_i\left(x_1\right), \tag{2_2}$$

$$i=1,2,;\quad x_1\in\left[a_1,b_1\right]$$

or

$$\begin{cases}\alpha_1\left(x_1\right)u\left(x_1,\gamma_1\left(x_1\right)\right)+\alpha_2\left(x_1\right)\times\\ \times u\left(a_1+b_1-x_1,\gamma_2\left(a_1+b_1-x_1\right)\right)=\alpha\left(x_1\right),\\ \beta_1\left(x_1\right)\left[\dfrac{\partial u\left(x\right)}{\partial x_2}-i\dfrac{\partial u\left(x\right)}{\partial x_1}\right]\Bigg|_{x_2=\gamma_1(x_1)}+\\ +\beta_2\left(x_1\right)\left[\dfrac{\partial u\left(\eta\right)}{\partial \eta_2}-i\dfrac{\partial u\left(\eta\right)}{\partial \eta_1}\right]\Bigg|_{\substack{\eta_2=\gamma_2(\eta_1)\\ \eta_1=a_1+b_1-x_1}}=\beta\left(x_1\right),\quad x_1\in\left[a_1,b_1\right],\end{cases} \tag{2_3}$$

where $x_2=\gamma_k\left(x_1\right), k=1,2; x_1\in\left[a_1,b_1\right]$ are the equations of a part of the boundaries Γ denoted by $\Gamma_k,\ k=1,2$, obtained from orthogonal projection of domain D onto x_1 parallel to x_2.

Let the following restrictions hold.

1. The plane bounded domain D is convex in the direction of x_2, the boundary $\Gamma=\bar{D}\diagdown D$ is a Lyapunov line.

2. Boundary conditions (2_1) or (2_2) are independent, $\alpha_{ij}\left(x_1\right),\ \beta_{ij}\left(x_1\right)\ i,j=1,2;\ x_1\in\left[a_1,b_1\right]$ belong to some Holder class, $\alpha_i\left(x_1\right),\ \beta_i\left(x_1\right)\ i,j=1,2;\ x_1\in\left[a_1,b_1\right]$ are continuous functions, $\varphi_i\left(x_1\right),\ i=1,2;\ x_1\in\left[a_1,b_1\right]$ are continuously-differentiable functions satisfying the conditions $\varphi_i\left(a_1\right)=\varphi_i\left(b_1\right)=0,\ i=1,2.$

3. The coefficients of the boundary condition (2_3) $\beta_j\left(x_1\right),\ j=1,2;\ x_1\in\left[a_1,b_1\right]$ belong to some Holder class, $\beta\left(x_1\right)\ x_1\in\left[a_1,b_1\right]$ is a continuously differentiable function satisfying the condition $\beta\left(a_1\right)=\beta\left(b_1\right)=0$, $\alpha_j\left(x_1\right)\ j=1,2;\ x_1\in\left[a_1,b_1\right]$ and $\alpha\left(x_1\right)\ x_1\in\left[a_1,b_1\right]$ are continuous functions.

Main relations. As is known, the fundamental solution of Laplace two-dimensional equation is of the form [7]:

$$U\left(x-\xi\right)=\frac{1}{2\pi}\ln\left|x-\xi\right|=\frac{1}{2\pi}\ln\sqrt{\left(x_2-\xi_2\right)^2+\left(x_1-\xi_1\right)^2} \tag{3}$$

i.e. the following relation is valid

$$\Delta_x U(x-\xi) \equiv \sum_{j=1}^{2} \frac{\partial^2 U(x-\xi)}{\partial x_j^2} = \delta(x-\xi) = \delta(x_1-\xi_1)\,\delta(x_2-\xi_2) \tag{4}$$

Multiplying equation (1) by fundamental solution (3) and integrating the obtained expression in the domain D, applying the Ostrogradsky-Gauss formula [7], we get the green second formula

$$\int_{\Gamma} \left[u(x) \frac{dU(x-\xi)}{dv_x} - \frac{du(x)}{dv_x} U(x-\xi) \right] dx = \begin{cases} u(\xi), \ \xi \in D, \\ \frac{1}{2} u(\xi), \ \xi \in \Gamma, \\ 0, \ \ \xi \notin \overline{D}, \end{cases} \tag{5}$$

where v_x is the external normal to the boundary Γ at the point x.

Further, similar to [4]-[6] we get Green's second formula in the form:

$$\int_{\Gamma} \left[\frac{du(x)}{dv_x} \frac{\partial U(x-\xi)}{dx_1} - \frac{du(x)}{d\tau_x} \frac{\partial U(x-\xi)}{dx_2} \right] dx = \begin{cases} \frac{\partial u(\xi)}{\partial \xi_1}, \ \xi \in D, \\ \frac{1}{2} \frac{\partial u(\xi)}{\partial \xi_1}, \ \xi \in \Gamma, \\ 0, \ \ \xi \notin \overline{D}, \end{cases} \tag{6}$$

$$\int_{\Gamma} \left[\frac{du(x)}{dv_x} \frac{\partial U(x-\xi)}{dx_2} - \frac{du(x)}{d\tau_x} \frac{\partial U(x-\xi)}{dx_1} \right] dx = \begin{cases} \frac{\partial u(\xi)}{\partial \xi_2}, \ \xi \in D, \\ \frac{1}{2} \frac{\partial u(\xi)}{\partial \xi_2}, \ \xi \in \Gamma, \\ 0, \ \ \xi \notin \overline{D}, \end{cases} \tag{7}$$

where τ_x is a tangential direction to the boundary Γ at the point x.

This proves

Theorem 1. *Under condition 1, each solution of equation (1) determined in D satisfies main relations (5)-(7).*

Necessary conditions. Note that the second expressions of relation (5)-(7) are necessary conditions. These conditions are of the form:

$$u(\xi_1, \gamma_k(\xi_1)) = -\frac{1}{\pi} \sum_{\substack{j=1 \\ j \neq k}}^{2} (-1)^j \int_{a_1}^{b_1} u\left(x_1, \gamma_j(x_1)\right) \times$$

$$\times \frac{\gamma_j'(x_1)(x_1-\xi_1) - \left(\gamma_j(x_1) - \gamma_k(\xi_1)\right)}{(x_1-\xi_1)^2 + \left(\gamma_j(x_1) - \gamma_k(\xi_1)\right)^2} dx_1 -$$

$$-\frac{(-1)^k}{\pi} \int_{a_1}^{b_1} \frac{u(x_1, \gamma_k(x_1))}{1+\gamma_k'^2(\sigma_k)} \frac{\gamma_k'(x_1) - \gamma_k'(\sigma_k)}{x_1-\xi_1} dx_1 -$$

$$-\frac{1}{2\pi} \sum_{j=1}^{2} \int_{a_1}^{b_1} \ln \left[(x_1-\xi_1)^2 + \left(\gamma_j(x_1) - \gamma_k(\xi_1)\right)^2 \right] \times$$

$$\times \left.\frac{du(x)}{dv_{jx}}\right|_{x_2=\gamma_j(x_1)}\sqrt{1+\gamma_j'^2(x_1)}dx_1,\quad k=1,2;\ \xi_1\in(a_1,b_1) \tag{8}$$

$$\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_k(\xi_1)} = -\frac{1}{\pi}\sum_{\substack{j=1\\ j\neq k}}^{2}(-1)^j\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_j(x_1)}\times$$

$$\times\frac{\gamma_j'(x_1)(x_1-\xi_1)-\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)}{(x_1-\xi_1)^2+\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)^2}dx_1-$$

$$-\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_k(x_1)}\frac{\gamma_k'(x_1)-\gamma_k'(\sigma_k)}{x_1-\xi_1}\frac{dx_1}{1+\gamma_k'^2(\sigma_k)}+$$

$$+\frac{1}{\pi}\sum_{\substack{j=1\\ j\neq k}}^{2}(-1)^j\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_j(x_1)}\frac{(x_1-\xi_1)+\gamma_j'(x_1)\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)}{(x_1-\xi_1)^2+\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)^2}dx_1+$$

$$+\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_k(x_1)}\frac{\gamma_k'(x_1)-\gamma_k'(\sigma_k)}{x_1-\xi_1}\frac{\gamma_k'(\sigma_k)}{1+\gamma_k'^2(\sigma_k)}dx_1+$$

$$+\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_k(x_1)}\frac{dx_1}{x_1-\xi_1},\quad k=1,2;\quad \xi_1\in(a_1,b_1) \tag{9}$$

$$\left.\frac{\partial u(x)}{\partial \xi_2}\right|_{x_2=\gamma_k(x_1)} = -\frac{1}{\pi}\sum_{\substack{j=1\\ j\neq k}}^{2}(-1)^j\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_j(x_1)}\times$$

$$\times\frac{\gamma_j'(x_1)(x_1-\xi_1)-\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)}{(x_1-\xi_1)^2+\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)^2}dx_1-$$

$$-\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_k(x_1)}\frac{\gamma_k'(x_1)-\gamma_k'(\sigma_k)}{x_1-\xi_1}\frac{1}{1+\gamma_k'^2(\sigma_k)}dx_1-$$

$$-\frac{1}{\pi}\sum_{\substack{j=1\\ j\neq k}}^{2}(-1)^j\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_j(x_1)}\frac{(x_1-\xi_1)+\gamma_j'(x_1)\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)}{(x_1-\xi_1)^2+\left(\gamma_j(x_1)-\gamma_k(\xi_1)\right)^2}dx_1+$$

$$+\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_k(x_1)}\frac{\gamma_k'(x_1)-\gamma_k'(\sigma_k)}{x_1-\xi_1}\frac{\gamma_k'(\sigma_k)}{1+\gamma_k'^2(\sigma_k)}dx_1-$$

$$-\frac{(-1)^k}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_k(x_1)}\frac{dx_1}{x_1-\xi_1},\quad k=1,2;\quad \xi_1\in(a_1,b_1) \tag{10}$$

where $\sigma_k=\sigma_k(x_1,\xi_1)$ for $k=1,2$ is between x_1 and ξ_1.

Thus, we establish the statement.

Theorem 2. *Under condition 1, the boundary value of each solution of equation (1) determined in the domain D and boundary value of its derivatives satisfy necessary conditions (8)-(10).*

Regularization of singularities. From the obtained necessary conditions (9)-(10) we form the following linear combination:

$$\overline{\alpha}_{i1}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_1(\xi_1)}+\overline{\alpha}_{i2}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\xi_2=\gamma_1(\xi_1)}+$$

$$+\overline{\beta}_{i1}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_2(\xi_1)}+\overline{\beta}_{i2}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\xi_2=\gamma_2(\xi_1)}=$$

$$\overline{\alpha}_{i1}(\xi_1)\frac{-1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\xi_1}+\overline{\alpha}_{i2}(\xi_1)\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\xi_1}+$$

$$+\overline{\beta}_{i1}(\xi_1)\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}\frac{dx_1}{x_1-\xi_1}+\overline{\beta}_{i2}(\xi_1)\frac{-1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}\frac{dx_1}{x_1-\xi_1}+...=$$

$$=\frac{-1}{\pi}\int_{a_1}^{b_1}\left[\overline{\alpha}_{i2}(x_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}-\overline{\alpha}_{i1}(x_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}-\right.$$

$$\left.-\overline{\beta}_{i2}(x_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}+\overline{\beta}_{i1}(x_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_2(x_1)}\right]\frac{dx_1}{x_1-\xi_1}+...,$$

where the dots means the sum of nonsingular terms [4]-[6], the coefficients are arbitrary functions. Taking into account (2_1), we choose these coefficients in the following way:

$$\overline{\alpha}_{i2}(x_1)=\alpha_{i1}(x_1),\quad -\overline{\alpha}_{i1}(x_1)=\alpha_{i2}(x_1),$$
$$-\overline{\beta}_{i2}(x_1)=\beta_{i1}(x_1),\quad \overline{\beta}_{i1}(x_1)=\beta_{i2}(x_1).$$

Then, allowing for (2_1) the previous expressions will take the form:

$$-\overline{\alpha}_{i2}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_1(\xi_1)}+\alpha_{i1}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\xi_2=\gamma_1(\xi_1)}+$$

$$+\beta_{i2}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_2(\xi_1)}-\beta_{i1}(\xi_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\xi_2=\gamma_2(\xi_1)}=$$

$$=\frac{1}{\pi}\int_{a_1}^{b_1}\left[\alpha_{i1}(x_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}+\alpha_{i2}(x_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}+\right.$$

$$\left.+\beta_{i1}(x_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}+\beta_{i2}(x_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_2(x_1)}\right]\frac{dx_1}{x_1-\xi_1}+...,=$$

$$= \frac{1}{\pi} \int_{a_1}^{b_1} \left[\varphi_i (x_1) - \alpha_i (x_1) u (x_1, \gamma_1 (x_1)) - \beta_i (x_1) u (x_1, \gamma_1 (x_1)) \right] \times$$

$$\times \frac{dx_1}{x_1 - \xi_1} + ..., \ i = 1, 2; \ \ \xi_1 \in (a_1, b_1) \tag{11}$$

If $\varphi_i (x_1)$, $i = 1, 2$; $x_1 \in [a_1, b_1]$ are differentiable functions vanishing at the end of the interval (a_1, b_1), i.e.

$$\varphi_i (a_1) = \varphi_i (b_1) = 0, \ \ i = 1, 2,$$

then the first term in the right hand side of (11) is regular [8]. Concerning the remaining two terms in the right hand side of (11), for their regularization it is enough to take into account necessary conditions (8) and change integration.

So, we get the following statement:

Theorem 3. *Regular relations (11) hold under conditions 1, 2.*

Fredholm property. Combining the given boundary conditions (2_1) with the obtained regular relation (11), it is easy to see that under the condition

$$\det \begin{pmatrix} \alpha_{11}(x_1) & \alpha_{12}(x_1) & \beta_{11}(x_1) & \alpha_{12}(x_1) \\ \alpha_{21}(x_1) & \alpha_{22}(x_1) & \beta_{21}(x_1) & \beta_{22}(x_1) \\ -\alpha_{12}(x_1) & \alpha_{11}(x_1) & \beta_{12}(x_1) & -\beta_{11}(x_1) \\ -\alpha_{22}(x_1) & \alpha_{21}(x_1) & \beta_{22}(x_1) & -\beta_{21}(x_1) \end{pmatrix} \neq 0 \tag{12}$$

the following statement is true.

Theorem 4. *Under the condition 1.,2 and (12) boundary condition (1)-(2_1) is Fredholm.*

Really, under conditions (12), from relations (2_1), (11) for the vector column

$$\left(\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}, \ \left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}, \ \left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}, \ \left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)} \right)^T$$

we get a normal system of Fredholm second kind integral equations with a nonsingular kernel.

Taking into account regularity of necessary conditions (8), we get a system of Fredholm second kind integral equations for the unknowns of the vector column

$$\left(u (x_1, \gamma_1 (x_1)), u (x_1, \gamma_2 (x_1)), \ \left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}, \right.$$

$$\left. \left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}, \ \left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_2(x_1)}, \ \left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_2(x_1)} \right)^T.$$

Now pass to boundary value problem (1), (2_2).

Necessary conditions. Coming back to necessary conditions (8)-(10), for $k = 2$ we accept the following transformation

$$\xi_1 = a_1 + b_1 - \eta_1,$$

and in the right hand side both for $k=1$ and $k=2$ whose terms contain $\gamma_2(x_1)$ we make a substitution

$$x_1 = a_1 + b_1 - \zeta_1$$

and have:

$$u(\xi_1,\gamma_1(\xi_1)) = \frac{1}{\pi}\int_{a_1}^{b_1} \frac{u(x_1,\gamma_1(x_1))}{1+\gamma_1'^2(\sigma_1)} \frac{\gamma_1'(x_1)-\gamma_1'(\sigma_1(x_1,\xi_1))}{(x_1-\xi_1)}dx_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1} \frac{u(a_1+b_1-\zeta_1,\gamma_2(a_1+b_1-\zeta_1))}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,\xi_1))}\times$$

$$\times\frac{\gamma_2'(a_1+b_1-\zeta_1)(a_1+b_1-\zeta_1-\xi_1)-(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))}{(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2}d\zeta_1-$$

$$-\frac{1}{2\pi}\int_{a_1}^{b_1}\ln\left[(x_1-\xi_1)^2+(\gamma_1(x_1)-\gamma_1(\xi_1))^2\right]\left.\frac{du(x)}{dv_{1x}}\right|_{x_2=\gamma_1(x_1)}\sqrt{1+\gamma_1'^2(x_1)}dx_1-$$

$$-\frac{1}{2\pi}\int_{a_1}^{b_1}\ln\left[(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2\right]\times$$

$$\times\left.\frac{du(x)}{dv_{2x}}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\sqrt{1+\gamma_2'^2(a_1+b_1-\zeta_1)}d\zeta_1,$$

$$u(a_1+b_1-\eta_1,\gamma_2(a_1+b_1-\eta_1)) =$$

$$=-\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-\zeta_1,\gamma_2(a_1+b_1-\zeta_1))}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,a_1+b_1-\eta_1))}\frac{\gamma_2'(a_1+b_1-\zeta_1)-\gamma_2'(\sigma_2)}{\eta_1-\xi_1}d\zeta_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}u(x_1,\gamma_1(x_1))\frac{\gamma_1'(x_1)(x_1-a_1-b_1+\eta_1)-(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))}{(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2}dx_1-$$

$$-\frac{1}{2\pi}\int_{a_1}^{b_1}\ln\left[(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2\right]\times$$

$$\times\left.\frac{du(x)}{dv_{1x}}\right|_{x_2=\gamma_1(x_1)}\sqrt{1+\gamma_1'^2(x_1)}dx_1-$$

$$-\frac{1}{2\pi}\int_{a_1}^{b_1}\ln\left[(\eta_1-\zeta_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_2(a_1+b_1-\eta_1))^2\right]\times$$

$$\times\left.\frac{du(x)}{dv_{2x}}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\sqrt{1+\gamma_2'^2(a_1+b_1-\zeta_1)}d\zeta_1,\ \xi_1,\eta_1\in(a_1,b_1) \qquad (13)$$

$$\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\xi_2=\gamma_1(\xi_1)}=\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)-\gamma_1'(\sigma_1(x_1,\xi_1))}{x_1-\xi_1}\times$$

$$\times\frac{dx_1}{1+\gamma_1'^2(\sigma_1(x_1,\xi_1))}-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{\gamma_2'(a_1+b_1-\zeta_1)(a_1+b_1-\zeta_1-\xi_1)-(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))}{(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2}d\zeta_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)-\gamma_1'(\sigma_1(x_1,\xi_1))}{x_1-\xi_1}\frac{\gamma_1'(\sigma_1(x_1,\xi_1))}{1+\gamma_1'^2(\sigma_1(x_1,\xi_1))}dx_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{(a_1+b_1-\zeta_1-\xi_1)+\gamma_2'(a_1+b_1-\zeta_1)(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))}{(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2}d\zeta_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\xi_1},$$

$$\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}=-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{\gamma_2'(a_1+b_1-\zeta_1)-\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{\eta_1-\xi_1}\times$$

$$\times\frac{d\zeta_1}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)(x_1-a_1-b_1+\eta_1)-(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))}{(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2}dx_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{\gamma_2'(a_1+b_1-\zeta_1)-\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{\eta_1-\xi_1}\times$$

$$\times\frac{\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}d\zeta_1-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_2(x_1)}\times$$

$$\times\frac{(x_1-a_1-b_1+\eta_1)-\gamma_1'(x_1)(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))}{(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2}dx_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{dx_1}{x_1-\xi_1},\quad \xi_1,\eta_1\in(a_1,b_1) \tag{14}$$

$$\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\xi_2=\gamma_1(\xi_1)}=\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)-\gamma_1'(\sigma_1(x_1,\xi_1))}{x_1-\xi_1}\times$$

$$\times\frac{dx_1}{1+\gamma_1'^2(\sigma_1(x_1,\xi_1))}-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{\gamma_2'(a_1+b_1-\zeta_1)(a_1+b_1-\zeta_1-\xi_1)-(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))}{(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2}d\zeta_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)-\gamma_1'(\sigma_1(x_1,\xi_1))}{x_1-\xi_1}\frac{\gamma_1'(\sigma_1(x_1,\xi_1))}{1+\gamma_1'^2(\sigma_1(x_1,\xi_1))}dx_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{(a_1+b_1-\zeta_1-\xi_1)+\gamma_2'(a_1+b_1-\zeta_1)(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))}{(a_1+b_1-\zeta_1-\xi_1)^2+(\gamma_2(a_1+b_1-\zeta_1)-\gamma_1(\xi_1))^2}d\zeta_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\xi_1},$$

$$\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}=-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\times$$

$$\times\frac{\gamma_2'(a_1+b_1-\zeta_1)-\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{\eta_1-\xi_1}\times$$

$$\times\frac{d\zeta_1}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{\gamma_1'(x_1)(x_1-a_1-b_1+\eta_1)-(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))}{(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2}dx_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{\gamma_2'(a_1+b_1-\zeta_1)-\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{\eta_1-\xi_1}\times$$

$$\times\frac{\gamma_2'(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}{1+\gamma_2'^2(\sigma_2(a_1+b_1-\zeta_1,\ a_1+b_1-\eta_1))}d\zeta_1+$$

$$+\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{(x_1-a_1-b_1+\eta_1)+\gamma_1'(x_1)(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))}{(x_1-a_1-b_1+\eta_1)^2+(\gamma_1(x_1)-\gamma_2(a_1+b_1-\eta_1))^2}dx_1-$$

$$-\frac{1}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{d\zeta_1}{x_1-\zeta_1},\quad \xi_1,\eta_1\in(a_1,b_1) \tag{15}$$

Regularization of singularities. From the obtained necessary conditions (14)-(15) we form the following linear combination:

$$\overline{\alpha}_{i1}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}+\overline{\alpha}_{i2}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}+$$

$$+\overline{\beta}_{i1}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}+\overline{\beta}_{i2}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}=$$

$$=-\frac{\overline{\alpha}_{i1}(\eta_1)}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\eta_1}+\frac{\overline{\alpha}_{i2}(\eta_1)}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{x_2=\gamma_1(x_1)}\frac{dx_1}{x_1-\eta_1}-$$

$$-\frac{\overline{\beta}_{i1}(\eta_1)}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{dx_1}{\zeta_1-\eta_1}+\frac{\overline{\beta}_{i2}(\xi_1)}{\pi}\int_{a_1}^{b_1}\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\frac{dx_1}{\zeta_1-\eta_1}+...=$$

$$=\frac{1}{\pi}\int_{a_1}^{b_1}\left[\overline{\alpha}_{i2}(\zeta_1)\left.\frac{\partial u(\xi)}{\partial x_1}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}-\overline{\alpha}_{i1}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}+\right.$$

$$\left.+\overline{\beta}_{i2}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}-\overline{\beta}_{i1}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\right]\frac{d\zeta_1}{x_1-\zeta_1}+...,$$

where the dots means the sum of nonsingular terms.

Assuming

$$\begin{aligned}\overline{\alpha}_{i2}(\zeta_1)&=\alpha_{i1}(\zeta_1),\quad -\overline{\alpha}_{i1}(\zeta_1)=\alpha_{i2}(\zeta_1),\\ \overline{\beta}_{i2}(\zeta_1)&=\beta_{i1}(\zeta_1),\quad -\overline{\beta}_{i1}(\zeta_1)=\beta_{i2}(\zeta_1),\end{aligned}$$

and taking into account under the sign of integral the boundary condition (2_2), form the last relation we get

$$-\alpha_{i2}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}+\alpha_{i1}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}+$$

$$-\beta_{i2}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}-\beta_{i1}(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}=$$

$$=\frac{1}{\pi}\int\limits_{a_1}^{b_1}\left[\alpha_{i1}(\zeta_1)\left.\frac{\partial u(\xi)}{\partial x_1}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}+\alpha_{i2}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}+\right.$$

$$\left.+\beta_{i1}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}+\beta_{i2}(\zeta_1)\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\right]\frac{d\zeta_1}{\zeta_1-\eta_1}+...=$$

$$=\frac{1}{\pi}\int\limits_{a_1}^{b_1}\left[\varphi_i(\zeta_1)-\alpha_i(\zeta_1)u(\zeta_1,\gamma_1(\zeta_1))-\beta_i(\zeta_1)u(a_1+b_1-\zeta_1,\gamma_2(a_1+b_1-\zeta_1))\right]\times$$

$$\times\frac{d\zeta_1}{\zeta_1-\eta_1}+...,\ i=1,2;\quad \eta_1\in(a_1,b_1)\,. \tag{16}$$

This establishes the following statement:

Theorem 5. *Regular relations (16) hold under conditions 1.2.*

Indeed, the regularity of the first term in the right hand side of (16) is established in [8]. And the regularity of two terms are obtained similar to (11) after substitution from necessary conditions (13) and permutation of the obtained double integrals.

Fredholm property. Combining boundary conditions (2_2) with regular relation (16), allowing for definability of

$$\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}},\ \left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}},\ \left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}},\ \left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}},$$

we get the following sufficient condition

$$\det\begin{pmatrix}\alpha_{11}(x_1) & \alpha_{12}(x_1) & \beta_{11}(x_1) & \alpha_{12}(x_1)\\ \alpha_{21}(x_1) & \alpha_{22}(x_1) & \beta_{21}(x_1) & \beta_{22}(x_1)\\ -\alpha_{12}(x_1) & \alpha_{11}(x_1) & -\beta_{12}(x_1) & \beta_{11}(x_1)\\ -\alpha_{22}(x_1) & \alpha_{21}(x_1) & -\beta_{22}(x_1) & \beta_{21}(x_1)\end{pmatrix}\neq 0. \tag{17}$$

This establishes

Theorem 6. *Under conditions 1.,2 and (17) boundary value problem (1)-(2_2) is Fredholm.*

Finally, pass to boundary value problem (1)-(2_3). Proceeding form necessary conditions (14),(15) we form the linear combination:

$$-\beta_1(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}-i\beta_1(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}-$$

$$-\beta_2(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}-i\beta_2(\eta_1)\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}=$$

$$=\frac{1}{\pi}\left[\beta_1(\zeta_1)\left(\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}-i\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_1(x_1)\\ x_1=\zeta_1}}\right)+\right.$$

$$+\beta_2(\zeta_1)\left(\left.\frac{\partial u(x)}{\partial x_2}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}} - i\left.\frac{\partial u(x)}{\partial x_1}\right|_{\substack{x_2=\gamma_2(x_1)\\ x_1=a_1+b_1-\zeta_1}}\right)\Bigg]\frac{d\zeta_1}{\zeta_1-\eta_1}+...$$

Taking into account the second boundary condition form (2_3) we get:

$$-i\left[\beta_1(\eta_1)\left(\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}} - i\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_1(\xi_1)\\ \xi_1=\eta_1}}\right)-\right.$$

$$\left.-i\beta_2(\eta_1)\left(\left.\frac{\partial u(\xi)}{\partial \xi_2}\right|_{\substack{\xi_2=\xi_2(x_1)\\ \xi_1=a_1+b_1-\eta_1}} - i\left.\frac{\partial u(\xi)}{\partial \xi_1}\right|_{\substack{\xi_2=\gamma_2(\xi_1)\\ \xi_1=a_1+b_1-\eta_1}}\right)\right]=\frac{1}{\pi}\int_{a_1}^{b_1}\frac{\beta(\zeta_1)}{\zeta_1-\eta_1}d\zeta_1+...,$$

or

$$-i\beta(\eta_1)=\frac{1}{\pi}\int_{a_1}^{b_1}\frac{\beta(\zeta_1)}{\zeta_1-\eta_1}d\zeta_1+... \tag{18}$$

Thus, proceeding from the second boundary condition (2_3) after regularization of necessary conditions we get the Fredholm first kind integral equation (18).

Theorem 7. *Under conditions 1.,3. boundary value problem (1),(2_3) is not Fredholm.*

We have obtained the same result for the Cauchy-Riemann equation [1] as well.

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