MATHEMATICS

Akbar B. ALIYEV, Anar A. KAZIMOV

GLOBAL SOLVABILITY AND THE BEHAVIOR OF SOLUTIONS FOR CAUCHY PROBLEM FOR SYSTEMS OF THREE SEMILINEAR HYPERBOLIC EQUATIONS WITH DISSIPATION

Abstract

In this paper we study the global solvability and behavior of the solutions for the Cauchy problem for systems of three semilinear dissipative equations with the nonlinear parts consisting of the sums of functions of two variables. We find conditions when the nonlinear part ensures the existence of global solutions. We also investigate the matter of absence of global solutions for systems of three hyperbolic inequalities. From this, in particular, it follows that the conditions imposed on the growth of the nonlinear part in the theorem on the existence of global solutions are essential.

1. Global solvability of the Cauchy problem for semilinear hyperbolic equations with dissipation has been studied in the papers [5-10,13-14]. In these works, sufficient conditions for the growth of the nonlinear part ensuring the existence of global solutions are obtained. A lot of works have been dedicated to the matter of absence of global solutions for hyperbolic inequalities (eg see[1-4,11,12,15,16]).

The existence of global solutions for the special case of the systems of two semilinear hyperbolic equations was studied in [17]. One of these equations is a fourth-order equation and the other a second-order equation.

2. Formulation of the problem and main results

In the domain $R_+ \times R_n$ consider the Cauchy problem for systems of semilinear hyperbolic equations with dissipations:

$$\left.\begin{aligned} u_{1tt} + u_{1t} + (-1)^{l_1} \Delta^{l_1} u_1 &= \sum_{i,j\in j} f_{1ij}(u_i, u_j), \\ u_{2tt} + u_{2t} + (-1)^{l_2} \Delta^{l_2} u_2 &= \sum_{i,j\in j} f_{2ij}(u_i, u_j), \\ u_{3tt} + u_{3t} + (-1)^{l_3} \Delta^{l_3} u_3 &= \sum_{i,j\in j} f_{3ij}(u_i, u_j) \end{aligned}\right\} \qquad (1)$$

with initial conditions

$$u_i(0,x) = \varphi_i(x), \quad u_{i_t}(0,x) = \psi_i(x), \quad x \in R^n, \qquad (2)$$

where $J = \{(i,j), i < j; i,j = 1,2,3\}$.

Assume that the following conditions hold:

1^0. $l_1 \geq l_2 \geq l_3 \geq \frac{n}{2}$;

2^0. $f_{1ij}(\cdot), f_{2ij}(\cdot), f_{3ij} \in C^1(R^2)$, $(i,j) \in J$;

$$3^0.\ |f_{kij}(u_i, u_j)| \le c\,|u_i|^{\alpha_{kij}} \cdot |u_i|^{\beta_{kij}}, \tag{3}$$

where

$$\begin{gathered} \alpha_{kij} + \beta_{kij} > \tfrac{2}{m_k} \\ \tfrac{\alpha_{kij}}{l_i m_i} + \tfrac{\beta_{kij}}{l_j m_j} > \tfrac{2}{n} + \tfrac{r_{kij}}{m_k}, (i,j) \in J, k = 1,2,3. \end{gathered} \tag{4}$$

Here

$$r_{kij} = r_k\left(l_i, l_j, \alpha_{kij}, \beta_{kij}, m_k\right) =$$

$$= \begin{cases} \frac{1}{l_i}, & \alpha_{kij} \ge \frac{2}{m_k},\ \beta_{kij} \ge 0, \\ \frac{m_k \alpha_{kij}}{2l_i} + \frac{2 - m_k \alpha_{kij}}{2l_j}, & 0 \le \alpha_{kij} < \frac{2}{m_k}, \beta_{kij} > 0. \end{cases} \tag{5}$$

We introduce the following notation:

$$U^{l_i}_{\delta,m_i} = \Big\{(u,v) \cdot u \in W_2^{l_i}(R) \cap L_{m_i}(R^n), v \in L_2(R^n) \cap L_{m_i}(R^n),$$

$$\|u\|_{W_2^{l_i}(R^n)} + \|u\|_{Lm_i(R^n)} + \|v\|_{L_2(R^n)} + \|v\|_{L_{m_i}(R^n)} < \delta\Big\}, \quad i = 1,2.$$

We prove the following main theorem.

Theorem 1. *Suppose that conditions 1-3 are satisfied. Then there exists a real number* $\delta_0 > 0$*, such that for any* $(\varphi_i, \psi_i) \in U^{l_i}_{\delta_0,m_i} i = 1,2$ *problem (1), (2) has a unique solution*

$$u = (u_1, u_2, u_3) :\in C\left([0,\infty); W_2^{l_1}(R^n) \times W_2^{l_2}(R^n) \times W_2^{l_3}\right) \cap$$

$$\cap C\left([0,\infty); L_2(R^n) \times L_2(R^n) \times L_2(R^n)\right),$$

and for u_1, u_2, u_3 *the following estimates hold:*

$$\sum_{|\alpha|=r} \|D^\alpha u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n\left(\frac{1}{m_k} - \frac{1}{2}\right) + r}{2l_k}}, \quad r = 0,1,...,1, \tag{6}$$

$$\|D_i u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\min\left(1 + \frac{n}{2l_k}\left(\frac{1}{m_k} - \frac{1}{2}\right), \gamma_{kij}\right)}, \tag{7}$$

where

$$\gamma_k = \min_{(i,j)\in J}\left(\frac{n}{2}\left(\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j}\right) - \frac{r_{kij}}{m_k}\right), \quad k = 1,2,3.$$

Now consider a system of hyperbolic inequalities

$$\left.\begin{aligned} u_{1tt} + u_{1t} + (-1)^{l_1}\Delta^{l_1} u_1 \ge c_{11}|u_1|^{p_{11}} + c_{12}|u_2|^{p_{12}} + c_{13}|u_3|^{p_{13}} \\ u_{2tt} + u_{2t} + (-1)^{l_2}\Delta^{l_2} u_2 \ge c_{21}|u_1|^{p_{21}} + c_{22}|u_2|^{p_{22}} + c_{23}|u_3|^{p_{23}} \\ u_{3tt} + u_{3t} + (-1)^{l_3}\Delta^{l_3} u_3 \ge c_{31}|u_1|^{p_{31}} + c_{32}|u_2|^{p_{32}} + c_{33}|u_3|^{p_{33}} \end{aligned}\right\} \tag{8}$$

with initial conditions:

$$u_i(0,x) = \varphi_i(x), \quad u_{i_t}(0,x) = \psi_i(x), \quad i = 1,2,3. \tag{9}$$

We assume that

$$c_{ij} \geq 0 \quad i,j = 1,2,3. \tag{10}$$

and one of the following conditions is satisfied:

1) Let

$$c_{kk} > 0, \quad 1 < p_{kk} \leq 1 + \frac{2l_k}{n}, k = 1,2,3, \tag{11}$$

and

$$\sum_{k=1}^{3} \int_{R^n} [\varphi_k(x) + \psi_k(x)]\, dx \geq 0. \tag{12}$$

2) There exist such $i,j \in \{1,2,3\}, i \neq j$ that $c_{ij} > 0, c_{ji} > 0$ and

$$1 < p_{ij} < +\infty, 1 < p_{ji} < +\infty,$$

$$\max(p_{ij}, p_{ji}) \leq \frac{2}{n} \min\left\{\frac{l_j p_{ij}}{p_{ij}-1}, \frac{l_j p_{ij}}{p_{ij}-1}\right\}$$

$$\int_{R^n} \left[\varphi_i(x) + \varphi_j(x) + \psi_i(x) + \psi_j(x)\right] dx \geq 0$$

$$1 < p_{kk} \leq 1 + \frac{2l_k}{n}, \int_{R^n} [\varphi_k(x) + \psi_k(x)]\, dx \geq 0 \quad k \notin (i,j).$$

4) Let $c_{1j_1} > 0,\ c_{1j_2} > 0,\ c_{1j_3} > 0$ and

$$\max(p_{1j_1}, p_{2j_2}, p_{3j_3}) \leq \frac{2}{n} \min\left\{\frac{l_1 p_{1j_1}}{p_{1j}-1}, \frac{l_2 p_{2j_2}}{p_{2j_2}-1}, \frac{l_3 p_{3j_3}}{p_{3j_3}-1}\right\}$$

$$\sum_{k=1}^{3} \int_{R^n} \left[\varphi_{j_k}(x) + \psi_{j_k}(x)\right] dx \geq 0, \tag{13}$$

where $(j_1, j_2, j_3) = (2,3,1)$ or $(j_1, j_2, j_3) = (3,1,2)$.

Definition. *The weak solution of inequalities (8) with initial data (9), where* $\varphi_i(.) \in W_1^{l_i}(R^n), \psi_i(.) \in L_1(R^n)$ *is called functions* (u_1, u_2, u_3) *such that*

$$u_1, |u_1|^{p_{1j}}, u_2, |u_2|^{p_{2j}}, u_3, |u_3|^{p_{3j}}, j = 1,2,3$$

belong to $L^1_{loc}\left(R^{n+1}\right)$ *and satisfy the inequalities*

$$\int_0^\infty \int_{R^n} u_i(t,x) \left[\zeta_{i_{tt}}(t,x) - \zeta_{i_t}(t,x) + (-1)^{l_i} \Delta^{l_i} \zeta_i(t,x)\, dxdt\right] -$$

$$-\int_{R^n} [\varphi_i(x) + \psi_i(x)]\, \zeta_i(0,x)\, dx + \int_{R^n} \varphi_i(x) \frac{\partial \zeta_i(0,x)}{\partial t} dx \geq$$

$$\geq c_{i1}\int\limits_0^\infty\int\limits_{R^n}|u_1|^{p_{1i}}\zeta_{i_t}(t,x)\,dxdt+c_{i2}\int\limits_0^\infty\int\limits_{R^n}|u_2|^{p_{2i}}\zeta_i(t,x)\,dxdt+$$

$$+c_{i1}\int\limits_0^\infty\int\limits_{R^n}|u_2|^{p_{3i}}\zeta_i(t,x)\,dxdt, \qquad (14)$$

for any function $\varsigma_i(t,x)\geq 0$ *with compact support from the class* $C^{2,l_i}_{t,x}([0,\infty)\times R^n)$ $i=1,2,3.$

Theorem 2. *Suppose that one of the conditions 1) -3) is satisfied. Then (8), (11) has no nontrivial global solution.*

3. Proof of theorems

Local solvability.In the Hilbert space $H=L_2(R^n)\times L_2(R^n)\times L_2(R^n)$ we write problem (1), (2), as the Cauchy problem

$$\left.\begin{array}{c} y''+y'+Ay=F(y)\\ y(0)=y_0, y'(0)=y \end{array}\right\}, \qquad (15)$$

where

$$y=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix},\quad y_0=\begin{pmatrix}\varphi_1\\\varphi_2\\\varphi_3\end{pmatrix},\quad y_1=\begin{pmatrix}\psi_1\\\psi_2\\\psi_3\end{pmatrix}.$$

A is a linear operator in H defined by the equalities

$$D(A)=W_2^{2l_1}(R^n)\times W_2^{2l_2}(R^n)\times W_2^{2l_3}(R^n),$$

$$A=\begin{pmatrix}(-1)^{l_1+1}\Delta^{l_1}+1 & 0 & 0\\ 0 & (-1)^{l_2+1}\Delta^{l_2}+1 & 0\\ 0 & 0 & (-1)^{l_3+1}\Delta^{l_3}+1\end{pmatrix};$$

$$F(y)=\begin{pmatrix}\sum\limits_{(i,j)\in J}f_{1ij}(u_i,u_j)+u_1\\ \sum\limits_{(i,j)\in J}f_{2ij}(u_i,u_j)+u_2\\ \sum\limits_{(i,j)\in J}f_{3ij}(u_i,u_j)+u_3\end{pmatrix}$$

is a non- linear operator acting from $H\left(A^{1/2}\right)$ to H.

A is a self-adjoint positive definite operator, and conditions 1 and 2 imply that the nonlinear operator $F(\cdot)$ satisfies the local Lipschitz condition,i.e.

$$\|F(y_1)-F(y_2)\|_H\leq c(r)\left\|A^{1/2}(y_1-y_2)\right\|_H,$$

where $c(\cdot)\in C(R_+)$, $c(r)\geq 0,$ $r=\sum\limits_{i=1}^{2}\left\|A^{\frac{1}{2}}y_i\right\|_H.$

By the theorem of solvability of the Cauchy problem for the operator differential equation we have the following local solvability theorem (see [19]) for problem (15).

Theorem 3. *Let the conditions 1.2 be satisfied. Then for any* $y_0 \in D\left(A^{1/2}\right) = W_2^{l_1}(R^n) \times W_2^{l_2}(R^n) \times W_2^{l_3}(R^n)$ *there exists* $T' \in (0,\infty)$ *such that problem (19) (20) has a unique solution* $y \in C\left([0,T'], D\left(A^{1/2}\right)\right) \cap C^1([0,T], H)$.

If T_0 *is the length of the maximum interval of the existence of solutions* $y \in C\left([0,T_0], D\left(A^{1/2}\right)\right) \cap C^1([0,T_0], H)$ *then one of the following statements is true:*

1) $T_0 = +\infty$

2) If $T_0 < +\infty$, *then* $\lim\limits_{t\to T_0-0}\left(\left\|A^{1/2}y(t)\right\| + \|y'(t)\|\right) = +\infty$.

It follows that, if the a priori estimate

$$\left\|A^{1/2}y(t)\right\| + \|y'(t)\| \le c, t \in [0,\infty) \tag{16}$$

holds then problem (15) has a global solution.

Proof of Theorem 1. By Theorem 3 and (16), to prove Theorem 1 we need to get a priori estimate:

$$\sum_{k=1}^{3}\left\|\nabla^{l_k}u_k(t,\cdot)\right\|_{L_2(R^n)} + \|D_t u_k(t,\cdot)\|_{L_2(R^n)} \le c,\ t\in[0,\infty)\ k=1,2,3. \tag{17}$$

Using Fourier transform, we obtain the following inequality (see [13])

$$\|u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)} E_k(\varphi_k,\psi_k) +$$
$$+c\int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\left[\sum_{(i,j)\in J}\|f_{kij}(u_i(\tau,\cdot),u_j(\tau,\cdot))\|_{L_{m_k}(R^n)} +\right.$$
$$\left.+\sum_{(i,j)\in J}\|f_{kij}(u_i,(\tau,\cdot),u_j(\tau,\cdot))\|_{L_2(R^n)}\right]d\tau \tag{18}$$

and

$$\sum_{|\alpha|=l_k}\|D^\alpha u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{1}{2}} E_k(\varphi_k,\psi_k) +$$
$$+c\int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{1}{2}}\left[\sum_{(i,j)\in J}\|f_{kij}(u_i(\tau,\cdot),u_j(\tau,\cdot))\|_{L_{m_k}(R^n)} +\right.$$
$$\left.+\sum_{i,j\in J}\|f_{kij}(u_i,(\tau,\cdot),u_j(\tau,\cdot))\|_{L_2(R^n)}\right]d\tau, \tag{19}$$

$$\|D_t u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-1} E_k(\varphi_k,\psi_k) +$$
$$+c\int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-1}\left[\sum_{(i,j)\in J}\|f_{kij}(u_i(\tau,\cdot),u_j(\tau,\cdot))\|_{L_{m_k}(R^n)} +\right.$$

$$+\sum_{(i,j)\in J}\left\|f_{kij}\left(u_i,(\tau,\cdot),u_j(\tau,\cdot)\right)\right\|_{L_2(R^n)}\Bigg]d\tau, \qquad (20)$$

where $1\le m_k\le 2, k=1,2,3, t\in[0,T_0)$.

Further, using the Holder inequality and conditions 1-3 , we obtain that

$$\left\|f_{kij}\left(u_1,u_2,u_3\right)\right\|_{L_2(R^n)}\le c\left\|u_i\right\|^{\alpha_{kij}}_{L_{p_{kij}\alpha_{kij}}(R^n)}\cdot\left\|u_j\right\|^{\beta_{kij}}_{L_{q_{kij}\beta_{kij}}(R^n)}, \qquad (21)$$

where $p_{kij}>1$, $q_{kij}>1$, $\frac{1}{p_{kij}}+\frac{1}{q_{kij}}=1$(if $\alpha_{kij}=0$, then $p_{kij}=\infty$, $q_{kij}=1$, if $\beta_{kij}=0$ then $p_{kij}=1$, $q_{kij}=\infty$).

By applying the multiplicative inequality (see [20]), from (21) we obtain that

$$\left\|f_{kij}\left(u_i,u_j\right)\right\|_{L_{m_k}(R^n)}\le$$

$$\le\left\|u_i\right\|^{(1-\theta_{1kij})\alpha_{k,i,j}}_{L_2(R^n)}\cdot\left\|\nabla^{l_i}u_i\right\|^{\theta_{1kij}\alpha_{k,i,j}}_{L_2(R^n)}\cdot\left\|u_j\right\|^{(1-\theta_{2kij})\beta_{k,i,j}}_{L_2(R^n)}\cdot\left\|\nabla^{l_j}u_j\right\|^{\theta_{2kjj}\beta_{k,i,j}}_{L_2(R^n)}, \qquad (22)$$

where

$$\theta_{1kij}=\frac{n}{l_i}\left(\frac{1}{2}-\frac{1}{p_{kij}\alpha_{k,i,j}m_k}\right),\quad \theta_{2kij}=\frac{n}{l_j}\left(\frac{1}{2}-\frac{1}{p_{kij}\alpha_{k,i,j}m_k}\right). \qquad (23)$$

In a similar way we find that

$$\left\|f_{kij}\left(u_i,u_j\right)\right\|_{L_2(R^n)}\le c\left\|u_i\right\|^{\left(1-\theta'_{i_1kij}\right)\alpha_{kij_1}}_{L_2(R^n)}\cdot\left\|\nabla^{l_i}u_i\right\|^{\left(1-\theta'_{i_1}\right)\alpha_{kij_1}}_{L_2(R^n)}\times$$

$$\times\left\|u_j\right\|^{(1-\theta'_{2kij})\beta_{kij_1}}_{L_2(R^n)}\cdot\left\|\nabla^{l_i}u_j\right\|^{\theta'_{2kij}\beta_{kij}}_{L_2(R^n)}, (24)$$

where

$$\theta'_{1kij}=\frac{n}{2l_j}\left(1-\frac{1}{p_k\alpha_{k,i,j}m_k}\right),\theta'_{2kij}=\frac{n}{2l_k}\left(1-\frac{1}{q_k\beta_{k,i,j}m_k}\right). \qquad (25)$$

We introduce the following notation

$$X_k(t)=(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\left\|u_k(t,\cdot)\right\|_{L_2(R^n)}$$

$$Y_k(t)=(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\left\|\nabla_{l_k}u_k(t,\cdot)\right\|_{L_2(R^n)}.$$

From (18) –(24) we obtain that

$$X_k(t)\le cE_k(\varphi_k,\psi_k)+(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\int_0^t(1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}\times$$

$$\times\sum_{k=1}^{3}\left(F_{1k}(\tau)+F_{2k}(\tau)\right)d\tau, \qquad (26)$$

$$Y_k(t) \le cE_k(\varphi_k,\psi_k) + (1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}} \int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{1}{2}} \times$$

$$\times \sum_{k=1}^{3} \left(F_{1k}(\tau) + F_{2k}(\tau)\right) d\tau, \tag{27}$$

where

$$F_{1k}(\tau) = \sum_{(i,j)\in J} (1+\tau)^{-\gamma_{kij}} X_i^{(1-\theta_{1ki})\alpha_{kij}}(\tau) X_j^{(1-\theta_{2ki})\beta_{kij}} Y_i^{\theta_{1ki}\alpha_{kij}}(\tau) Y_j^{\theta_{2ki}\beta_{kij}}(\tau),$$

$$F_{2k}(\tau) = \sum_{(i,j)\in J} (1+\tau)^{-\gamma'_{kij}} X_i^{(1-\theta'_{1ki})\alpha_{kij}}(\tau) X_j^{(1-\theta'_{2ki})\beta_{kij}} Y_i^{\theta'_{1ki}\alpha_{kij}}(\tau) Y_j^{\theta'_{2ki}\beta_{kij}}(\tau),$$

$$\gamma_{kij} = \frac{n}{2l_i}\left(\frac{1}{m_i}-\frac{1}{2}\right)(1-\theta_{1kij})\alpha_{kij} + \frac{n}{2l_j}\left(\frac{1}{m_j}-\frac{1}{2}\right)(1-\theta_{2kij})\beta_{kij} +$$

$$+\left(\frac{n}{2l_i}\left(\frac{1}{m_i}-\frac{1}{2}\right)+\frac{1}{2}\right)\theta_{1kij}\alpha_{kij} + \left(\frac{n}{2l_j}\left(\frac{1}{m_j}-\frac{1}{2}\right)+\frac{1}{2}\right)\theta_{2kij}\beta_{kij}, \tag{28}$$

$$\gamma'_{kij} = \frac{n}{2l_i}\left(\frac{1}{m_i}-\frac{1}{2}\right)(1-\theta'_{1kij})\alpha_{kij} + \frac{n}{2l_j}\left(\frac{1}{m_j}-\frac{1}{2}\right)(1-\theta'_{2kij})\beta_{kij} +$$

$$+\left(\frac{n}{2l_i}\left(\frac{1}{m_i}-\frac{1}{2}\right)+\frac{1}{2}\right)\theta'_{1kij}\alpha_{kij} + \left(\frac{n}{2l_j}\left(\frac{1}{m_j}-\frac{1}{2}\right)+\frac{1}{2}\right)\theta'_{2kij}\beta_{kij}. \tag{29}$$

From (23), (25) ,(28) and (29) we have

$$\gamma_{kij} = \frac{n}{2}\left[\left(\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j}\right) - \frac{1}{m}\left(\frac{1}{l_i p_{kij}} + \frac{1}{l_j q_{kij}}\right)\right]$$

$$\gamma'_{kij} = \frac{n}{2}\left[\left(\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j}\right) - \frac{1}{2m_k}\left(\frac{1}{l_i p_{kij}} + \frac{1}{l_j q_{kij}}\right)\right].$$

It is obvious that $\gamma'_{kij} > \gamma_{kij}\quad k = 1,2,3,$

Proposition 1: The exponents $p_{kij} > 1$ and $q_{kij} > 1$, $(i,j) \in J_k$, $k = 1,2,3$ can be chosen so that the inequalities

$$\gamma_{kij} > 1, \quad k = 1,2,3,\ (i,j) \in J$$

are satisfied.

Therefore, by Proposition 1 and Segal's Lemma (see [18]) we obtain the following inequalities

$$(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)} \int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)} (1+\tau)^{-\gamma_{kij}} d\tau \le c, \quad t \in [0,T_0); \tag{30}$$

$$(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)} \int_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)} (1+\tau)^{-\gamma'_{kij}} d\tau \le c, \quad t \in [0,T_0); \tag{31}$$

$$(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\int\limits_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\times$$

$$\times (1+\tau)^{-\gamma_{kij}}\, d\tau \le c, \quad t\in[0,T_0)\,; \tag{32}$$

$$(1+t)^{\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\int\limits_0^t (1+t-\tau)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)+\frac{1}{2}}\times$$

$$\times (1+\tau)^{-\gamma'_{kij}}\, di \le c, \quad t\in[0,T_0)\,. \tag{33}$$

By taking into account (30)-(33) , from (26)-(27) we obtain the inequality

$$Z(t) \le c_1\eta + c_2 Z^q(t)\,, \quad t\in[0,T_0)\,, \tag{34}$$

where

$$Z(t) \le \sum_{k=1}^{3} \sup_{0\le S\le t}\left[X_k(s)+Y_k(s)\right],$$

$$q = \max_{(i,j)\in J_k;\ k=1,2,3}\left(\alpha_{kij}+\beta_{kij}\right), \quad \eta=\sum_{k=1}^{3} E_k(\varphi_k,\psi_k)\,.$$

(34) implies that for sufficiently small η we have

$$Z(t)\le M, \quad t\in[0,T_0)\,. \tag{35}$$

Therefore

$$\sum_{\|\alpha\|=l_k}\|D^\alpha u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{1}{2}}, \quad k=1,2,3,\ t\in[0,T_0)\,. \tag{36}$$

$$\|u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)}, \quad k=1,2,3. \tag{37}$$

By using the theorem on intermediate derivatives, we obtain

$$\sum_{\|\alpha\|=r}\|D^\alpha u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\frac{n}{2l_k}\left(\frac{1}{m_k}-\frac{1}{2}\right)-\frac{r}{2l_k}},$$

$$r=0,1,...,l_k,\ k=1,2,3. \tag{38}$$

Then, using (37) and (38), from (20) we obtain that

$$\|D_t u_k(t,\cdot)\|_{L_2(R^n)} \le c(1+t)^{-\min\left(\frac{n}{2l_k}\left(\frac{1}{m_i}-\frac{1}{2}\right),\gamma_k\right)}, \tag{39}$$

where

$$\gamma_k=\min_{(i,j)\in J}\frac{n}{2}\left[\left(\frac{\alpha_{kij}}{l_i m_i}+\frac{\beta_{kij}}{l_j m_j}\right)-\frac{r_{kij}}{m_k}\right], \quad k=1,2,3.$$

Therefore, for sufficiently small η the Cauchy problem (1)-(2) has a global solution, i.e. $T_0=+\infty$.

Proof of Theorem 2. We will prove the theorem by the method of test functions(see[1]). First consider the case 1). In this case,from (27) have that

$$u_{ktt} + u_{kt} + (-1)^{l_k} \Delta^{l_k} u_k \geq c_{kk} |u_k|^{p_k}, \ k = 1, 2, 3. \tag{40}$$

This follows from [13] that (40) with initial conditions (10) has no nontrivial global solution .

Consider the case 3). In this case from (35) obtain

$$\max (p_{1j_1}, p_{2j_2}, p_{3j_3}) \leq \frac{2}{n} \inf \left\{ \frac{l_1 p_{1j_1}}{p_{1j_1} - 1}, \frac{l_2 p_{2j_2}}{p_{2j_2} - 1}, \frac{l_3 p_{3j_3}}{p_{3j_3} - 1} \right\}$$

$$\sum_{k=1}^{3} \int_{R_n} \left[\varphi_{j_k}(x) + \psi_{j_k}(x) \right] dx \geq 0$$

where

$$(j_1, j_2, j_3) = (2, 3, 1)$$

or

$$(j_1, j_2, j_3) = (3, 1, 2) .$$

Let $(j_1, j_2, j_3) = (3, 1, 2)$. The case $(j_1, j_2, j_3) = (2, 3, 1)$ can be proved similarly.

The test functions ξ_1, ξ_2 are chosen as follows

$$\xi_1(t, x) = \xi_2(t, x) = \xi(t, x) = h\left(\frac{t^\chi + |x|^\mu}{d^2} \right),$$

where

$$h(\cdot) \in C_\circ^\infty(R_+), \quad 0 \leq h(r) \leq 1, \ h(r) = \begin{cases} 0, & r \geq 2, \\ 1, & 0 \leq r \leq 1, \end{cases}$$

$\mu > 0$, $\chi > 1$ and $d > 0$ are some parameters that will be chosen later. From the definition $\xi(t, x)$ it follows that

$$\frac{\partial \xi(0, x)}{\partial t} = 0. \tag{41}$$

Applying Holder's inequality from (18) and (41), obtain

$$c_{31} \int_0^\infty \int_{R^n} |u_1|^{p_{31}} \xi(t, x)\, dxdt +$$

$$+ c_{12} \int_0^\infty \int_{R^n} |u_2|^{p_{12}} \xi(t, x)\, dxdt + c_{23} \int_0^\infty \int_{R^n} |u_3|^{p_{23}} \xi(t, x)\, dxdt +$$

$$+ \sum_{i=1}^{3} \int_{R_n} (\varphi_i(x) + \psi_i(x))\, \xi(0, x)\, dx \leq \left(\int_0^\infty \int_{R^n} |u_1|^{p_{31}} \xi(t, x)\, dxdt \right)^{1/p_{31}} \times$$

$$\times\left[\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{31}}{p_{31}}}\left(|\xi_{tt}|^{p'_{31}}+|\xi_t|^{p'_{31}}+\left|\Delta^{l_1}\xi\right|^{p'_{31}}\right)dxdt\right]^{1/p_{31}}+$$

$$+\left(\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\,\xi\,(t,x)\,dxdt\right)^{1/p_{12}}\times$$

$$\times\left[\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{12}}{p_{12}}}\left(|\xi_{tt}|^{p_{12}}+|\xi_t|^{p_{12}}+\left|\Delta^{l_2}\xi\right|^{p_{12}}\right)dxdt\right]^{1/p_{12}}+$$

$$+\left(\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\,\xi\,(t,x)\,dxdt\right)^{1/p_{12}}\times$$

$$\times\left[\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{23}}{p_{23}}}\left(|\xi_{tt}|^{p_{23}}+|\xi_t|^{p_{23}}+\left|\Delta^{l_3}\xi\right|^{p_{23}}\right)dxdt\right]^{1/p_{23}}.$$

From this using the Cauchy - Young inequality, we obtain

$$(c_{31}-\varepsilon)\int_0^\infty\int_{R^n}|u_1|^{p_{31}}\,\xi\,(t,x)\,dxdt+(c_{12}-\varepsilon)\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\,\xi\,(t,x)\,dxdt+$$

$$+(c_{23}-\varepsilon)\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\,\xi\,(t,x)\,dxdt+\sum_{i=1}^{3}\int_{R_n}(\varphi_i\,(x)+\psi_i\,(x))\,\xi\,(0,x)\,dx\le$$

$$\le\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{31}}{p_{31}}}|\xi_{tt}|^{p'_{31}}\,dxdt+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{31}}{p_{31}}}|\xi_t|^{p'_{31}}\,dxdt+$$

$$+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{31}}{p_{31}}}\left|\Delta^{l_i}\xi\right|^{p'_{31}}dxdt+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{12}}{p_32}}|\xi_{tt}|^{p'_{12}}\,dxdt+$$

$$+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{12}}{p_{12}}}|\xi_t|^{p'_{12}}\,dxdt+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{12}}{p_{12}}}\left|\Delta^{l_2}\xi\right|^{p'_{12}}dxdt+$$

$$+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{23}}{p_{23}}}|\xi_{tt}|^{p'_{23}}\,dxdt+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{23}}{p_{23}}}|\xi_t|^{p'_{23}}\,dxdt+$$

$$+\frac{1}{\varepsilon}\int_0^\infty\int_{R^n}\xi^{-\frac{p'_{33}}{p_{23}}}\left|\Delta^{l_3}\xi\right|^{p'_{23}}dxdt=$$

$$=\frac{1}{\varepsilon}\left[I_{1,p_{31}}+I_{2,p_{31}}+I_{3,p_{31}}\right]+\frac{1}{\varepsilon}\left[I_{1,p_{21}}+I_{2,p_{21}}+I_{3,p_{21}}\right]+$$

$$+\frac{1}{\varepsilon}\left[I_{1,p_{23}}+I_{2,p_{23}}+I_{3,p_{23}}\right], \tag{42}$$

where $\varepsilon<\min\left(c_{31},c_{12},c_{23}\right)$.

We substitute the variable in the right side of inequality (42)

$$t=d^{\frac{2}{\chi}}\tau,\quad x_k=d^{\frac{2}{\mu}}y_k,\quad k=1,2,...,n.$$

Further, by denoting

$$\Omega=\left\{(\tau,y)\in R_+\times R^n,\ \tau^\chi+|y|^\mu\leq 2\right\},\quad \rho\left(y,\tau\right)=\tau^\chi+|y|^\mu$$

we have

$$I_{1,p_{31}}\leq a_{1,p_{31}}d^{-\sigma_{31}-\frac{2p_{31}}{\chi}};$$
$$I_{2,p_{31}}\leq a_{2,p_{31}}d^{-\sigma_{31}};$$
$$I_{3,p_{31}}\leq a_{3,p_{31}}d^{-r_{31}};$$
$$I_{1,p_{12}}\leq a_{1,p_{12}}d^{-\sigma_{12}-\frac{2p_{12}}{\chi}};$$
$$I_{2,p_{12}}\leq a_{2,p_{12}}d^{-\sigma_{12}};$$
$$I_{3,p_{12}}\leq a_{3,p_{12}}d^{-r_{12}},$$
$$I_{1,p_{23}}\leq a_{1,p_{23}}d^{-\sigma_{23}-\frac{2p_{23}}{\chi}};$$
$$I_{2,p_{23}}\leq a_{2,p_{23}}d^{-\sigma_{23}};$$
$$I_{3,p_{23}}\leq a_{3,p_{23}}d^{-r_{23}},$$

where

$$a_{1,p_{ji}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ji}}{p_{ji}}}\left(\frac{\partial^2}{\partial\tau^2}(hop)\right)^{p'_{ji}}dyd\tau;$$

$$a_{2,p_{ji}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ji}}{p_{ji}}}\left(\frac{\partial}{\partial\tau}(hop)\right)^{p'_{ji}}dyd\tau;$$

$$a_{3,p_{ji}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ji}}{p_{ji}}}\left(\Delta^{l_1}(hop)\right)^{p'_{ji}}dyd\tau;$$

$$a_{1,p_{ij}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ij}}{p_{ij}}}\left(\frac{\partial^2}{\partial\tau^2}(hop)\right)^{p_{ij}}dyd\tau;$$

$$a_{2,p_{ij}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ij}}{p_{ij}}}\left(\frac{\partial^2}{\partial\tau^2}(hop)\right)^{p_{ij}}dyd\tau;$$

$$a_{3,p_{ij}}=\int\limits_{\Omega}\int(hop)^{-\frac{p'_{ij}}{p_{ij}}}\left(\Delta^{l_k}(hop)\right)^{p_{ij_1}}dyd\tau;$$

$$\sigma_{ij}=\frac{2p'_{ij}}{\chi}-\frac{2}{\chi}-\frac{2n}{\mu};$$

$$r_{ij}=\frac{4p'_{ij}l_i}{\mu}-\frac{2}{\chi}-\frac{2n}{\mu};$$
$$\sigma_{ji}=\frac{2p'_{ji}}{\chi}-\frac{2}{\chi}-\frac{2n}{\mu};$$
$$r_{ji}=\frac{4p_{ij}l_i}{\mu}-\frac{2}{\chi}-\frac{2n}{\mu},$$

and (i,j) is (3,1) or (1,2) or (2,3)

$$\max\left(p_{1j_1},p_{2j_2},p_{3j_3}\right)\leq\frac{2}{n}\inf\left\{\frac{l_1p_{1j_1}}{p_{1j_1}-1},\frac{l_2p_{2j_2}}{p_{2j_2}-1},\frac{l_3p_{3j_3}}{p_{3j_3}-1}\right\}.$$

Now, we choose $\mu>0$, $\chi>1$ such that $\mu=\theta\chi$, $\chi>1$, where

$$\theta\in\left[\max\left(\frac{n}{p'_{1j_1}-1},\frac{n}{p'_{2j_2}-1},\frac{n}{p'_{3j_3}-1}\right),\right.$$
$$\left.\min\left(2l_1p'_{1,j_1}-n,2l_2p'_{2,j_2}-n,2l_3p'_{3,j_3}-n\right)\right].$$

Then

$$\sigma_{ij}=\frac{2}{\mu}\left[\left(p'_{ij}-1\right)\theta-n\right]\geq 0;$$
$$r_{ij}=\frac{2}{\mu}\left[p'_{ij}l_j-n-\theta\right]\geq 0;$$
$$\sigma_{ji}=\frac{2}{\mu}\left[\left(p'_{ji}-1\right)\theta-n\right]\geq 0;$$
$$r_{ji}=\frac{2}{\mu}\left[p'_{ji}l_i-n-\theta\right]\geq 0.$$

As $d\to\infty$, in inequality (42) we have

$$(c_{31}-\varepsilon)\int_0^\infty\int_{R^n}|u_1|^{p_{31}}\xi(t,x)\,dxdt+(c_{12}-\varepsilon)\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\xi(t,x)\,dxdt+$$
$$+(c_{23}-\varepsilon)\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\xi(t,x)\,dxdt+\sum_{i=1}^3\int_{R_n}(\varphi_i(x)+\psi_i(x))\,dx\leq c+\infty.$$

Taking into account (13), we have

$$\int_0^\infty\int_{R^n}|u_1|^{p_{31}}\,dxdt+\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\,dxdt+\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\,dxdt\leq+\infty. \quad (43)$$

Further from (14) we obtain that

$$\sum_{i=1}^3\int_{|x|<2d^2}[\varphi_i(x)+\psi_i(x)]\,h\left(\frac{|x|^\mu}{d^2}\right)dx+c_{31}\int_0^\infty\int_{R^n}|u_1|^{p_{31}}\xi dxdt+$$

$$+c_{12}\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\,\xi dxdt+c_{23}\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\,\xi dxdt\le$$

$$\le\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_1|^{p_{31}}\,dxdt\right)^{1/p_{31}}\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|\phi_1(t,x)|^{p'_{32}}\,dxdt\right)^{1/p'_{31}}+$$

$$+\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_2|^{p_{12}}\,dxdt\right)^{1/p_{12}}\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|\phi_2(t,x)|^{p'_{12}}\,dxdt\right)^{1/p'_{12}}+$$

$$+\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_3|^{p_{23}}\,dxdt\right)^{1/p_{23}}\left(\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|\phi_3(t,x)|^{p'_{23}}\,dxdt\right)^{1/p'_{23}}$$

where

$$\phi_i(t,x)=\xi_{tt}-\xi_t+(-1)^{l_i}\Delta^{l_i}\xi,\ i=1,2,3.$$

From (44) it follows that

$$\lim_{d\to+\infty}\left[\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_1|^{p_{31}}\,dxdt\times\right.$$

$$\left.\times\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_2|^{p_{12}}\,dxdt\int\limits_{d^2\le t^\chi+|x|^\mu\le 2d^2}\int|u_3|^{p_{23}}\,dxdt\right]=0$$

Using (45) from (44) we have

$$\sum_{i=1}^{3}\int_{R^n}(\varphi_i(x)+\psi_i(x))\,dx+c_{31}\int_0^\infty\int_{R^n}|u_1|^{p_{31}}\,dxdt+$$

$$+c_{12}\int_0^\infty\int_{R^n}|u_2|^{p_{12}}\,dxdt+c_{23}\int_0^\infty\int_{R^n}|u_3|^{p_{23}}\,dxdt\le 0.$$

In view of (13) from here it follows that

$$u_1(t,x)=u_2(t,x)=u_3(t,x)=0.$$

Appendix

Proof of proposition. Let $\alpha_{kij}>2$, $\beta_{kij}=0$. In this case $p_{kij}=1$, and $q_{kij}=\infty$. From definition r_{kij} we have:

$$\frac{\alpha_{kij}}{l_i m_i}>\frac{2}{n}+\frac{1}{l_i m_k},$$

therefore

$$\gamma_k = \frac{n}{2}\left(\frac{\alpha_{kij}}{l_i m_i} - \frac{1}{l_i m_m}\right) > 1.$$

Let $\alpha_{kij} \geq \frac{2}{m_k}$, $\beta_{kij} > 0$. Then

$$\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j} - \frac{2}{n} - \frac{r_{kij}}{m_k} = \delta > 0; \quad (i,j) \in J_k, k = 1,2,3.$$

From here we obtain that

$$\gamma_k = \frac{n}{2}\left[\frac{2}{n} + \frac{r_{kij}}{m_k} + \delta - \left(\frac{1}{p_{kij} l_i m_k} + \frac{1}{p_{kij} l_j m_k}\right)\right] =$$

$$= 1 + \frac{n\delta}{2} + \frac{n}{2m_k}\left(\frac{1}{l_i} - \frac{1}{p_{kij} l_i} - \frac{1}{q_{kij} l_j}\right) = 1 + \frac{n\delta}{2} - \frac{n}{2q_{kij}}\left(\frac{1}{l_j} - \frac{1}{l_i}\right).$$

If we choose

$$q_{kij} > \frac{l_i - l_j}{l_i l_j \delta},$$

then we obtain that $\gamma_k > 1, k = 1,2,3$.

Suppose now that $0 < \alpha_{kij} < \frac{2}{m_k}$, $\beta_{kij} > 0$. In this case

$$\frac{\alpha_{kij}}{l_i m_i} + \frac{\beta_{kij}}{l_j m_j} - \frac{2}{n} - \frac{r_{kij}}{m_k} = \delta > 0, k = 1,2,3,$$

where

$$r_{kij} = \frac{m_k \alpha_{kij}}{2l_i} + \frac{2 - m_k \alpha_{kij}}{2l_j}.$$

Hence we see that

$$\gamma_1 = 1 + \frac{n\delta}{2} + \frac{n}{2}\left[\frac{r_{kij}}{m_k} - \frac{1}{m_k}\left(\frac{1}{p_{kij} l_i} + \frac{1}{q_{kij} l_j}\right)\right].$$

We choose $p_{kij} > 1$, $q_{kij} > 1$ as follows

$$\frac{1}{p_{kij}} = \frac{\alpha_{kij} m_k}{2} - \varepsilon m_k, \quad \frac{1}{q_{kij}} = \frac{2 - \alpha_{kij} m_k}{2} + \varepsilon m_k,$$

where $\varepsilon > 0$ is sufficiently small. Then, taking into account the expression r_{kij}, we obtain,that

$$\gamma_k = 1 + \frac{n\delta}{2} > 1.$$

Acknowledgment. This work was supported by the Science Development Foundation under the President of the Republic of Azerbaijan – Grant No EIF-2011-1(3)-82/18-1.

References

[1]. Mitidieri E., Pokhozhaev S.I. *A priori estimates and the absence of solutions to nonlinear partial differential equations and inequalities.* Tr. Mat. Inst Steklova, 2001, 234, pp.1-384.

[2]. Veron L., Pohozaev S. I. *Blow-up results for nonlinear hyperbolic inequalities.* AnnScuola Norm. Super. Pisa. Cl. Sci., 2001, Ser 4, v. 29, No 2, pp. 393-420.

[3]. Fujita H. *On the blowing-up of solutions of the Cauchy problem for* $ut = \Delta u + u1$ J.Fac. Sci Univ. Tokyo, Sect I.,1966, vol. 13, pp.109-124.

[4]. Hayakawa K. *On the nonexistence of global solutions of some semilinear parabolic equations.* Proc. Japan Acad, 1973, v.49, pp.503-505.

[5]. Ikehata B.R., Miataka Y., Nakataka Y. *Decay estimates of solutions of dissipative wave equations in Rn with lower power nonlinearities.* J. Math. Soc Japan, 2004, vol. 56, No 2, pp. 365-373.

[6] . Matsumura A. *On the asymptotic behavior of solution of semi linear wave equation.* Publ. RiMS, Kyoto Univ. 1976, 12, pp. 169-189.

[7]. Nakao Hayashi, E. I. Kaikina, P. I. Naumkin. *Damped wave equation with a critical nonlinearity.* Transactions of the American Mathematical Society, 2005, vol. 358, No 3.

[8]. Ono K., *Global existence and asymptotic behavior of solutions to the wave equations with linear dissipation.* Publ. RIMS, 1996, v.32, pp. 401-414.

[9]. Todorova G., Yordanov B. *Critical Exponent for a nonlinear wave equation with damping.* Journal of Differential Equations. 2001, 174, pp. 464-489.

[10]. Nakao H., Ono K. *Global existence to the Cauchy problem of the semilinear dissipativewave equations.* Math. Z., 1993, vol. 214, pp.325-342.

[11]. Zhag Q. S. *A blow-up result for a nonlinear wave equation with damping.* The critical case C. R. Acad sei. Paris, Serie I, 2001, 333, pp. 109-114.

[12]. Galakhov E.I. *Blow-Up of solutions of some non-linear wave equations.* Different. Uravneniya, 2009, v. 45, No 1, pp. 60-68.

[13]. Aliev A.B, Kazymov A.A.,*Global weak solutions of the Cauchy problem for semilinear pseudo-hyperbolic equations,differential Equations,* 2009, 45 (2), pp.1-14. Original.Russian published in Differential'nye Uravneniya, 2009, vol.45, No.2, pp.169-179.

[14]. Aliev A.B., Lichaei B.H. *Existence and non-existence of global solutions of the Cauchy problem for higher order semilinear pseado-hyperbolic equations.* Nonlinear Analysis, 2010, vol, 72, No 7,8, pp.3275-3288.

[15]. Laptev G.G. *Absence of solutions of differential inequalities and system of hyperbolic type in conic domains.* Izvestiya Mathematics, 2002, 66, 6, pp. 65-90.

[16]. Del Santo D.,Georgiev V., Mitidieri E. *Global existence of the solutions and formation of singularities for a class of hyperbolic systems* // Geometrical optics and related topics/ Eds.F.Colmbini, N.Lerner .Boston etc: Birkhuser, 1997, pp.117-140 (Progr. Nonlinear Diff. Equat. and Apll; v.32).

[17]. Aliev A.B Mammadzada K.S., Lichaei B.H. *Existence of a weak solution of the Cauchy problem for systems of semilinear hyperbolic equations.* Transactions of

National Academy of sciences of Azerbaijan,ser.of phys.-tech. and math. Sciences, 2010, v.XXX, No 1, pp.13-24.

[18]. Segal I.E. *Dispersion for non-linear realistic equations* II // Ann. Sci. Egole Norm.Sup. 1968, v. 4, No 1, pp. 459-497.

[19]. Aliev A.B., *Solvability "in the Large of the Cauchy problem for quasilinear equations of hyperbolic type.* Dokl.Acad.Nauk SSSR, 1978, 240(2), pp. 249-252.

[20]. Besov O.V, Ilin V.P., Nikolski S.M. *Integral Representation of Functions and Embedding* Theorem,V.H. Wilson and Sons, Washington, DC, 1978.

Akbar B. Aliyev, Anar A. Kazimov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F. Agayev str., 370141, Baku, Azerbaijan.
Tel.: (99412) 539 47 20 (off)

Received January 14, 2011; Revised April 04, 2011