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SCATTERING DATA OF STURM-LIOUVILLE OPERATOR WITH SPECTRAL PARAMETER IN DISCONTINUITY CONDITION

Abstract

In the paper, scattering data of Sturm-Liouville operator with a spectral parameter in the discontinuity condition is introduced and some properties of these data are studied.

In the Hilbert space $L_2(-\infty;+\infty)$ consider the operator L generated by the differential equation

$$-y'' + q(x)\,y = \lambda^2 y, \tag{1}$$

and the conditions

$$y(a+0) = y(a-0),$$

$$y'(a+0) - y'(a-0) = \lambda\beta y(a), \tag{2}$$

where $\beta, a \in (-\infty, +\infty)$, $\beta \neq 0$, λ is a complex parameter, $q(x)$ is a real-valued function and satisfies the condition

$$\int_{-\infty}^{+\infty} (1+|x|)\,|q(x)|\,dx < +\infty. \tag{3}$$

In the paper, the scattering data of the operator L is introduced and some properties of these data are studied.

The case $\beta = 0$ was considered in the papers [1], [2]. Such a problem for Sturm-Liouville not self-adjoint operator on the axis was studied in [3], for a quadratical bundles of Sturm-Liouville operators in [4], [5] and etc.

Denote by $e^{\pm}(x,\lambda)$ the solution of problem (1) - (2) possessing the asymptotics $(\operatorname{Im}\lambda \geq 0)$ $e^{+}(x,\lambda) \sim e^{i\lambda x}$ as $x \to +\infty$, $e^{-}(x,\lambda) \sim e^{-i\lambda x}$as $x \to +\infty$. It is known that [6] the solutions $e^{\pm}(x,\lambda)$ exist, are unique, regular (with respect to λ) in the half plane $\operatorname{Im}\lambda > 0$ and continuous up to $\operatorname{Im}\lambda = 0$ boundary. Furthermore, the functions $e^{\pm}(x,\lambda)$ admit the representations,

$$e^{+}(x,\lambda) = e_0^{+}(x,\lambda) + \int_{x}^{+\infty} K^{+}(x,t)\,e^{i\lambda t}dt,$$

$$e^{-}(x,\lambda) = e_0^{-}(x,\lambda) + \int_{-\infty}^{x} K^{-}(x,t)\,e^{-i\lambda t}dt,$$

where $e_0^{\pm}(x,\lambda)$ are the solutions of problem (1)-(2) for $q(x)=0$:

$$e_0^{\pm}(x,\lambda)=\begin{cases} e^{\pm i\lambda x}, \pm x>\pm a \\ \left(1+\frac{i\beta}{2}\right)e^{\pm i\lambda x}-\frac{i\beta}{2}e^{\pm i\lambda x(2a-x)}, \ \pm x<\pm a. \end{cases}$$

From he real-value property of the function $q(x)$ it follows that for real λ together with $e^{+}(x,\lambda)$ and $e^{-}(x,\lambda)$ the solutions of equation (1) are also $\overline{e^{+}(x,\lambda)}$ and $\overline{e^{-}(x,\lambda)}$ (the dash over the function here and in sequel denotes a complex conjugation). Since the Wronskian of the two solutions $y_1(x)$ and $y_2(x)$ of equation (1)

$$W\{y_1(x),y_2(x)\}=y_1'(x)y_2(x)-y_1(x)y_2'(x)$$

is independent of x, it coincides with its own value as $x\to+\infty$ or $x\to-\infty$. Therefore

$$W\left[e^{+}(x,\lambda),\overline{e^{+}(x,\lambda)}\right]=$$

$$=\lim_{x\to+\infty}\left[e'^{+}(x,\lambda)\overline{e^{+}(x,\lambda)}-e^{+}(x,\lambda),\overline{e'^{+}(x,\lambda)}\right]=2i\lambda,$$

$$W\left[e^{-}(x,\lambda),\overline{e^{-}(x,\lambda)}\right]=$$

$$=\lim_{x\to-\infty}\left[e'^{-}(x,\lambda)\overline{e^{-}(x,\lambda)}-e^{-}(x,\lambda),\overline{e'^{-}(x,\lambda)}\right]=-2i\lambda. \tag{4}$$

Consequently for $\lambda\neq0$ the pairs $e^{+}(x,\lambda),\overline{e^{+}(x,\lambda)}$ and $e^{-}(x,\lambda),\overline{e^{-}(x,\lambda)}$ form two fundamental systems of the solutions of equation (1). Therefore for $\lambda\in R^*=(-\infty,+\infty)\setminus\{0\}$ the following representations hold:

$$e^{+}(x,\lambda)=b(\lambda)e^{-}(x,\lambda)+a(\lambda)\overline{e^{-}(x,\lambda)}, \tag{5}$$

$$e^{-}(x,\lambda)=-\overline{b}(\lambda)e^{+}(x,\lambda)+a(\lambda)\overline{e^{+}(x,\lambda)}, \tag{6}$$

and from (4)

$$a(\lambda)=\frac{1}{2i\lambda}W\left[e^{+}(x,\lambda),e^{-}(x,\lambda)\right], \quad \lambda\in R^* \tag{7}$$

$$b(\lambda)=-\frac{1}{2i\lambda}W\left[e^{+}(x,\lambda),\overline{e^{-}(x,\lambda)}\right], \quad \lambda\in R^*. \tag{8}$$

Further, from (4) and (5) we have

$$2i\lambda=W\left[e^{+}(x,\lambda),\overline{e^{+}(x,\lambda)}\right]=|b(\lambda)|^2W\left[e^{-}(x,\lambda),\overline{e^{-}(x,\lambda)}\right]+$$

$$+|a(\lambda)|^2W\left[\overline{e^{-}(x,\lambda)},e^{-}(x,\lambda)\right]=\left\{|a(\lambda)|^2-|b(\lambda)|^2\right\}2i\lambda.$$

Consequently,

$$|a(\lambda)|^2-|b(\lambda)|^2=1,\lambda\in R^* \tag{9}$$

Assume

$$u^{-}(x,\lambda)=\frac{1}{a(\lambda)}e^{+}(x,\lambda), \quad u^{+}(x,\lambda)=\frac{1}{a(\lambda)}e^{-}(x,\lambda),$$

$$r^{-}(\lambda)=\frac{b(\lambda)}{a(\lambda)},\quad r^{+}(\lambda)=-\frac{\overline{b(\lambda)}}{a(\lambda)},\quad t(\lambda)=\frac{1}{a(\lambda)}.$$

Then we can write equalities (5) and (6) in the form

$$u^{-}(x,\lambda)=r^{-}(\lambda)e^{-}(x,\lambda)+\overline{e^{-}(x,\lambda)},$$

$$u^{+}(x,\lambda)=r^{+}(\lambda)e^{+}(x,\lambda)+\overline{e^{+}(x,\lambda)}.$$

Hence we get the asymptotic formulae

$$u^{-}(x,\lambda)=t(\lambda)e^{i\lambda x}+o(1),\quad x\to+\infty,$$

$$u^{-}(x,\lambda)=r^{-}(\lambda)e^{-i\lambda x}+e^{i\lambda x}+o(1),\quad x\to-\infty,$$

$$u^{+}(x,\lambda)=t(\lambda)e^{-i\lambda x}+o(1),\quad x\to-\infty,$$

$$u^{+}(x,\lambda)=r^{+}(\lambda)e^{i\lambda x}+e^{-i\lambda x}+o(1),\quad x\to+\infty.$$

The solutions $u^{\pm}(x,\lambda)$ are called eigen functions of the left $(u^{-}(x,\lambda))$ and the right $(u^{+}(x,\lambda))$ scattering problems, the coefficients $r^{-}(\lambda)$, $r^{+}(\lambda)$ and $t(\lambda)$ are called the left and right reflection factors and the conversion factor of the operator L, respectively.

Since the solutions $e^{+}(x,\lambda)$ and $e^{-}(x,\lambda)$ admit analytic continuation in the half-plane $\operatorname{Im}\lambda>0$, it follows from formula (7) that the function $a(\lambda)$ also admits analytic continuation to the half-plane $\operatorname{Im}\lambda>0$ by the same formula. Thus,

$$a(\lambda)=\frac{1}{2i\lambda}W\left[e^{+}(x,\lambda),e^{-}(x,\lambda)\right],\quad \lambda\in\{\operatorname{Im}\lambda\geq 0\}\backslash 0. \tag{10}$$

Clarify the distribution of the zeros of the function $a(\lambda)$ in the half-plane $\operatorname{Im}\lambda\geq 0$. From (9) it follows that $a(\lambda)\neq 0$ for $\lambda\in R^{*}$.

Lemma 1. *The function $a(\lambda)$ may have in the half-plane $\operatorname{Im}\lambda>0$ only finitely many zeros.*

Proof. Assume the contrary: let the function $a(\lambda)$ in the half-plane $\operatorname{Im}\lambda>0$ have infinitely many zeros $\lambda_k, k=1,2,\ldots$. By (10), from the equality $a(\lambda_k)=0$ it follows that the functions $e^{+}(x,\lambda_k)$ and $e^{-}(x,\lambda_k)$ are linearly dependent:

$$e^{+}(x,\lambda_k)=c_k e^{-}(x,\lambda_k),\quad x\in(-\infty,+\infty). \tag{11}$$

Since $e^{\pm}(x,\lambda)\sim e^{\pm i\lambda x}$, $x\to\pm\infty$, it follows from (11) that equation (1) for $\lambda=\lambda_k$ has the non-trivial solution

$$y_k(x)=e^{+}(x,\lambda_k)=c_k e^{-}(x,\lambda_k)\in L_2(-\infty,+\infty),$$

i.e. λ_k is an eigen value of the operator L. From the equation

$$-y_k''+q(x)y_k=\lambda^2 y_k,$$

and from the conditions

$$y_k(a+0)=y_k(a-0),$$

$$y_k'(a+0) - y_k'(a-0) = \lambda_k \beta y_k(a),$$

we have

$$\lambda_k^2 (y_k, y_k) - \lambda_k \beta |y_k(a)|^2 - \Phi(y_k) = 0$$

or

$$\lambda_k = \frac{\beta |y_k(a)|^2 \pm \sqrt{\beta^2 |y_k(a)|^4 + 4\Phi(y_k)}}{2(y_k, y_k)}, \tag{12}$$

where the scalar product $(.,.)$ is taken in the space $L_2(-\infty, +\infty)$, the functional $\Phi(.)$ is determined as follows:

$$\Phi(y) = \int_{-\infty}^{\infty} \left\{ |y'(x)|^2 + q(x) |y(x)|^2 \right\} dx.$$

From formula (12) and non-real property of the numbers λ_k it follows that the equalities

$$\Phi(y_k) < 0, \quad k = 1, 2, ... \tag{13}$$

should be fulfilled.

Further, since the numbers λ_k $(k = 1, 2, 3...)$ are pair-wise different, then from the asymptotic formula $y_k(x) = e^{i\lambda_k x}[1 + o(1)]$, $x \to +\infty$ it follows that the system of functions $\{y_k(x)\}$ is linearly independent. Thus, from the domain of definition of the functional Φ there exists an infinite-dimensional linear variety on which inequality (13) is fulfilled. According to [2] (see theorems 13 and 28) hence is should follow that the minimal close operator L_0 generated in the space $L_2(-\infty, +\infty)$ by the differential expression $-\frac{d^2}{dx^2} + q(x)$, (L_0 is a self – adjoint operator) has infinitely many negative eigen values that is not valid under condition (3) on $q(x)$ (see [2]). The obtained contradiction proves the lemma.

So the function $a(\lambda)$ may have finitely many zeros lying in the half-plane $\operatorname{Im}\lambda > 0$. Denote them by $\lambda_1, ..., \lambda_n$. Let m_k be multiplicity of the roots λ_k of the equation $a(\lambda) = 0$, i.e.

$$\left.\frac{d^j}{d\lambda^j} a(\lambda)\right|_{\lambda=\lambda_k} = 0, \quad \left.\frac{d^{m_k}}{d\lambda^{m_k}} a(\lambda)\right|_{\lambda=\lambda_k} \neq 0,$$

$$j = 1, 2, ..., m_k - 1, \quad k = 1, 2, ..., n. \tag{14}$$

From relation $a(\lambda_k)$ we have

$$e^-(x, \lambda_k) = \chi_{k,0}^+ e^+(x, \lambda_k), \ \chi_{k,0} \neq 0. \tag{15}$$

Further, if $\overset{\circ}{a}(\lambda_k) = 0$, i.e.

$$W\left[\dot{e}^-(x, \lambda_k), e^+(x, \lambda_k)\right] + W\left[e^-(x, \lambda_k), \dot{e}^+(x, \lambda_k)\right] = 0,$$

then taking into account (15), hence get

$$W\left[\dot{e}^-(x, \lambda_k) - \chi_{k,0}^+, \dot{e}^+(x, \lambda_k), e^+(x, \lambda_k)\right] = 0.$$

Consequently,

$$\dot{e}^{-}(x,\lambda_k)-\chi^{+}_{k,0},\dot{e}^{+}(x,\lambda_k)=\chi^{+}_{k,1},e^{+}(x,\lambda_k)$$

or

$$\dot{e}^{-}(x,\lambda_k)=\chi^{+}_{k,0},\dot{e}^{+}(x,\lambda_k)+\chi^{+}_{k,1},e^{+}(x,\lambda_k).$$

Procceding from the remaining relations of (14), arguing in the same way, we prove the following lemma.

Lemma 2. *There exists the chains of the numbers* $\left\{\chi^{+}_{k,0},\chi^{+}_{k,1},...,\chi^{+}_{k,m_k-1}\right\}$ *such that the following equalities are valid:*

$$\frac{1}{j!}\frac{d^j}{d\lambda^j}e^{-}(x,\lambda)\bigg|_{\lambda=\lambda_k}=\sum_{s=0}^{j}\chi^{+}_{k,j-s}\,\frac{1}{s!}\frac{d^s}{d\lambda^s}e^{+}(x,\lambda)\bigg|_{\lambda=\lambda_k} \tag{16}$$

$j=0,1,...,m_k-1;\ k=1,2,...,n,$ *where* $\chi^{+}_{k,0}\neq 0.$

It is seen from (16) that the conversion matrix from the vector

$$\left(e^{+}(x,\lambda_k),...,\frac{1}{(m_k-1)!}\frac{d^{m_k-1}}{d\lambda^{m_k-1}}e^{+}(x,\lambda_k)\right)^T$$

to the vector $\left(e^{-}(x,\lambda_k),...,\frac{1}{(m_k-1)!}\frac{d^{m_k-1}}{d\lambda^{m_k-1}}e^{-}(x,\lambda_k)\right)$ is of the form

$$\begin{pmatrix} \chi^{+}_{k,0} & 0 & 0 & ... & 0 \\ \chi^{+}_{k,1} & \chi^{+}_{k,0} & 0 & ... & 0 \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ \chi^{+}_{k,m_k-1} & \chi^{+}_{k,m_k-2} & \chi^{+}_{k,m_k-3} & ... & \chi^{+}_{k,0} \end{pmatrix}$$

Therefore, from (16) we find

$$\frac{1}{j!}\frac{d^j}{d\lambda^j}e^{+}(x,\lambda)\bigg|_{\lambda=\lambda_k}=\sum_{s=0}^{j}\chi^{-}_{k,j-s}\,\frac{1}{s!}\frac{d^s}{d\lambda^s}e^{-}(x,\lambda)\bigg|_{\lambda=\lambda_j}, \tag{17}$$

where the chains of the numbers $\left\{\chi^{-}_{k,0},\chi^{-}_{k,1},...,\chi^{-}_{k,m_k-1}\right\}$ and $\left\{\chi^{+}_{k,0},\chi^{+}_{k,1},...,\chi^{+}_{k,m_k}\right\}$ are connected with the relations

$$\chi^{+}_{k,0}\ \chi^{-}_{k,0}=1,$$

$$\chi^{\pm}_{k,j}=\frac{(-1)^j}{\left(\chi^{\pm}_{k,0}\right)^{j+1}}\begin{vmatrix} \chi^{\pm}_{k,1} & \chi^{\pm}_{k,0} & 0 & ... & 0 \\ \chi^{\pm}_{k,2} & \chi^{\pm}_{k,1} & \chi^{\pm}_{k,0} & ... & 0 \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ \chi^{\pm}_{k,j} & \chi^{\pm}_{k,j-1} & \chi^{\pm}_{k,j-2} & ... & \chi^{\pm}_{k,1} \end{vmatrix}. \tag{18}$$

Call the collection of the quantities

$$\left\{r^{-}(\lambda),\lambda_k,\chi^{-}_{k,j}\left(j=\overline{0,m_k-1},k=1,...,n\right)\right\}$$

and

$$\left\{r^{+}(\lambda),\lambda_k,\chi^{+}_{k,j}\left(j=\overline{0,m_k-1},k=1,...,n\right)\right\}$$

the left and right scattering data of the operator L.

Now, study some properties of the scattering data.At first we prove the following lemma:

Lemma 3: *The coefficients* $a(\lambda)$, $b(\lambda)$ *determined by formula (7), (8) admit the following representations:*

$$a(\lambda)=\left(1+\frac{i\beta}{2}\right)-\frac{1}{2i\lambda}\left\{\left(1+\frac{i\beta}{2}\right)\int\limits_{-\infty}^{\infty}q(t)\,dt+\int\limits_{0}^{+\infty}A(t)\,e^{i\lambda t}dt\right\},$$

$$b(\lambda)=\frac{i\beta}{2}e^{2i\lambda a}+\frac{1}{2i\lambda}\int\limits_{-\infty}^{+\infty}B(t)\,e^{-i\lambda t}dt,$$

where $A(t)\in L_1(0,+\infty)$, $B(t)\in L_1(-\infty,+\infty)$.

Proof. Without loss of generality, assume $a>0$. Differentiating and integrating by parts the representations for the solutions $e^{+}(x,\lambda)$ and $e^{-}(x,\lambda)$ and using the properties of the kernels of these representations [6], we get (for $x<a$)

$$e^{+\prime}(x,\lambda)=i\lambda\left(1+\frac{i\beta}{2}\right)e^{i\lambda x}+\frac{i\beta}{2}i\lambda e^{i\lambda(2a-x)}+$$

$$+\left[K^{+}(2a-x+0)-K^{+}(2a-x-0)\right]e^{i\lambda(2a-x)}-K^{+}(x,x)\,e^{i\lambda x}+$$

$$+\int\limits_{x}^{+\infty}K_x^{+\prime}(x,t)\,e^{i\lambda t}dt=i\lambda\left(1+\frac{i\beta}{2}\right)e^{i\lambda x}+\frac{i\beta}{2}i\lambda e^{i\lambda(2a-x)}+$$

$$+\frac{i\beta}{4}\left\{\int\limits_{x}^{a}q(\xi)\,d\xi-\int\limits_{a}^{+\infty}q(\xi)\,d\xi\right\}e^{i\lambda(2a-x)}-$$

$$-\frac{1}{2}\left(1+\frac{i\beta}{2}\right)\int\limits_{x}^{+\infty}q(\xi)\,d\xi e^{i\lambda x}+\int\limits_{x}^{+\infty}K_x'(x,t)\,e^{i\lambda t}dt,$$

$$e^{-\prime}(x,\lambda)=-i\lambda e^{-\lambda x}+K^{-}(x,x)\,e^{-i\lambda x}+$$

$$+\left[K^{-}(2a-x+0)-K^{-}(2a-x-0)\right]e^{-i\lambda(2a-x)}+\int\limits_{-\infty}^{x}K_x^{-\prime}(x,t)\,e^{-i\lambda t}dt,$$

$$\int\limits_{x}^{+\infty}K^{+}(x,t)\,e^{i\lambda t}dt=-\left[K^{+}(2a-x+0)-K^{+}(2a-x-0)\right]\frac{e^{i\lambda(2a-x)}}{i\lambda}-$$

$$-K^{+}(x,x)\frac{e^{i\lambda x}}{i\lambda}-\frac{1}{i\lambda}\int\limits_{x}^{+\infty}K_t^{+\prime}(x,t)\,e^{i\lambda t}dt,$$

$$\int\limits_{-\infty}^{x}K^{-}(x,t)\,e^{-i\lambda t}dt=\frac{K^{-}(x,x)}{i\lambda}e^{-i\lambda x}+\frac{1}{i\lambda}\int\limits_{-\infty}^{x}K_t^{-\prime}(x,t)\,e^{-i\lambda t}dt.$$

Therefore,

$$e^{+\prime}(0,\lambda)\,e^{-}(0,\lambda)=i\lambda\left(1+\frac{i\beta}{2}\right)+\frac{i\beta}{2}i\lambda e^{i\lambda 2a}-\left[\left(1+\frac{i\beta}{2}\right)+\frac{i\beta}{2}e^{i\lambda 2a}\right]\frac{1}{2}\int\limits_{-\infty}^{0}q(t)\,dt+$$

$$+\frac{i\beta}{4}\left\{\int\limits_{0}^{a}q(t)\,dt-\int\limits_{a}^{+\infty}q(t)\,dt\right\}e^{i\lambda 2a}-\frac{1}{2}\left(1+\frac{i\beta}{2}\right)\int\limits_{a}^{+\infty}q(t)\,dt+\int\limits_{a}^{+\infty}A_1(t)\,e^{i\lambda t}dt,$$

$$e^{+}(0,\lambda)\,e^{-\prime}(0,\lambda)=-i\lambda\left[\left(1+\frac{i\beta}{2}\right)-\frac{i\beta}{2}e^{i\lambda 2a}\right]+\frac{1}{2}\int\limits_{-\infty}^{0}q(t)\,dt\left(1+\frac{i\beta}{2}-\frac{i\beta}{2}e^{i\lambda 2a}\right)-$$

$$-i\lambda\left\{-\frac{i\beta}{4}\left(\int\limits_{0}^{a}q(t)\,dt-\int\limits_{a}^{+\infty}q(t)\,dt\right)\frac{e^{i\lambda x}}{i\lambda}-\right.$$

$$\left.-\frac{1}{2i\lambda}\left(1+\frac{i\beta}{2}\right)\int\limits_{0}^{+\infty}q(t)\,dt\right\}+\int\limits_{a}^{+\infty}A_2(t)\,e^{i\lambda t}dt.$$

Taking into account these relations in formula (7), we find

$$2i\lambda a(\lambda)=e^{+\prime}(0,\lambda)\,e^{-}(0,\lambda)-e^{+}(0,\lambda)\,e^{-\prime}(0,\lambda)=$$

$$=2i\lambda\left(1+\frac{i\beta}{2}\right)-\left(1+\frac{i\beta}{2}\right)\int\limits_{-\infty}^{+\infty}q(t)\,dt+\int\limits_{0}^{\infty}A(t)\,dt,$$

where $A(t)=A_1(t)-A_2(t)$.

Hence we find the representation for $a(\lambda)$.

The representation for $b(\lambda)$ is proved in the same way.

From this lemma it follows that the reflection coefficients satisfy the asymptotic equalities

$$r^{\pm}(\lambda)=r_0^{\pm}(\lambda)+O\left(\frac{1}{\lambda}\right),\quad(\lambda\to\pm\infty),$$

where

$$r_0^{\pm}(\lambda)=\mp\frac{i\beta}{2+i\beta}e^{\mp 2i\lambda a}$$

are the reflection coefficients of the operator L when $q(x)\equiv 0$. Further, from (9) we have $|a(\lambda)|>|b(\lambda)|\ (\lambda\in R^{*})$, consequently the inequality

$$\left|r^{\pm}(\lambda)\right|<1,\lambda\in R^{*}$$

is valid.

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