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SEMIGROUPS OF HOMEOMORPHISMS AND DIMENSION

Abstract

In this paper connection between semigroups of homeomorphisms of some topological spaces and dimension of these spaces is studied.

1.1. Let X be a topological space, $H(X)$ be the semigroup of all homeomorphic mappings of X into itself, $H'(X)$ be a subsemigroup of the semigroup $H(X)$, such that if $f,g \in H'(X)$ and $gX \subseteq fX$ then $f^{-1}g \in H'(X)$. Obviously, $H(X)$ itself is such a semigroup.

Proposition 1.2. If there exists a homeomorphism $b \in H'(X)$, such that for each homeomorphism $c \in H'(X)$ there exists a homeomorphism $a_c \in H'(X)$ satisfying the condition $bX \subseteq a_c cX$, then b belongs to the minimal ideal of the semigroup $H'(X)$.

Proof. We will show, that b belongs to each ideal of $H'(X)$. Let $I(X)$ be an ideal of $H'(X)$ and $c \in I(X)$. As $c \in I(X)$ so $a_c c \in I(X)$. Thus $b = a_c c\left((a_c c)^{-1} b\right) \in a_c cH' \subseteq IH' \subseteq I$. Therefore b belongs to each ideal of $H'(X)$. Hence, the intersection of all ideals of $H'(X)$is not empty and it is the minimal ideal of $H'(X)$.

1.3. Let X be a locally compact Hausdorff space having an open base $\{\mathcal{B}_\alpha\}_{\alpha\in A}$ under the condition that every element of this base is homeomorphic to X. Besides, for each point $\xi \in X$ there exists a neighbourhood V_ξ, such that for every element $\mathcal{B}_\alpha$ of this base there exists an open homeomorphic mapping $a_\alpha : X \to X$ under the condition that $V_\xi \subseteq a_\alpha \mathcal{B}_\alpha$. We will denote the class of all such spaces $\overline{\mathcal{L}}$. $OH(X)$ is the semigroup of all open homeomorphic mappings X into itself. $OH_k(X)$ is the subsemigroup of $OH(X)$ consisting of all such $a \in OH(X)$ that there exists compact $K_a \subseteq X$ under the condition that $aX \subseteq K_a$. Obviously, $OH_k(X)$ is an ideal of $OH(X)$. From proposition 1.2 it follows that $OH(X)$ has the minimal ideal [1]. We will denote it $I_m(X)$. Let D_X be such a subsemigroup of $OH(X)$ that $I_m(X) \subseteq D_X \subseteq OH(X)$. In particular, open subsets of the finite dimensional Euclidean spaces and the cube $D^r, r \geq \aleph_0$ [2], [3], [4] belong to the class $\overline{\mathcal{L}}$. Let R be a right ideal of D_X. We will denote sR the set $\bigcup\limits_{a\in R} aX$.

Lemma 1.4. *Let $\{R_k\}_{k=1}^n$ be a finite system of right ideals of D_X*

$$\bigcap_{k=1}^n R_k \neq \varnothing \leftrightarrow \bigcap_{k=1}^n sR_k \neq \varnothing$$

Lemma 1.5. *Let* $a, b \in D_X$ *and* $c \in I_m(X)$ *be a solution of the equation* $ax = b$, *then* $b \in I_m(X)$ *and* $\overline{bX} \subseteq aX$.

Lemma 1.6. *Let* $\xi \in X$ *and* V_ξ *is an arbitrary neighbourhood of* ξ. *There exists an element* $b \in I_m(X)$, *such that* $\xi \in bX \subseteq V_\xi$.

Lemma 1.7. *Let* $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *be a maximal centered system of principal right ideals of* $I_m(X)$. *There exists a unique point* $\xi \in X$, *such that if* $a \in I_m(X)$, $\xi \in aX$, *then* $aI_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. *Besides,* $\xi = \bigcap\limits_\alpha \overline{a_\alpha X}$.

Proof. It is clear that $s\left(a_\alpha I_m^1\right) = a_\alpha X$. From lemma 1.4 it follows that the system $\{a_\alpha X\}_{\alpha\in A}$ will be a centered system of sets of X. Let us consider all intersections of finite number elements of the system $\{a_\alpha X\}_{\alpha\in A}$. We will denote this new system $\{\Omega_\beta\}_{\beta\in B}$. It is obvious that $\{\Omega_\beta\}_{\beta\in B}$ will be the centered system of open sets of X. As $I_m(X) \subseteq OH_k(X)$ so the system $\left\{\bar\Omega_\beta\right\}_{\beta\in B}$ will be a centered system of compact subsets of X.

Hence $\bigcap\limits_\beta \bar\Omega_\beta \neq \varnothing$. Let us take a point $\xi \in \bigcap\limits_\beta \bar\Omega_\beta$. If $a \in I_m$ and $\xi \in aX$ then it is clear that $aX \cap \Omega_\beta \neq \varnothing$ for each $\beta \in B$. Let us choose a finite subsystem $\left\{a_k I_m^1\right\}_{k=1}^n$ of the system $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. There exists $\Omega_{\beta'}, \beta' \in B$, such that $\Omega_{\beta'} = \bigcap\limits_{k=1}^n a_k X$. That's why $aX \cap \bigcap\limits_{k=1}^n a_k X \neq \varnothing$. From lemma 1.4 it follows that $aI_m^1 \cap \bigcap\limits_{k=1}^n a_k I_m^1 \neq \varnothing$ and $aI_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$.

If $\xi' \neq \xi$ and $\xi' \in \bigcap\limits_\beta \bar\Omega_\beta$ then there exists neighbourhoods $V_{\xi'}$ and V_ξ, such that $V_{\xi'} \cap V_\xi = \varnothing$ and elements $b, c \in I_m$, such that $\xi \in bX \subset V_\xi$, $\xi' \in cX \subset V_{\xi'}$. It is clear that $bI_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. At the same way we can prove that $cI_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. From the other side as $V_{\xi'} \cap V_\xi = \varnothing$ so $cI_m^1 \cap bI_m^1 = \varnothing$ and it follows that $cI_m^1 \notin \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. Contradiction.

We will say that the system $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ corresponds to the point ξ.

Lemma 1.8. *Let* $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *be a maximal centered system of principal right ideals of* $I_m(X)$, *such that it corresponds to the point* ξ. *If* $\left\{a_\beta I_m^1\right\}_{\beta\in B}$ *is another maximal centered system of* $I_m(X)$, *such that it also corresponds to* ξ *then for any elements* $a_{\alpha'} I_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *and* $a_{\beta'} I_m^1 \in \left\{a_\beta I_m^1\right\}_{\beta\in B}$ *under the condition* $a_{\alpha'} I_m^1 \cap a_{\beta'} I_m^1 = \varnothing$ *there don't exist elements* $a_{\alpha''} I_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *and* $a_{\beta''} I_m^1 \in \left\{a_\beta I_m^1\right\}_{\beta\in B}$, *such that solutions of equations* $a_{\alpha'} x = a_{\alpha''}$ *and* $a_{\beta'} x = a_{\beta''}$ *belong to* $I_m(X)$.

Lemma 1.9. *Let* $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *be a maximal centered system which corresponds to* ξ. *If* $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ *is an arbitrary maximal centered system of* $I_m(X)$ *which corresponds to* $\xi' \neq \xi$, *then there exist elements* $a_{\alpha'} I_m^1$, $a_{\alpha''} I_m^1 \in \left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ *and* $a_{\gamma'} I_m^1$, $a_{\gamma''} I_m^1 \in \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$, *such that*

1. $a_{\alpha'} I_m^1 \cap a_{\gamma'} I_m^1 = \varnothing$

2. Solutions of equations $a_{\alpha'} x = a_{\alpha''}$ *and* $a_{\gamma'} x = a_{\gamma''}$ *belong to* $I_m(X)$.

Lemma 1.10. *Let $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ be a maximal centered system which corresponds to ξ. A subsystem $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ of $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ satisfies to the condition: 1) every maximal centered extension of $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ corresponds to ξ if and only if for each neighbourhood V_ξ of ξ there exists such a subsystem $\left\{a_k I_m^1\right\}_{k=1}^n \subset \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ that $\bigcap\limits_{k=1}^n a_k X \subseteq V_\xi$.*

Let us denote $\{S_i\}_{i\in I}$ the family of all maximal centered systems which correspond to ξ. We will consider all subsystems belonging to each element of $\{S_i\}_{i\in I}$ and satisfying to the condition 1). The family of all such subsystems we'll denote $\{s_j\}_{j\in J}$.

Lemma 1.11. *A subsystem $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma} \in \{s_j\}_{j\in J}$ satisfies to the condition:*

2) for each element $a_{\gamma'} I_m^1 \in \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ and any subsystem $\left\{b_\theta I_m^1\right\}_{\theta\in\Theta} \in \{s_j\}_{j\in J}$ there exists such a finite subsystem $\left\{b_k I_m^1\right\}_{k=1}^n \subset \left\{b_\theta I_m^1\right\}_{\theta\in\Theta}$ that $\bigcap\limits_{k=1}^n b_k I_m^1 \subseteq a_{\gamma'} I_m^1$ if and only if $\xi \in a_\gamma X$ for every $\gamma\in\Gamma$.

Lemma 1.12. *A subsystem $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma} \in \{s_j\}_{j\in J}$ and satisfying to the condition 2) we will call B- subsystem. A subsystem of right ideals $\{R_\theta\}_{\theta\in\Theta}$ of D_X we will call B-system of right ideals if for each B- subsystem $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma} \in \{s_j\}_{j\in J}$ there exists a finite subsystem $\left\{a_k I_m^1\right\}_{k=1}^n \subseteq \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ and $R_{\theta'} \in \{R_\theta\}_{\theta\in\Theta}$, such that $\bigcap\limits_{k=1}^n a_k I_m^1 \subseteq R_{\theta'}$.*

Lemma 1.13. *A system of right ideals $\{R_\theta\}_{\theta\in\Theta}$ of D_X is a B-system of right ideals of D_X if and only if $\bigcup\limits_\theta sR_\theta = X$.*

Proof. Let $\xi \in X$ and $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ be a maximal centered system which corresponds to ξ. If $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ is a B- subsystem of $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ then $\xi \in a_\gamma X$ for each $\gamma\in\Gamma$. If $\{R_\theta\}_{\theta\in\Theta}$ is a B- system of right ideals of semigroup D_X then there exists $R_{\theta'} \in \{R_\theta\}_{\theta\in\Theta}$ and a finite subsystem $\left\{a_k I_m^1\right\}_{k=1}^n \subseteq \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$, such that $\bigcap\limits_{k=1}^n a_k I_m^1 \subseteq R_{\theta'}$. It's clear that $\xi \in \bigcap\limits_{k=1}^n a_k X$. For points $a_k^{-1}\xi$, $k=1,2,...,n$ there exist elements $b_k \in I_m$ under the condition that $a_k^{-1}\xi \in b_k X$, $k=1,2,...,n$. Let us consider $\bigcap\limits_{k=1}^n a_k b_k X$. Let us take $c \in I_m$, such that $\xi \in cX \subseteq \bigcap\limits_{k=1}^n a_k b_k X$. It is clear that $a_k^{-1} cX \subseteq b_k X$ and $b_k\left(b_k^{-1} a_k^{-1} c\right) = a_k^{-1} c \in I_m$, $k=1,2,...,n$. Hence $c = a_k a_k^{-1} c \in a_k I_m^1$, $k=1,2,...,n$, $c \in \bigcap\limits_{k=1}^n a_k I_m^1$, $c \in R_{\theta'}$, $cX \subseteq sR_{\theta'}$. That's why $\xi \in sR_{\theta'}$ and $\bigcup\limits_\theta sR_\theta = X$.

Let us consider a system of right ideals of D_X such that $\bigcup\limits_\theta sR_\theta = X$. Let $\xi \in X$ then ξ belongs to some $sR_{\theta'}$. There exists $a \in R_{\theta'}$, such that $\xi \in aX$. Let $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$ be a maximal centered system which corresponds to ξ. $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ is a B- subsystem of $\left\{a_\alpha I_m^1\right\}_{\alpha\in A}$. As $\left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ is a B- subsystem so $\xi \in a_\gamma X$ for each $\gamma\in\Gamma$. For the point $a^{-1}\xi$ there exists an element $b \in I_m$, such that $a^{-1}\xi \in bX$.

The set abX is a neighbourhood of ξ. There exists a finite subsystem $\left\{a_kI_m^1\right\}_{k=1}^n \subset \left\{a_\gamma I_m^1\right\}_{\gamma\in\Gamma}$ under the condition that $\bigcap\limits_{k=1}^n a_kX \subseteq abX$. Let $c \in \bigcap\limits_{k=1}^n a_kI_m^1$. It is clear that $cX \subseteq a_kX$, $k = 1, 2, ..., n$ and $cX \subseteq \bigcap\limits_{k=1}^n a_kX$. Hence $cX \subseteq abX$, $a^{-1}cX \subseteq bX$, $b\left(b^{-1}a^{-1}c\right) = a^{-1}c \in I_m$, $c = a\left(a^{-1}c\right) \in aI_m^1 \subseteq R_{\theta'}$. That's why $\bigcap\limits_{k=1}^n a_kI_m^1 \subseteq R_{\theta'}$.

Theorem 1.14. *Let $X \in \overline{\mathcal{L}}$. $\dim X = n$ if and only if it is possible to refine in each finite $B-$ system of right ideals of D_X a finite B- system of right ideals of D_X of multiplicity $\leq n+1$ and besides, for some finite B- system of right ideals of D_X, multiplicity refined in it each finite B- system of right ideals of D_X is $\geq n+1$.*

Proof. Let $\dim X = n$ and $\{R_p\}_{p=1}^q$is a finite B- system of right ideals of D_x. It follows from lemma 1.13 that $\bigcup\limits_{p=1}^q sR_p = X$. Let us refine in $\{sR_p\}_{p=1}^q$ combinatorially a finite open covering $\{\Omega\}_{p=1}^q$ of multiplicity $\leq n+1$. Let us take for each point $\xi \in \Omega_{p'}$ an element $a_\xi \in R_p$, such that $\xi \in a_\xi X$. For the point $a_\xi^{-1}\xi$ there exists an element $b_\xi \in I_m$ under the condition that $a_\xi^{-1}\xi \in b_\xi X$. It is clear that $\xi \in a_\xi b_\xi X \cap \Omega_{p'}$. There exists an element $c_\xi \in I_m^1$, such that $\xi \in c_\xi X \subseteq a_\xi b_\xi X \cap \Omega_{p'}$. It follows that $a_\xi^{-1}c_\xi X \subseteq b_\xi X$, $b_\xi\left(b_\xi^{-1}a_\xi^{-1}c_\xi\right) = a_\xi^{-1}c_\xi \in I_m$ and $c_\xi = a_\xi\left(a_\xi^{-1}c_\xi\right) \in a_\xi I_m^1 \subseteq R_{p'}$. Let us denote $R'_{p'}$ the right ideal $\bigcup\limits_\xi c_\xi D_X$of D_X. Obviously, $R'_{p'} \subseteq R_{p'}$ and $sR'_{p'} = \Omega_{p'}$. We'll denote $mult\sigma$ the multiplicity of the system of sets σ. It follows from lemma 1.4 that $mult\left\{R'_p\right\}_{p=1}^q = mult\{\Omega\}_{p=1}^q \leq n+1$. As $\dim X = n$ so there exists a finite open covering $\{\Omega\}_{p=1}^q$ of X such that the multiplicity of any refined in it finite open covering is $\geq n+1$. Let $\xi \in \Omega_{p'}$, $1 \leq p' \leq q$. There exists $a_\xi \in I_m$, such that $\xi \in a_\xi\ X \subseteq \Omega_{p'}$. We will denote $R_{p'}$ the right ideal $\bigcup\limits_{\xi\in\Omega_{p'}} a_\xi D_X^1$. It is clear that $\bigcup\limits_{p=1}^q sR_p = X$. It follows from lemma 1.13 that the system $\{R_p\}_{p=1}^q$ is a B- system of right ideals of D_X. Let $\left\{\tilde{R}_k\right\}_{k=1}^m$ be a B-system of right ideals refined in it. Assume that $mult\left\{\tilde{R}_k\right\}_{k=1}^m < n+1$ and $\left\{R'_p\right\}_{p=1}^q$ is an enlargement of $\left\{\tilde{R}_k\right\}_{k=1}^m$ relatively to $\{R_p\}_{p=1}^q$. The system $\left\{R'_p\right\}_{p=1}^q$ is a B-system of right ideals of D_X refined in $\{R_p\}_{p=1}^q$ combinatorially and $mult\left\{R'_p\right\}_{p=1}^q < n+1$. Let us denote $\Omega'_p$the set sR'_p, $1 \leq p \leq q$. As $\left\{R'_p\right\}_{p=1}^q$ is a B-system so $\bigcup\limits_{p=1}^q \Omega'_p = \bigcup\limits_{p=1}^q sR'_p = X$. Besides, follows from $R'_p \subseteq R_p$ that $sR'_p \subseteq sR_p$, $\Omega'_p \subseteq \Omega_p$, $1 \leq p \leq q$. It follows from lemma 1.4 that $mult\left\{\Omega'_p\right\}_{p=1}^q = mult\left\{sR'_p\right\}_{p=1}^q = mult\left\{R'_p\right\} < n+1$.

Let $\{\Omega_p\}_{p=1}^q$ be a finite open covering of X and $\xi \in \Omega_{p'}$, $1 \leq p' \leq q$. Let us take $a_\xi \in I_m$, such that $\xi \in a_\xi X \subseteq \Omega_{p'}$. We'll denote $R_{p'}$ the right ideal $\bigcup\limits_{\xi\in\Omega_{p'}} a_\xi D_X^1$ of D_X. It's clear that $sR_{p'} = \Omega_{p'}$ and $\bigcup\limits_{p=1}^q sR_p = \bigcup\limits_{p=1}^q \Omega_p = X$. That's why $\{R_p\}_{p=1}^q$ is a finite B-system of right ideals of D_X. Let $\{R'_k\}_{k=1}^m$ be a finite B- system of right ideals of D_X of the multiplicity $\leq n+1$ refined in $\{R_p\}_{p=1}^q$. As $\{R'_k\}_{k=1}^m$ is a B-system so $\bigcup\limits_{k=1}^{m} sR'_k = X$. If $R'_{k'} \in \{R'_k\}_{k=1}^m$, $1 \leq k' \leq m$ then there exists $R_{p'} \in \{R_p\}_{p=1}^q$, such that $R'_{k'} \subseteq R_{p'}$. It follows that $sR'_{k'} \subseteq sR_{p'} = \Omega_{p'}$. Besides, as $mult\,\{R'_k\}_{k=1}^m \leq n+1$ and $mult\,\{R'_k\}_{k=1}^m = mult\,\{sR'_k\}_{k=1}^m$ so $mult\,\{sR'_k\}_{k=1}^m \leq n+1$. Let us take such a finite B-system $\{R_p\}_{p=1}^q$ of right ideals of D_X that the multiplicity of each finite B-system of right ideals of D_X refined in it is $\geq n+1$. As $\{R_p\}_{p=1}^q$ is a B- system so $\bigcup\limits_{p=1}^{q} sR_p = X$. We'll denote Ω_p the set sR_p. Let $\{\Omega'_k\}_{k=1}^n$ be a finite open covering of X refined in $\{\Omega_p\}_{p=1}^q$ and $\Omega'_{k'} \in \{\Omega'_k\}_{k=1}^n$, $1 \leq k' \leq n$. Let us take $\Omega_{p'} \in \{\Omega_p\}_{p=1}^q$, such that $\Omega'_{k'} \subseteq \Omega_{p'}$. As $\Omega_{p'} = sR_{p'}$ so there exists for each point $\xi \in \Omega'_{k'}$ an element $a_\xi \in R_{p'}$ under the condition that $\xi \in a_\xi X$. There exists for each open set $\Omega'_{k'} \cap a_\xi X$ an element $b_\xi \in I_m$, such that $\xi \in b_\xi X \subseteq \Omega'_{k'} \cap a_\xi X$, the equation $a_\xi x = b_\xi$ is solved and $x = a_\xi^{-1} b_\xi \in I_m$. It follows that $s\left(b_\xi D_X^1\right) = b_\xi X \subseteq \Omega'_{k'}$, $b_\xi = a_\xi \left(a_\xi^{-1} b_\xi\right)$, $b_\xi D_X^1 = \left(a_\xi^{-1} b_\xi\right) D_X^1 \subseteq a_\xi D_X^1 \subseteq R_{p'}$. Hence the right ideal $\bigcup\limits_{\xi \in \Omega'_{k'}} b_\xi D_X^1 \subseteq R_{p'}$ and $s\left(\bigcup\limits_{\xi \in \Omega'_{k'}} b_\xi D_X^1\right) = \Omega'_{k'}$. Let us denote $R'_{k'}$ the right ideal $\bigcup\limits_{\xi \in \Omega'_{k'}} b_\xi D_X^1$. Analogously, we will construct for each Ω'_k, $k = 1, 2, ..., m$ a right ideal R'_k. It is clear that the system of right ideals $\{R'_k\}_{k=1}^m$ will be refined in the system of right ideals $\{R_p\}_{p=1}^q$ and $\bigcup\limits_{k=1}^{m} sR'_k = \bigcup\limits_{k=1}^{m} \Omega'_k = X$. That's why the system $\{R'_k\}_{k=1}^m$ is a finite B- system of right ideals of D_X refined in the system $\{R_p\}_{p=1}^q$. As $mult\,\{R'_k\}_{k=1}^n \leq n+1$ so $mult\,\{\Omega'_k\}_{k=1}^m = mult\,\{sR'_k\}_{k=1}^m = mult\,\{R'_k\}_{k=1}^m \geq n+1$. Therefore the multiplicity of each open covering $\{\Omega'_k\}_{k=1}^n$ refined in $\{\Omega_p\}_{p=1}^q$ is $\geq n+1$. Hence $\dim X = n$. The theorem is proved.

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