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A FLUTTER OF A PLATE ARRANGED ON WEDGE CHEEK

Abstract

Assume that a surface of a wedge occupies the domain

$$V = \{0 \leq x < \infty, y = \pm kx, . |z| < \infty\}.$$

Then a thin elastic plate occupies the part $\Gamma = \{x_1 \leq x \leq x_2\}$ *of the plate* $y = kx$.

In the paper the flutter of this plate in supertonic gas flow whose velocity vector is directed along the axis x, *is investigated.*

Imagine a surface of a wedge that occupies the domain

$$V = \{0 \leq x < \infty, y = \pm kx, . |z| < \infty\}$$

in a Cartesian coordinates; a part Γ of the plane $y = kx$ is occupied by a thin elastic plate $\Gamma = \{x_1 \leq x \leq x_2\}$.

Investigate the flutter of this plate in supersonic gas flow whose velocity vector is directed along the axis x. On the example of the problem that we consider, mechanical effects that arise from new addends in surplus pressure are estimated basing on the results of the works $[1-4]$. In the considered case, instead of [5] we get the expression

$$\Delta p = \frac{2}{\gamma+1}\left(\rho^0 D^2 - \gamma\rho^0\right) - \frac{4\rho^0 D}{\gamma+1}\left(1 + 2\varepsilon - \varepsilon a(D)\right)\left(\frac{\partial w}{\partial t} + \upsilon\frac{\partial w}{\partial x}\right) -$$

$$-\frac{\rho^0 Dx}{\upsilon}\left(1 - \varepsilon\frac{12a(D)}{\gamma(\gamma+1)}\right)\left(\frac{\partial^2 w}{\partial t^2} + 2\upsilon\frac{\partial^2 w}{\partial t\partial x} + \upsilon\frac{\partial^2 w}{\partial x^2}\right). \quad (1)$$

Formula (1) is derived under assumption of cylindrical bending of the plate; we'll assume that it is valid in the general case $w = w(x, y, t)$ as well. For describing plate's motion, we use one of the variants of the theory of [5]; this is T.Karman equations

$$D_0\Delta^2 w = hL(w, \Phi) + q(x, y, t),$$

$$\Delta^2\Phi = -\frac{E}{2}L(w, w); \quad (2)$$

here w and Φ are deflection and stress function, respectively, the operator L is of the form

$$L(u, \upsilon) = \frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial^2 \upsilon}{\partial y^2} + \frac{\partial^2 u}{\partial y^2}\cdot\frac{\partial^2 \upsilon}{\partial x^2} - 2\frac{\partial^2 u}{\partial x\partial y}\cdot\frac{\partial^2 \upsilon}{\partial x\partial y}.$$

System (2) should be complemented by appropriate boundary conditions. The lateral load $q(x, y, t)$ consists of the sum of two addends: inertia forces $\rho h\partial^2 w/\partial t^2$ and Δp; we substitute it into (2)and get mathematical model of plate's vibrations. Estimate the addends in the right part of expression (1).

We compare the "additional mass"

$$\frac{\rho^0 Dx}{\upsilon}\left(1-\varepsilon\frac{12a\left(D\right)}{\gamma\left(\gamma+1\right)}\right)$$

with linear mass of the plate ρh; the cofactor in parenthesis is of unit order, $D\sim \upsilon tg\alpha$, therefore we get $tg\alpha = k$

$$\frac{\rho^0 Dx}{\upsilon}\left(1-\varepsilon\frac{12a\left(D\right)}{\gamma\left(\gamma+1\right)}\right)\frac{1}{\rho h}\sim\frac{\rho^0 xtg\alpha}{\rho h}.$$

For real limits of change of parameters this relation will be of order $10^{-1}\sim 10^{-3}$, therefore, in the first approximation we can ignore the summand with "additional mass".

Estimate the order of the second addend in the last parenthesis in (1). Typical time (period) of plate's vibrations t_0 is found from the relation

$$\frac{Eh^3}{\nu_2\left(1-\nu^2\right)l_0^4}\sim\frac{\rho h}{t_0^2}, t_0\sim\frac{3l^2}{c_0 h}, c_0^2=\frac{E}{\rho}.$$

Therefore we have

$$2\upsilon\frac{\partial^2 w}{\partial t\partial x}/\upsilon^2\frac{\partial^2 w}{\partial x^2}\sim\frac{c_0 h}{\upsilon l}.$$

Since $c_0\sim 5\cdot 10^3 m/\sec.$, $\upsilon = Ma_0\sim 10^3 m/\sec.$ for $M\sim 3$, $h/l\sim 10^{-2}\div 10^{-3}$ we get $c_0 h/\left(\upsilon l\right)\sim 10^{-1}\div 10^{-2}$. Consequently, we can also ignore this summand in the first approximation.

Represent Δp in (1) by the sum $\Delta p+\Delta p_g$:

$$\Delta p_c=\frac{2}{\gamma+1}\left(\rho^0 D^2-\gamma p^0\right)$$

Δp_g is a "dynamical" part of pressure. After the made estimations we finally get

$$\Delta p_g=-\frac{4\rho^0 D}{\gamma+1}\left(1+2\varepsilon-a\varepsilon\left(D\right)\right)\left(\frac{\partial w}{\partial t}+\upsilon\frac{\partial w}{\partial x}\right)-$$

$$-\rho^0 D\upsilon x\left(1-\varepsilon\frac{12a\left(D\right)}{\gamma\left(\gamma+1\right)}\right)\frac{\partial^2 w}{\partial x^2}. \tag{3}$$

The first summand is traditional "piston" formula with a coefficient that depends on the flow velocity in a very complicated way, the second summand has the sense of contractive normal force in the mean plane of plate and obviously, may have essential influence on vibrations character and flutter's critical velocity. The solutions cited below affirm this quality conclusion.

In the first example we consider a flutter of a plate in a simple linear statement; the aim is to clarify the role of the last summand in (3).

The vibration equation is of the form

$$D_0\Delta^2 w=\Delta p_g=\rho h\frac{\partial^2 w}{\partial t^2}, \tag{4}$$

the boundary conditions will be revised later.

By l_0 we denote typical size of the plate, $t_0 = l_0^2\sqrt{\rho h/D_0}$, introduce dimensionless coordinates x/l_0, y/l_0 and leave previous denotation for them. As usual, assume $w = W(x,y)\exp(wt)$ and substitute them into (4); Allowing for (3) we get

$$\Delta^2 W + A_2 M^2 \chi \frac{\partial^2 w}{\partial x^2} + A_1 M^2 \frac{\partial W}{\partial x} = \lambda W, \qquad (5)$$

$$\lambda + w^2 + A_0 M w = 0. \qquad (6)$$

Here we introduce the denotation

$$A_0 = \frac{8\gamma\sqrt{3(1-\nu^2)}}{\gamma+1}\frac{p_0 c_0 l_0^2}{E a_0 h^2}\left(1 + 2\varepsilon - \varepsilon a^*(tg\beta)\right),$$

$$A_1 = \frac{48\gamma(1-\nu^2)}{\gamma+1}\frac{p_0 l_0^3}{E h^3}\left(1 + 2\varepsilon - \varepsilon a^*(tg\beta)\right),$$

$$A_2 = 12\gamma(1-\nu^2)\frac{p_0 l_0^3}{E h^3} tg\beta\left(\lambda - \varepsilon\frac{12 a^*(tg\beta)}{\gamma(\gamma+1)}\right),$$

$$a^*(tg\beta) = 1 + \frac{2}{(\gamma-1)M^2 tg^2\beta}.$$

"Shock wave slope" $tg\beta$ is found from the equation

$$tg\beta = tg\alpha + \varepsilon a^*(tg\beta)\, tg\beta, \ \varepsilon = \frac{\gamma-1}{\gamma+1}. \qquad (7)$$

It is convenient to make substitution $Mtg\beta = u$, $Mtg\alpha = u_0$. Then (7) is a quadratic equation

$$2u^2 - (\gamma+1)u_0 u - 2 = 0 \qquad (8)$$

with respect to u. Consequently, we finally have a problem on eigen values λ in (5) under homogeneous boundary conditions. In complex domain λ, the stable vibrations domain is interior to the parabola

$$A_0^2 M^2 \operatorname{Re}\lambda = (Jm\lambda)^2. \qquad (9)$$

Consequently, the problem is to determine an eigen value that first will find itself on stability parabola; the velocity that corresponds to it will be critial velocity of the flutter.

On a simple example of a rectangular plate simply supported by all edges, we'll obtain some results of quality character. Assume that one of the sides of the plate is parallel to the edge of the wedge and is at dimensionless distance x_0; the domain S is occupied by the plate: $S = 0 \le x \le 1/\beta_0$; $0 \le y \le 1$. In the choosen system of coordinates, the problem (5) is written in the form:

$$\Delta^2 W + A_2 M^2 (x_0 + x)\frac{\partial^2 W}{\partial x^2} + A_1 M^2 \frac{\partial W}{\partial x} = \lambda W, \qquad (10)$$

$$x = 0,\ x = 1/\beta_0 : W = 0, \frac{\partial^2 W}{\partial x^2} = 0$$

$$y = 0,\ y = 1 :\ W = 0, \frac{\partial^2 W}{\partial y^2} = 0.$$

It is known that [7, 8, 9] for $\beta_0 \sim 1$, two-term aproximation of Galerkin's method in this statement of problem at $A_2 = 0$, for critical velocity of a flutter will give result of accebtable accuracy: therefore we put

$$W = (c_1 \sin \beta_0 \pi x + c_2 \sin 2\beta_0 \pi x) \sin \pi y.$$

After ordinary procedure, for the coeffisients c_1,c_2 we get a homogeneous linear system whose roots of characteristic equation is written in the form

$$\lambda_{1,2} = \lambda' \pm i\lambda''$$

$$\lambda' = \pi^4 \left(1 + 5\beta^2 + \frac{17}{2}\beta_0^4\right) - \frac{5}{2}\pi^2\beta_0^2 A_2 M^2 \left(x_0 + \frac{1}{2\beta_0}\right)$$

$$\lambda'' = \left\{ \left(\frac{16\beta_0}{3}\right)^2 B_1 B_2 A_1^2 M^4 - \left[3\pi^4\beta_0^2 \left(2+5\beta_0^2\right) - 3\pi^2\beta_0^2 A_2 M^2 \left(x_0 + \frac{1}{2\beta_0}\right)\right]^2 \right\}^{\frac{1}{2}} \quad (11)$$

$$B_1 = 1 - 8A_2/\left(3A_1\right); \; B_2 = 1 + 2A_2/\left(3A_1\right).$$

Hence we get an equation of stability parabola

$$A_0 M^2 \lambda' = \left(\lambda''\right)^2. \quad (12)$$

Determine M_0 by approximate criterion $\lambda'' = 0$ that corresponds to flutter's critical velocity ignoring aerodynamic damping ($A_0 = 0$); from we (11) get

$$\left[\sqrt{B_1 B_2} + \frac{9}{16}\beta_0\pi^2 \left(x_0 + \frac{1}{2\beta_0}\right)\frac{A_2}{A_1}\right] A_1 M_0^2 = \frac{9}{16}\pi^4\beta_0 \left(2 + 5\beta_0^2\right). \quad (13)$$

The estimates show that for $a^* \sim 1$ the expression in square brackets is greater than unit; the case of piston theory $A_2 = 0$, $B_1 = B_2 = 1$ and (13) yield

$$M_0^* = \frac{3\pi^2}{4\sqrt{A_1}}\sqrt{\beta_0 \left(2 + 5\beta_0^2\right)} > M_0.$$

Thus, the piston theory will give excessive estimations of flutter's critical velocity by approximate criterion $JmA = 0$. Quality analysis (for $\beta_0 = 1$, $x_0 = 0$) shows that exact criterium (12) also is reduced to the inequality $M_{cr} < M_{cr}^*$, where M_{cr}^* responds to the case $A_2 = 0$. The role of the last summand in expression (3) for ΔP_g increases for plates stretched in flow direction. In this case, one of the variants of geometric non-linear theory of plates, for example, Karman equations (2) should be used. Denote by w_0 a dimensionless bending referred to the parameter $x_1 = D_0/\left(Eh^2\right)$ and by Φ_0 a dimensionless stress function referred to the parameter $x_2 = \dfrac{D_0}{h}$. We determine the functions w_0 and Φ_0 from the Karman system for quasistatic loading:

$$\Delta P_{kco} = \frac{2}{\gamma + 1}\left(\rho_0 D - \gamma\rho_0\right) - \frac{4\rho_0 D\upsilon}{\gamma + 1}\left(1 + 2\varepsilon - \varepsilon a\left(D\right)\right)\frac{\partial w_0}{\partial x} -$$

$$-\rho_0 D\upsilon x \left(1 - \varepsilon\frac{12a\left(D\right)}{\gamma\left(\gamma + 1\right)}\right)\frac{\partial^2 w_0}{\partial x^2}.$$

Assume $w = w_0 + w_1$, $\Phi = \Phi_0 + \Phi_1$ subsitute them into (2) and linearize by small perturbations w_1, Φ_1. We'll not take into account the equation for perturbations Φ_1

$$\Delta^2 \Phi_1 + \frac{1}{2} L\left(w_0, W_1\right) = 0$$

and in the remaining equation we neglect influence of Φ_1 on W_1. As a result, we get a simplified linearized statement a problem on plate's flutter located on wedge cheek when it is streamlined by supersonic gas flow ($w = W \exp\left(wt\right)$)

$$\Delta^2 W + A_2 M^2 \frac{\partial^2 W}{\partial x^2} + A_1 M^2 \frac{\partial W}{\partial x} - L\left(\Phi_0, W\right) = \lambda W.$$

Together with homogeneous boundary conditions, this composes an eigen value problem. Cite the results of concrete calculations on definition of critical velocity of rectangular plate's flutter. Geometry of the plate: dimension in the direction of flow (axis x) is l_1; in the direction of axis y is l_2; thickness $h = 2 \cdot 10^{-3} m$. Mechanical characteristics of the material: $E = 2 \cdot 10^{11} Pa$, $\rho = 8 \cdot 10^6 \widetilde{a}/i^3$, $\nu = 0.3$. Flow's parameters: $\rho^0 = 10^5 Pa$, $a_0 = 330 m/\sec$, $\gamma = 1.4$. Distance $x_0 = 1m$. The parameters: lenght of plate's sides l_1, l_2, half-opening angle of the wedge α varied. All calculations were conducted in Galerkin's two-term approximation

$$W = \left(c_1 \sin \frac{\pi\left(x - x_0\right)}{l_1} + c_2 \sin \frac{2\pi\left(x - x_0\right)}{l_1}\right) \sin \frac{\pi y}{l_2}$$

The values of critical velocity (in Mach numbers) that were determined from the Karman system are cited without simplifications for different values of l_1, l_2, and α.

Table 1

α, degree	$l_1 = 0,25$ $l_2 = 0,3$	$l_1 = 0,25$ $l_2 = 0,25$	$l_1 = 0,3$ $l_2 = 0,25$	$l_1 = 0,35$ $l_2 = 0,25$
10	16,9	14,4	12,7	11,16
15	15,1	11,7	9,7	8,5
20	15,1	10,7	7,9	6,9

For comparison, critical values of M obtained by the linear theory are given in table 2.

Table 2

α, degree	$l_1 = 0,25$ $l_2 = 0,3$	$l_1 = 0,25$ $l_2 = 0,25$	$l_1 = 0,35$ $l_2 = 0,25$
10	8,2	8,8	7,2
15	6,5	6,8	5,3
20	5,3	5,4	4,3

As is seen, in calculations by non-linear theory, essentially great critical velocities are obtained. Stabilizing influence of stretching stresses in mean plane manifests itself. On the other hand, in linear theory we reveal non-monotonic dependence of M_{cr} on plate's sizes. Non-linear theory doesn't shows such an effect. A flutter problem of the strip $l_2 \to \infty$ with sizes $l_1 = 0.25$, $h = 10^{-3}$; $x_0 = 2$ is solved. The remaining parameters are the same. The results are in table 3.

Table 3

α, degree	8	10	12
M_{cr}	18.1	17.8	18.4

Interesting conclusions follow from the results represented in this table. The plate is twice thin than in previous calculations, but distance x_0 is twice long; as a result, the values of M_{cr} are the same as in table 1. The role of parameter x_0 manifests itself. Secondly, minimum M_{cr} for $a = 10^0$ is obviously seen.

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