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# A PROBLEM ON CRACK NUCLEATION UNDER INNER COMPRESSION OF CYLINDRICAL BODIES

**Abstract**

*A problem of fracture mechanics on nucleation of crack type defect in plunger pair bushing is considered. It is assumed that under repeated reciprocating motion of a plunger there happens crack nucleation and failure of materials of pair elements embryonic cracks are simulated by bridged prefracture strips that are considered as areas of weakened interpartide bonds of the matherial. It is accepted that the inner boundary of the bushing is close to the annular one and has a rough surface.*

Contact deformation of cylindrical bodies of close radii under inner compression is considered. It is assumed that the surfaces of the bodies in the contact area is rough. We assume that the outer cylinder (bushing) is an unrestricted plate with an opening close to anular one wherein elastic cylinder (shaft) is inserted. Concentrated power $P_0$ pressing it to the hole's boundary and concentrated pair whose moment is determined from the cylinder's limit equilibrium condition under the action of Coulomb friction forces is applied to the center of the shaft.

For determining contact pressure it is necessary to consider [1,2] a contact problem on pressing of a plunger into a bushing's surface involving wear. Let on some unknown part shaft with mechanical characteristics $G_1$ and $\mu_1$ be retained against internal surface of a bushing with mechanical characteristics $G$ (shear modulus) and $\mu$ (Poisson ratio).

The condition connecting replacements of a bushing and shaft is written in the form [1,2]

$$v_1 + v_2 = \delta(\theta) \quad \theta_1 \leq \theta \leq \theta_2. \tag{1}$$

Here $\delta(\theta)$ is a slip of surface point of the bushing and shaft determined by the form of inner surface of the bushing and pluhger, and also by the quantity of the pressing force $P_0$; $\theta_2 - \theta_1$ is quantity of contact angle (area).

In the contact zone, in addition to contact pressure there is a tangential stress $\tau_{r\theta}$ connected with contact pressure $p(\theta, t)$ by the Coulomb law

$$\tau_{r\theta}(\theta, t) = fp(\theta, t), \tag{2}$$

where $f$ is a friction coefficient of the pair "bushing-shaft".

Refer the contact pair bushing to polar system of coordinates $r\theta$, for that we choose an origin of coordinates at the center of circle $L$ of radius $R$.

We'll assume that inner contour of the bushing and external contour of the shaft are close to annular one.

Represent the boundary $L'$ of inner contour of the bushing in the form

$$r = \rho(\theta), \quad \rho(\theta) = R + \varepsilon H(\theta),$$

where $\varepsilon = R_{\max}/R$ is a small parameter; $R_{\max}$ is the greatesr height of the bulge (cavity) of unevennes of friction surface.

The coefficients of the Fourier series for the function $H(\theta)$ :

$$H(\theta) = \sum_{k=0}^{n} \left(a_k^0 \cos k\theta + b_k^0 \sin k\theta\right),$$

are found by means of profilogram of the treated surface of the bushing that describes each inner profile of the bushing.

In the similar way, the shaft's contour may be represeted as.

$$\rho_1(\theta) = R' + \varepsilon H_1(\theta), \quad H_1(\theta) = \sum_{k=0}^{n} \left(a_k^1 \cos k\theta + b_k^1 \sin k\theta\right).$$

It is assumed that the bushing and shaft wear is of abrasive character.

For displacements of the points of friction surface of the bushing we have

$$v_1 = v_{1y} + v_{1sh} + v_{1i}\ , \tag{3}$$

where $v_{1y}$ are elastic displacements of bushing's contact surface; $v_{1sh}, v_{1i}$ are displacements caused by removal of microbulges and by bushing surface wear, respectively.

Similarly, for displacemments of bushing's contact surface we have

$$v_2 = v_{2y} + v_{2sh} + v_{2i}\ . \tag{4}$$

Velocity of alternations of surface displacements in the course of bushing and shaft wear will be [2]

$$\frac{dv_{ju}}{dt} = K^{(j)} p(\theta, t) \qquad (j = 1, 2), \tag{5}$$

where $K^{(j)}$ is the wear coefficient of the bushing and shaft's material $(j = 1, 2)$ respectively.

As in the functioning process of the contact pair the bushing will be loaded by power load there will arise prefracture zones that will be simulated as areas of interparticle bonds of the material.

Interaction of the lips of thse areas is similated by introducing the bouds possessing the given deformation diagram between the lips of prefracture strips of bonds.

Physical nature of such bonds and dimensions of prefracture zones wherein interaction of interparticle bonds area lips are realized, depend on the form of the material.

Since the indicated zones are small compared with the remaining part of the bushing, one can mentally remove then and replace by sections whose surfafaces interact between themselves by some law corresponding to the action of the removed material.

It is assumed that the law of deformation of bonds is given. Notice that equation of deformation of bonds for different materials are considered in [3-5].

In the investigated case, arise of a defect (crack) is a process of going the prefracture area to the area of broken bonds between material's surface.

We'll assume that the prefracture strip is criented in the direction of maximal stretching stresses arising in the bushing.

Let's consider a prefracture strip of length $2l$ allocated on the segment $|x_1| \leq l$, $y_1 = 0$. At the center of the prefracture strip we locate an origin of local system of coordinates $x_1O_1y_1$ whose axis $x_1$, coincides with the line of the strip and makes an angle with the axis $x\,(\theta = 0)$. The prefracture strip lips interact so that this interaction (bonds between lips) restrains the defect (crack) nucleation.

For mathematical descriprion of interaction of prefracture strip lips, it is assumed that between the lips these are bonds whose deformation law is given. Under the action of external loads connecting the prefracture strip lips there will arise normal $q_{y_1}(x_1)$ and tangential $q_{x_1y_1}(x_1)$ tractions.

Consequently, normal and tangential stresses numerically equal $q_{y_1}(x_1)$ and $q_{x_1y_1}(x_1)$ respectively, well be applied to the prefracture strip lips. The quantities of these stresses are not known beforehand and are to be determined in the process of solution of the boundary value problem of fracture mechanics.

For determining the replacements $v_{1y}$ and $v_{1sh}$ it is necessary to solve the following problem of elasticity theory for a bushing:

$$\begin{gathered}\sigma_n = -p(\theta);\ \tau_{nl} = -fp(\theta)\ \text{ for } r = \rho \text{ in a contact area}\\ \sigma_n = 0;\ \tau_{nl} = 0 \text{ for } r = \rho \text{ outsid the contact area}\end{gathered} \tag{6}$$

on prefracture strip lips

$$\sigma_{y_1} = q_{y_1}(x_1);\quad \tau_{x_1y_1} = q_{x_1y_1}(x_1)\quad \text{for } |x_1| \leq l, \tag{7}$$

$n, t$ are natural coordinates; $\sigma_n$, $\sigma_t$ and $\tau_{nt}$ are stress tensor components.

In the similar way we state a problem of elasticity theory for determining displacements $v_{2y}$ and $v_{2sh}$ of contact surface of the shaft

$$\begin{gathered}\text{for } r = -\rho(\theta)\ \ \sigma_n = -p(\theta);\ \tau_{nt} = -fp(\theta)\ \text{ for } r = \rho \text{ in a contact area}\\ \sigma_n = 0;\ \tau_{nl} = 0 \text{ for } r = \rho \text{ outsid the contact area}\end{gathered} \tag{8}$$

Here the function $C(x_1, \sigma_1)$ may be considered as an effective compliance of tension dependent bonds; $\sigma_1$ is modulus of force vector in bonds $(u^+, -u^-)$ is tangential, $(v^+, -v^-)$ is normal constituent of the crack lips opening.

Using the calculation method stated in [6] we find boundary conditions at each approximation:

for zero approximation of the problem

$$\begin{gathered}\sigma_r^{(0)} = -p^{(0)}(\theta)\,;\ \tau_{n\theta}^{(0)} = -fp^{(0)}(\theta)\ \text{ for } r = R \text{ in the contact area}\\ \sigma_r^{(0)} = 0;\ \tau_{n\theta}^{(0)} = 0 \text{ for } r = R \text{ out of contact area}\end{gathered} \tag{10}$$

on the prefracture strip lips

$$\sigma_{y_2}^{(0)} = q_{y_1}^{(0)};\quad \tau_{x_1 y_1} = q_{x_1 y_1}^{(0)} \quad \text{for} \quad |x_1| \le l \tag{11}$$

for the first approximation of the problem

$$\begin{gathered}\sigma_r^{(1)} = N - p^{(1)}(\theta)\,;\quad \tau_{r\theta}^{(1)} = T - fp^{(1)}(\theta) \quad \text{for } r = R \text{ on the contact area}\\ \sigma_r^{(1)} = N;\quad \tau_{r\theta}^{(1)} = T \quad \text{for } r = R \text{ out the contact area}\end{gathered} \tag{12}$$

on the prefracture strip lips

$$\sigma_{y_1}^{(1)} = q_{y_1}^{(1)};\quad \tau_{x_1 y_1}^{(1)} = q_{x_1 y_1}^{(1)} \quad \text{for} \quad |x_1| \le l. \tag{13}$$

Here $N = -H(\theta)\dfrac{\partial \sigma_r^{(0)}}{\partial r} + 2\tau_{r\theta}^{(0)}\dfrac{1}{R}\dfrac{dH}{d\theta}$; for $r = R$

$$T = \left(\sigma_\theta^{(0)} - \sigma_r^{(0)}\right)\frac{1}{R}\frac{dH}{d\theta} - H(\theta)\frac{\partial \tau_{r\theta}^{(0)}}{\partial r}.$$

Similarly, we can write the boundary conditions at each approximation for a shaft.

Additional relation (9) accepts the following form:

$$\begin{gathered}\left(v^{(0)+}(x_1,0) - v^{(0)-}(x_1,0)\right) - i\left(u^{(0)+}(x_1,0) - u^{(0)-}(x_1,0)\right) =\\ = C\left(x_1, \sigma_1^{(0)}\right)\left[q_{y_1}^{(0)}(x_1) - iq_{x_1 y_1}^{(0)}(x_1)\right]\end{gathered} \tag{14}$$

at the first approximation

$$\begin{gathered}\left(v^{(1)+}(x_1,0) - v^{(1)-}(x_1,0)\right) - i\left(u^{(1)+}(x_1,0) - u^{(1)-}(x_1,0)\right) =\\ = C\left(x_1, \sigma_1^{(1)}\right)\left[q_{y_1}^{(1)}(x_1) - iq_{x_1 y_1}^{(1)}(x_1)\right].\end{gathered} \tag{15}$$

By means of Kolosov-Muskheleshvili formulae [7], we write the boundary conditions of the problem at zero approximation (10)-(11) for complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$. On annular boundaries of the bushing they will be of the form

$$\Phi^{(0)}(z) + \overline{\Phi^{(0)}(z)} - e^{2i\theta}\left[\overline{z}\Phi^{(0)'}(z) - \Psi^{(0)}(z)\right] = X^{(0)}(\theta) \tag{16}$$

$$z=\mathrm{Re}^{i\theta};\quad X^{(0)}(\theta)=\begin{cases}-(1-if)\,p^{(0)}(\theta) & \text{on a contact area}\\ 0 & \text{out of a contact area}\end{cases}$$

Boundary conditions on the strip lips will be writen as:

$$\Phi^{(0)}(t)+\overline{\Phi^{(0)}(t)}+\bar{t}\Phi^{(0)'}(t)+\Psi^{(0)}(t)=q_{y_1}^{(0)}+iq_{x_1y_1}^{(0)} \tag{17}$$

where $t$ is an affix of points of prefracture strip lips.

We look for the potentials $\Phi^{(0)}(z)$, , $\Psi^{(0)}(z)$, $\Phi_1^{(0)}(z)$, $\Psi_1^{(0)}(z)$, $\Phi_2^{(0)}(z)$, $\Psi_2^{(0)}(z)$ and in the form

$$\Phi^{(0)}(z)=\sum_{k=0}^{2}\Phi_k^{(0)}(z);\quad \Psi^{(0)}(z)=\sum_{k=0}^{2}\Psi_k^{(0)}(z) \tag{18}$$

$$\Phi_1^{(0)}(z)=\frac{1}{2\pi}\int_{-l}^{l}\frac{g_k^0(t)\,dt}{t-z_1}; \tag{19}$$

$$\Psi_1^{(0)}(z)=\frac{1}{2\pi}e^{-2i\alpha}\int_{-l}^{l}\left[\frac{\overline{g_k^0(t)}}{t-z_1}-\frac{\overline{T_k}e^{i\alpha}}{(t-z_1)^2}g^0(t)\right]dt;$$

$$T_1=te^{i\alpha}+z_1^0;\quad z_1=e^{-i\alpha}\left(z-z_1^0\right)$$

$$\Phi_2^{(0)}(z)=\frac{1}{2\pi}\int_{-l}^{l}\left[\left(-\frac{1}{z}-\frac{\overline{T_1}}{z-\overline{T_1}}\right)e^{i\alpha}g^0(t)+\overline{g^0(t)}\cdot e^{-i\alpha}\frac{1-T_1\overline{T_1}}{\overline{T_1}\left(1-z\overline{T_1}\right)^2}\right]dt$$

$$\Psi_2^{(0)}(z)=\frac{1}{2\pi z}\int_{-l}^{l}\left\{g^0(t)\left[\frac{1}{zT_1}-\frac{2}{z^2}-\frac{\overline{T_1}}{z\left(1-z\overline{T_1}\right)}+\frac{\overline{T_1^2}}{\left(1-z\overline{T_1}\right)^2}\right]e^{i\alpha}+\right.$$
$$\left.+e^{-i\alpha}\overline{g^0(t)}\left[-\frac{1}{1-zT_1}+\frac{1-T_1\overline{T_1}}{z\overline{T_1}\left(1-z\overline{T_1}\right)^2}-\frac{2\left(1-T_1\overline{T_1}\right)}{\left(1-z\overline{T_1}\right)^3}\right]\right\}dt. \tag{20}$$

Here $g^0(t)$ is a desired function characterizing the displacements in going through the prefracture strip.

For defining the potentials $\Phi_0^{(0)}(z)$ and $\Psi_0^{(0)}(z)$ we use N.I. Muskhelechvili method [7].

$$\Phi_0^{(0)}(z)=-\frac{1}{2\pi z}\int\frac{X^{(0)}(\sigma)\,d\sigma}{\sigma-z},\quad \sigma=e^{i\theta} \tag{21}$$

$$\Psi_0^{(0)}(z)=\frac{1}{z^2}\Phi_0^{(0)}(z)+\frac{1}{z^2}\overline{\Phi}_0^{(0)}(z)\left(\frac{1}{z}\right)-\frac{1}{z}\Phi_0^{(0)'}(z).$$

Satisfying the boundary condition on the prefracture strip lips by the functions (18)-(20), we find singular integral equation with respect to the function $g^0(x_1)$:

$$\int_{-l}^{l}\left[R(t,x_1)\,g^0(t)+S(t,x_1)\,\overline{g^0(t)}\right]dt=\pi\left[q_{y_1}^0-iq_{x_1y_1}^{(0)}+f^0(x_1)\right] \tag{22}$$

$$|x_1| \leq l$$

$$f^{(0)}(x_1) = -\left[\Phi_0^{(0)}(x_1) + \overline{\Phi_0^{(0)}(x_1)} + x_1\overline{\Phi_0^{(0)'}(x_1)} + \overline{\Psi_0^{(0)}(x_1)}\right].$$

To the singular integral equation for the inner prefracture strip at zero approximation we should add additional equality

$$\int_{l}^{l} g^{(0)}(t)\,dt = 0. \tag{23}$$

By means of the algebraization procedure [8,9], under the condition (23), the singular integral equation (22) is reduced to the system of $M$ complex algebraic equations for finding $M$ unknowns $g^{(0)}(t_m) = v^{(0)}(t_m) - iu^0(t_m)$ $(m = 1, 2, ..., M)$

$$\frac{1}{M}\sum_{m=1}^{M} l\left[g^{(0)}(t_m)R(lt_m, lx_r) + \overline{g^{(0)}(t_m)}S(lt_m, lx_r)\right] =$$

$$= f^{(0)}(x_r) + q_{y_1}^{(0)}(x_r) - iq_{x_1y_1}^{(0)}(x_r), \quad r = 1, 2, ..., M-1 \tag{24}$$

$$\sum_{m=1}^{M} g^{(0)}(t_m) = 0,$$

where $t_m = \cos\frac{2m-1}{2M}\pi$ $(m = 1, 2, ..., M)$; $x_r = \cos\frac{\pi r}{M}$ $(r = 1, 2, ..., M-1)$.

If in (24) we go over to complexly conjugated values, we get $M$ algebraic equations more. The right hand side of (24) contains unknown values of the tractions $q_{y_1}^{(0)}(x_r)$ and $q_{x_1y_1}^{(0)}(x_r)$ in bonds.

Additional condition (14) at zero approximation is the condition determining tractions in bonds arieung on prefracture strip lips

$$g^{(0)}(x_1) = \frac{2G}{i(i+k_b)}\frac{d}{dx_1}\left[C\left(x_1, \sigma_1^{(0)}\left(q_{y_1}^{(0)}(x_1) - iq_{x_1y_1}^{(0)}(x_1)\right)\right)\right], \tag{25}$$

where $k_b = 3-4\mu$ for plane deformation, $k_b = (3-\mu)/(1+\mu)$ for plane stress state.

For constructing missing algebraic equations for finding approximate values of the forces $q_{y_1}^{(0)}(x_r)$ and $q_{x_1y_1}^{(0)}(x_r)$ at the nodal points we require the conditions (25) to be fulfilled at nodal points For that we use the finite differences method.

We need two complex equations determining the dimensions of prefracture strip for closeness of the obtained system. Writing the stress finiteness conditions we find two missing equations more in the following form

$$\begin{aligned}\sum_{m=1}^{M}(-1)^m g^{(0)}(t_m)\,ctg\frac{2m-1}{4M}\pi = 0,\\ \sum_{m=1}^{M}(-1)^{M+m} g^{(0)}(t_m)\,ctg\frac{2m-1}{4M}\pi = 0.\end{aligned} \tag{26}$$

By means of complex potentials (18)-(20) and Kolosov-Muskheleshvili formulae [7] and integration of kinetic equation (5) wear of bushing's material at the zero approximation we find the displacements $v_1^{(0)}$ of bushing's contact surface. In the similar way, we find the solution of the elasticity theory problem for a shaft in the zero approximation. Using this solution and kinetic equation of shaft's material wear at zero approximation we find the displacements $v_2^{(0)}$ of the shaft's contact surface.

We substitute the found quantities $v_1^{(0)}$ and $v_2^{(0)}$ into the basic contact equation (1) at zero approximation.

For algebraization of the basic contact equation, the unknown functions of contact pressure at zero approximation are found in the form of expansions

$$p^{(0)}(\theta,t)=p_0^0(\theta)+tp_1^0(\theta)+...;$$

$$p_0^0(\theta)=\sum_{k=0}^{\infty}\left(\alpha_k^0\cos k\theta+\beta_k^0\sin k\theta\right);$$

$$p_1^0(\theta)=\sum_{k=0}^{\infty}\left(\alpha_k^1\cos k\theta+\beta_k^1\sin k\theta\right). \tag{27}$$

Sunstituting the relation in the basic contact equation at zero approximation, we get functionsl equations for sequential determination of $p_0^0(\theta)$, $p_1^0(\theta)$ and etc. For constructing algebraic system for finding $a_k,\beta_k$ we equate the coefficients for the same trigonometric functions in the left and right hand sides of the functional equation of the contact problem. We get an infinite algebraic system with respect to $\alpha_k^0\,(k=0,1,2,...)$, $\beta_k^0\,(k=1,2,...)$ and $a_k^1,\beta_k^1$ and etc.

Because of unknown quantities $\theta_1$ and $\theta_2$ the system of equations turns into nonlinear one. For determining the quantities $\theta_1$ and $\theta_2$ $\left(\theta_1=\theta_1^0+\varepsilon\theta_1^1+...;\ \theta_2=\theta_2^0+\varepsilon\theta_2^1+...\right)$ we have the condition:

for the zero approximation

$$p^{(0)}\left(\theta_1^0\right)=0;\qquad p^{(0)}\left(\theta_2^0\right)=0;$$

for the first approsimation

$$p^{(0)}\left(\theta_1^1\right)=0;\qquad p^{(0)}\left(\theta_2^1\right)=0.$$

The right hand sides of infinite algebraic systems with respect to $a_k,\beta_k$ a contain integrals of the unknown function $q^{(0)}(x_1)$. Thus, the infinite algebraic system with respect to $a_k,\beta_k$ and finite systems with respect to $g^{(0)}(x_1)$, $q_{y_1}^{(0)}(x_r)$ and $q_{x_1y_1}^{(0)}(x_r)$, $l$ are connected between themselves and they must be solved jointly. The conbined system of equations even for linear-elastic bonds became nonlinear because of unknown quantities $\theta_1,\theta_2,l$. For its solution at the zero approximation, the reduction and succussive approximations methods were used [9].

In the case non-linear law of deformation of bonds, for determiming tractions in bonds we also use iteration algorithm similar to the method of elastic solutions [10].

Nonlinear part of the bonds deformation curve is represented in the form of bilinear dependence [11] whose outgoing section corresponds to elastic deformation of bonds $(0 < V(x_1) < V_*)$ with maximal tension of bonds. For $V(x_1) < V_*$ the deformation law was describes by a nonlinear dependence determined by two points $(V_*, \sigma_*)$ and $(\delta_{cr},\ \sigma_{cr})$, moreover for $\sigma_{cr} \geq \sigma_*$ we have increasing linear dependece (linear hardening corresponds to elasticoplastic deformation of bonds).

After defining the desired quantities of the zero approximation we can construct the solution of the problem at the first approximation $N$ and $T$ are determined on the base of the obtained solution for $r = R$. The boundary conditions (12), (13) may be written in the form of a boundary value problem for finding complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$ that we seek in the form of (18) with obvious altermations. The further course of the solution is at the zero approximation. The obtained complex integral equation with respect to $g^{(1)}(t)$, $\overline{g^{(1)}(t)}$ under additional condition of type (23) by means of the algebtaization system is reduced to the system of $M$ algebraic equations for determining $N_0 \times M$ unknowns $g^{(1)}(t_m)$ $(m = 1, 2, ..., M)$.

The desired expansion coefficients of the contact pressure $p^{(1)}(\theta)$ the unknown values of tractions in bonds $q_{y_1}^{(0)}(x_1)$ and $q_{y_1}^{(0)}(x_r)$ are contained in the right hand side of this system.

Construction of missing equations for determining unknown tractions at the nodal poinds and prefracture zone sizes are realized as in the zero approxiation. A problem of theory of elasticity for a shaft at the first approximation is solved in the some way. Algebrazation of solving equation of the contact problem at the first approximation is carried out similar to the zero approximation. For that, the desired functions of contract pressure are represented in the form

$$p^{(1)}(\theta, t) = p_0^1(\theta) + tp_1^1(\theta) + ...;$$

$$p_0^1(\theta) = \alpha_{0,0}^1 + \sum_{k=0}^{\infty} \left(\alpha_{k,0}^1 \cos k\theta + \beta_{k,0}^1 \sin k\theta\right);$$

$$p_1^1(\theta) = \alpha_{0,1}^1 + \sum_{k=0}^{\infty} \left(\alpha_{k,1}^1 \cos k\theta + \beta_{k,1}^1 \sin k\theta\right);$$

As a result we get infinite linear algebraic systems with respect to $\alpha_{0,0}^1, \alpha_{k,0}^1, \beta_{k,0}^1$ $(k = 1, 2, ...)$ and $\alpha_{0,1}^1, \alpha_{k,1}^1, \beta_{k,1}^1$ $(k = 1, 2, ...)$ and etc.

The system of equations becomes nonlinear because of the unknown quantities $\theta_1^1$ and $\theta_2^1$. The constructed combined system of equations is closed and under the given functions $H(\theta)$ and $H_1(\theta)$ allows to find the contact pressure, tractions in bonds, prefracture strip sizes, stress-strain state, bushing and contact pair shaft wear by numerical calculations. The functions $H(\theta)$ and $H_1(\theta)$ describing roughness of internal surface of the bushing and shaft were considered as determined totality of unevenness of contours profile and also stationary random function with zero mean value and known variance.

As a rule, the greatest values of contact pressure are in the middle part of contact surface depending on the angle of contact and friction coefficient. Presence of friction forces in the contact zone leads to displacement of the graph contact pressure distribution to the contrary action of the moment.

Using the solution of the problem, calculate displacements on prefracture strip lips

$$-\frac{1+k_b}{2G}\int_{-l}^{x_1} g(x_1)\,dx_1 = v(x_1,0) - iu(x_1,0).$$

Assuming $x_1 = x_0$ applying change of variable, changing the integral by the sum, we find displacement vector on the prefracture strip lips for $x_1 = x_0$

$$V_0 = \sqrt{u^2+v^2} = \frac{1+k_b}{2G}\frac{\pi l}{M}\sqrt{A^2+B^2};$$

$$A = \sum_{m=1}^{M_1} v^0(t_m) + \varepsilon v^1(t_m); \quad B = \sum_{m=1}^{M} u^0(t_m) + \varepsilon u^1(t_m). \tag{28}$$

Here $M_1$ is the number of nodal points contained in the interval $(-l, x_0)$.

In the place of crack nucleation condition we accept the criterion of critical opening of prefracture strip lips. Considering relation (9), we can write the limit condition in the form

$$C(x_0, \sigma(x_0))\,\sigma(x_0) = \delta_{cr}, \tag{29}$$

where $\delta_{cr}$ is characteristics of resistance of bushing's material to crack initiation.

Soint solution of the combined algebraic system and conditions (29) makes possible to determine ultimate size of external load (contact pressure) the size of prefracture strip size for the limiting equilibrium state under which crack arises, under the given characteristics of crack resistance of the material

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