

# NECESSARY AND SUFFICIENT CONDITIONS FOR THE BOUNDEDNESS OF $B$ -RIESZ POTENTIAL IN MODIFIED $B$ -MORREY SPACES

## Abstract

*We consider the generalized shift operator, associated with the Bessel (Hankel) differential operator  $B = \frac{\partial^2}{\partial x^2} + \frac{\gamma}{x} \frac{\partial}{\partial x}$ ,  $\gamma > 0$ . The maximal operator  $M_\gamma$  ( $B$ -maximal operator) and the Riesz potential  $I_{\alpha, \gamma}$  ( $B$ -Riesz potential), associated with the generalized shift operator are investigated. At first, we prove that the  $B$ -maximal operator  $M_\gamma$  is bounded from the modified  $B$ -Morrey space  $\tilde{L}_{p, \lambda, \gamma}$  to  $\tilde{L}_{p, \lambda, \gamma}$  for all  $1 < p < \infty$  and  $0 \leq \lambda < 1 + \gamma$ .*

*We study the  $B$ -Riesz potential and their modified version in the modified  $B$ -Morrey space. We prove that  $B$ -Riesz potential  $I_{\alpha, \gamma}$ ,  $0 < \alpha < 1 + \gamma$  is bounded from the modified  $B$ -Morrey space  $\tilde{L}_{p, \lambda, \gamma}$  to  $\tilde{L}_{q, \lambda, \gamma}$  if and only if,  $\alpha / (1 + \gamma) \leq 1/p - 1/q \leq \alpha / (1 + \gamma - \lambda)$ ,  $1 < p < (1 + \gamma) / \alpha$ .*