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STABILITY OF A FILLED RIDGE CYLINDRICAL SHELL UNDER THE ACTION OF TIME DECREASING LOAD

Abstract

In the paper, within the bounds of the linear problem, dynamic critical load is investigated under the action of suddenly applied axial load to thin-shelled elastic systems. Formulae for determining critical load and critical value of critical stresses corresponding to the found critical time under the action of dynamically applied external load are found.

Under sudden application of loads calling contractive forces exceeding static critical values to thin-shelled elastic systems, there may appear motions characterized by monotone increase of deflections. Thereby, as it was first shown in the paper [1], the observed forms of stability loss not always may coincide with the forms that correspond to minimal static critical loads. Therefore, while solving the considered problems, there arises necessity to accept definite criterium of dynamic loss of stability. Usually, such criteria are formulated for imperfect system with initial deviations. The wide-used methods for determining dynamic critical load ignoring the external medium's influence were considered in the papers [2,3] in detail, where the solutions of a number of such type problems were reduced.

In the present paper within linear problem, as a dynamical stability loss criterium we accept analytic condition on possibility of intensive development of deflections under the action of suddenly applied axial load. Let's consider a filled closed cylindrical shell strengthned by a cross system of ribs, simply supported along ends under the action of axial contractive forces that in subcritical state reduce to homogeneous stress state characterized by contractive stress $\sigma_x = \frac{P}{2\pi Rh + kF_c}$, where P is axial force; h and R are thickness and radius of the shell, respectively; k is the amount of longitudinal ribs; F_c is area of cross section of longitudinal bar. It is assumed that wave character of force propagation may be ignored.

The formulae were found for determining critical time and the value of critical stresses that correspond to the found critical time under the action of dynamically applied external axial force.

Potential and kinetic energy for the considered cylindrical shell strengthened by a regular cross system of ribs are determined by the formulae [5]:

$$\Pi = \ni + A + K\ . \tag{1}$$

Here

$$\ni = \frac{Eh^3}{24\left(1-\nu^2\right)R^2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial \xi^2}+\frac{\partial^2 w}{\partial \theta^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \theta^2} - \left(\frac{\partial^2 w}{\partial \xi \partial \theta}\right)^2\right]\right\}\times$$

$$\times d\xi d\theta+\frac{h}{2ER^2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\left(\frac{\partial^2\varphi}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}\right)^2-2(1-\nu)\left[\frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2}-\left(\frac{\partial^2\varphi}{\partial\xi\partial\theta}\right)^2\right]\right\}\times$$

$$\times d\xi d\theta+\frac{1}{2R^3}\sum_{i=1}^{k}\left\{\int\limits_0^{\xi_1}\left[E_cI_{yc}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2+G_cI_{kp.c}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2\right]|_{\theta=\theta_i}\,d\xi\right\}+$$

$$+\frac{1}{2R^3}\sum_{i=1}^{k_1}\int\limits_0^{2\pi}\left[E_sI_{ys}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2+G_sI_{kp.s}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2\Big|_{\xi=\xi_j}\,d\theta+\right.$$

$$+\sigma_x h\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\frac{1}{E}\left(\frac{\partial^2\varphi}{\partial\theta^2}-\nu\frac{\partial^2\varphi}{\partial\xi^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial\xi}\right)^2\right\}d\xi d\theta+$$

$$+\sigma_x F_c R\sum_{i=1}^{k}\int\limits_0^{\xi_1}\left[\frac{\sigma_x}{E}-\frac{1}{2R^2}\left(\frac{\partial w}{\partial\xi}\right)^2|_{\theta=\theta_i}\right]d\xi;$$

$$K=\rho_0 hR^2\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left(\frac{\partial w}{\partial t}\right)^2 d\xi d\theta+\rho_c F_c R\sum_{i=1}^{k}\left(\frac{\partial w}{\partial t}\right)^2\Bigg|_{\theta=\theta_i}d\xi+$$

$$+\rho_s F_s R\sum_{i=1}^{k_1}\left(\frac{\partial w}{\partial t}\right)^2\Big|_{\xi=\xi_j}\,d\theta.$$

Here $\xi=\dfrac{x}{R}$, $\theta=\dfrac{y}{R}$; E_c, G_c are modulus of elasticity and shift of material of longitudinal ribs; k_1 is the amount of lateral ribs, respectively; σ_x are axial contractive stresses; u,ν,w are vector displacements components of the shell; ρ_0,ρ_c,ρ_s are densities of materials of the shell, longitudinal and lateral bars, respectively; $\theta_j=\dfrac{2\pi}{k_1}j$, E,ν are Young modulus and Poisson ratio of shell's material; $\xi_1=\dfrac{L_1}{r}$, L_1 is shell's length; $F_c,I_{yc},I_{kp.c},F_s,I_{xs},I_{kp.s}$ are areas and moments of inertia of cross section of longitudinal and lateral bar with respect to the axis ox and oz, and also inertia moment for torsion; t is time coordinate.

Influence of medium on the shell is determined as influence of external surface loads applied to the shell and is calculated as work performed by these loads when changing over the system from strain state to initial unstrained one and is represented in the form;

$$A=-R^2\int\limits_0^{\xi_1}\int\limits_0^{2\pi}q_z w d\xi d\theta. \tag{2}$$

Pasternak's model [6] is used for determining q_z. The essence of this model is that influence of medium on a shell on a contact surface is determined by the relation

$$q_z=\left(\widetilde{q}+\widetilde{q}_0\nabla^2\right)w=Kw, \tag{3}$$

where ∇^2 is Laplace's two dimensional operator.

Equation of continuity of strains is written in the form [5]:

$$\Delta\Delta\varphi = -ER\frac{\partial^2 w}{\partial \xi^2}\ . \tag{4}$$

Below, analytic criteria of dynamic stability loss are suggested for three typical laws of external load change and calculation formulae admitting to determine critical parameters of the applied transient load are obtained on these laws.

Deflection of the shell for stability loss is sought in the from

$$w = \sin d_m \xi \left[w_1(t)\cos n\theta + w_2(t)\sin n\theta \right], \tag{5}$$

where $w_1(t)$ and $w_2(t)$ are time functions to be determined. Substituting (5) into strain compatibility equation (4), and solving it with respect to φ, we find the expression for stress function

$$\varphi = -\frac{\sigma_x R^2\theta^2}{2} + E\frac{d_m^2 R}{(d_m^2+n^2)}(w_1\cos n\theta + w_2 \sin n\theta)\sin d_m\xi\ . \tag{6}$$

Substituting (5) and (6) into (3), (2) and (1), on the basis of second order Lagrange equation [6] we can get two independent ordinary differential equations with respect to shell deflection parameters $w_1(t)$ and $w_2(t)$, respectively. We can represent both of these equations in the form

$$\frac{d^2\overline{w}}{dt^2} + \omega_{mn}^2\left(1 - \frac{p}{p_{mn}}\right)\overline{w} = 0\ , \tag{7}$$

where under $\overline{w}$ one can understand $w_1(t)$ or $w_2(t)$ depending which term of expression (5) approximates deflection for stability loss; $\omega_{mn},\ p_{mn}$ are eigen frequency of vibrations of not loaded system, the parameters of critical values of statical longitudinal stresses that correspond to the considered deflection form and defined in the equation with respect to w_1, by the formulae:

$$\omega_{mn}^2 = \frac{E\Delta_{mn}}{(1-\nu^2)\,\rho_0 R^2}\frac{1}{1+2\overline{\rho}_c\overline{\gamma}_c\sigma_{1n}+2\overline{\rho}_s\overline{\gamma}_s\sigma_{2m}};\ p_{mn} = \frac{\Delta_{mn}}{d_m^2\,(1+2\overline{\gamma}_c\sigma_{1n})};$$

$$\Delta_{mn} = \frac{(1-\nu^2)\,d_m^4}{(d_m^2+n^2)^2} + a^2(d_m^2+n^2)^2 + \frac{\widetilde{q}-\widetilde{q}_0(d_m^2+n^2)(1-\nu^2)R^2}{Eh} +$$

$$+2\eta_c^{(1)}d_m^4\sigma_{1n} + 2\eta_{s1}^{(2)}(n^2-1)^2\sigma_{2m} + 2\mu_c^{(1)}\sigma_{2n}d_m^2 n^2;\ p = -\frac{\sigma_x(1-\nu^2)}{E};$$

$$\sigma_{1n} = \frac{1}{k}\sum_{i=1}^{k}\cos^2\frac{2\pi n}{k}i;\ \sigma_{2n} = \frac{1}{k}\sum_{i=1}^{k}\sin^2\frac{2\pi n}{k}i;\ \overline{\gamma}_c^{(1)} = \frac{F_c k}{2\pi R h};\ a^2 = \frac{h^2}{12R^2};$$

$$\overline{\rho}_c = \frac{\rho_c}{\rho_0};\ \eta_c^{(1)} = \frac{E_c(J_{yc}+h^2F_c)k}{2\pi R^2 h E};\ \mu_c^{(1)} = \frac{G_c}{E}(1-\nu^2)\overline{\mu}_c^{(1)};\ \overline{\mu}_c^{(1)} = \frac{J_{kp.c}k}{2\pi R^3 h}.$$

$$\sigma_{2m} = \frac{1}{k_1+1}\sum_{j=1}^{k_1}\sin^2\frac{2\pi n}{k_1+1}j;\ \eta_{s1}^{(2)} = \frac{E_s(I_{xs}+h_s^2F_s)(k_1+1)(1-\nu^2)}{EhL_1R^2}.$$

Equation (7) determines character of motion of a medium filled cylindrical shell strengthened by a cross system of ribs. Thereby a law of stress change in the system, i.e. law of external load change in time is of great importance.

In order to formulate criterium of dynamic stability loss of a medium-filled cylindrical shell strengthened by a cross system of ribs we can write equation (7) when only one of the considered loads functions, in the convenient form

$$\frac{d^2\overline{w}}{dt^2}+\omega_{mn}^2\left(1-\frac{\sigma}{\sigma_{mn}}\right)\overline{w}=0\ , \tag{8}$$

where $\sigma=\sigma(t)$ are stresses that correspond to axial dynamically applied external load; σ_{mn} is critical value of these stresses under statical loading that correspond to the considered bending form.

Assume that suddenly applied decreasing load acts on the shell. For linear law of suddenly applied load decrease, the change of contractive stresses in a shell is determined by the relation $\sigma=\sigma_0-\gamma t$, where σ_0 are stresses arising at load application time, γ is their decrease velocity.

A differential equation determining character of system's motion after substitution in (8) $\sigma=\sigma_0-\gamma t$, accepts the form

$$\frac{d^2\overline{w}}{dt^2}-(a_{mn}-b_{mn}t)\overline{w}=0,\ b_{mn}=\frac{\gamma\omega_{mn}^2}{\sigma_{mn}}\ . \tag{9}$$

We consider only those bending forms for which $\sigma_{mn}<\sigma_0$.

Assuming that beginning of motion is stipulated by deviation of the system whose amplitude equals C_0 for zero initial velocity, we can represent the solution of equation (9) in the form

$$w=C_0\left[1+\frac{a_{mn}t^2}{2}\left(1-\frac{b_{mn}t}{3a_{mn}}\right)+\frac{a_{mn}^2t^4}{4!}\left(1-\frac{4b_{mn}t}{5a_{mn}}\right)+...\right]. \tag{10}$$

The subsequent terms in square brackets are of bulky form and as they are not used in further statement we don't cite them here.

If suddenly applied load is constant $(\gamma=0)$, then $b_{mn}=0$ and instead of (10) we have the equation $\frac{d^2\overline{w}}{dt^2}-a_{mn}\overline{w}=0$,as a criterium of dynamic loss of stability determining the origin of intensive development of deflections we can accept $a_{mn}t^2=1$.

When $\gamma\neq 0$ by analogy as the indicated criteria we accept the condition according to which for time when contractive stresses decrease up to value σ_{mn}, the equation

$$\frac{a_{mn}t^2}{2}\left(1-\frac{b_{mn}t}{3a_{mn}}\right)=1 \tag{11}$$

should hold.

The time corresponding to decrease of contractive stresses up to value σ_{mn} are determined from the condition $\sigma=\sigma_0-\gamma t=\sigma_{mn}$:

$$t=\frac{\sigma_0-\sigma_{mn}}{\gamma}\ . \tag{12}$$

Substituting (12) into (11) with regard to accepted denotation for a_{mn} and b_{mn}, we find dependence between σ_0 and γ

$$\sigma_0 = \sigma_{mn} + \sqrt[3]{\frac{3\sigma_{mn}\gamma^2}{2\omega_{mn}^2}} \tag{13}$$

that determines composition of initial value of suddenly applied load and its drop velocity that in accordance with the accepted criterium of dynamic loss of stability is a boundary of safe loading of the system. In order to determine the least values of the cited quantities it is necessary to realize their minimization, by waving parameters. This enables to determine the least values of appropriate critical quantities that are accepted as computational. Substituting p_{mn} and ω_{mn} into (13), we can determine relation that determines a boundary of safe composition of initial values of time decreasing stresses and their drop velocities:

$$\sigma_{x0} = \frac{Ep_{mn}}{1-\nu^2} + \sqrt[3]{\frac{3\gamma^2\rho_0 R^2}{2d_m^2(1+2\overline{\gamma}_c^{(1)}\sigma_{1n})}\left(1+2\overline{\rho}_c\overline{\gamma}_c^{(1)}\sigma_{1n}+2\overline{\rho}_s\overline{\gamma}_s^{(2)}\sigma_{2m}\right)}\,. \tag{14}$$

It follows from 12) and (14) that the values of critical time and critical stress decrease according to increase of influence rigidity of a filler.

On the basis of analysis of expressions (14) we can deduce that under dynamical loading the size of hollows in longitudinal direction diminishes in comparison with their size on the case of static application of axial contractive force. Moreover, the great values of m correspond to minimal values of critical parameters and therefore, influence of discrete arrangement of annular ribs may significantly grow. It is interesting to notice that for the shells reinforced only in longitudinal direction $\left(\overline{\gamma}_s^{(2)} = 0\right)$, for $\overline{\rho}_c = 1$ the increase of critical stresses for decreasing (14) loads connected with velocity of their change is independent on the considered form of buckling on ribs parameters.

When analyzing general regularities of change of critical parameters of transient loads it is convenient to use relations obtained above and written in dimensionless form. For that we introduce the following denotation: $\psi = \dfrac{\sigma}{\sigma_e}$, $\gamma^* = \dfrac{\gamma}{\sigma_e\omega_{01}}$, where σ_e is minimal value of critical stresses of statical stability loss; ω_{01} is frequency of eigen vibrations of not loaded system by the form that corresponds to these stresses. Dependence of initial value of time increasing load on its drop velocity that characterizes their critical composition in dimensionless form has the form

$$\psi^* = \frac{\sigma_{mn}}{\sigma_e} + \sqrt[3]{\frac{3}{2}\frac{\sigma_{mn}}{\sigma_e}\frac{\omega_{01}^2}{\omega_{mn}^2}(\gamma^*)^2}\,. \tag{15}$$

For determining the least value of the quantity ψ^*, it is necessary to realize its minimization by waving parameters. On the basis of (15) and the results obtained for rapidly decreasing load, we establish dependence between its initial value and drop velocity

$$\psi_0 = \sqrt[3]{\frac{3(\gamma^*)^2}{2\overline{\mu}}} + \frac{\mu}{2} + \frac{1}{2\overline{\mu}}, \quad \overline{\mu} = d_m^2\sqrt{\frac{a^2}{1-\nu^2}}. \tag{16}$$

The curve $\psi_0(\gamma^*)$ constructed as a result of minimization with respect to $\overline{\mu}$ of expression (16) for definite values of γ^* is in figure 1. Using the curve described in fig. 1, we can determine if the given composition of initial value of axial force and its drop velocity is dangerous: if the point with coordinates equal calculated ψ_0 and γ^* lies under the line $\psi_0(\gamma^*)$, then according to the accepted criterium such a composition is not dangerous; otherwise, dynamic stability loss is possible.

Fig 1.

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